COMMISSIE VOOR HYDROLOGISCH ONDERZOEK T.N.O.
COMMITTEE FOR HYDROLOGICAL RESEARCH T.N.O.

VERSLAGEN EN MEDEDELINGEN No. 12
PROCEEDINGS AND INFORMATIONS No. 12

# WATER BALANCE STUDIES

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# I. THE DEVELOPMENT OF WATER BALANCE RESEARCH IN THE NETHERLANDS

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## 1. The hydrology of the top soil

Discussing the part the top soil is playing in the hydrological cycle, the soil layer under consideration has not to be confined to the ploughed or the sod layer, but will run generally from the soil surface to the groundwater table or, when the depth of the latter is more than say 2 meters, to about maximum rooting depth. This rooting depth is depending, besides on the groundwater level, on the kind of crop and on the profile.

The top layer of the soil defined in this way is from a hydrological point of view the intermediary between ground water and atmosphere. An intermediary however which is not only passing water upwards or downwards, but which is storing water during a certain period too. So the top soil is acting as a kind of hydrological buffer.

The water management of the top soil is a complicated one. Generally this layer consists of a three phases system (soil - water - air). Moisture transport through such a system happens by means of capillary conduction, diffusion or distillation. The quantitative way of handling the first two mechanisms was becoming much clearer during the last ten years. The meaning of the distillation however in a quantitative sense is still obscure.

On his upper end the top soil borders the atmosphere. In the day-time radiation energy from the sun and from the vault of heaven is absorbed here and converted into heat. This heat is used mostly by evaporation of water, the rest is warming up adjacent layers of the soil and the atmosphere. During the night the soil surface is cooling down due to the domination of the radiation of heat. By that, moisture from the atmosphere and from the top soil will condense in the soil surface, which is delivering heat. So a change of phase of water taking part in the hydrological cycle occurs in the interface between top soil and atmosphere. Besides, the greatest temperature gradients which are possible in soil and atmosphere appear on both sides of this interface. They are changing rather quickly and are influencing the moisture transport.

The buffering action of the top soil introduces the factor time. Rain water can be retained for a shorter or longer time and will disappear then into the

atmosphere or be added to the ground water. A small part will join in chemical, microbiological or physiological processes, which show their most manifold and intensive occurrence just in the upper layer of the top soil.

The factor that is complicating the hydrology of the top soil highly, is the vegetation. It is acting as a large and complex pumping system. The feature of this system is the extensive network of pump-barrels (roots), by which water can be withdrawn from all layers of the top soil. It is however subjected continuously to alterations, due to the origination, growth and death of roots. The growth of roots "towards the water" is very important if moisture cannot move towards the roots at great pF-values.

The evaporation from a soil surface overgrown with plants occurs by means of the leaves principally. These leaves possess in their stomata a regulating mechanism for the evaporation of water, which will be discussed in chapter II. Because evaporation from a soil surface bearing a vegetation, occurs as well from the leaves (transpiration) as directly from the soil surface (evaporation), it is named evapotranspiration usually.

The vegetation is influencing the hydrology of the top soil not only by the way uptake, transport out of the soil and transpiration of soil water take place, but also by its effect as an insulating layer for heat in the energy exchange between soil and atmosphere, which is affecting the moisture transport in the soil and the evaporation. Another effect of a plant cover is the interception of precipitation. A certain part of the rain for instance is not reaching the soil, but evaporates directly from the wet leaves and stems. In chapter IV this will be discussed. Especially in trees a part of the rain water catched by the tree top, is flowing down along branches and trunk, giving a heterogeneous distribution of the water entering the soil.

The remarks on the hydrology of the top soil mentioned above do not pretend giving a complete picture. They will make clear only that the problem is very complicated; reason why the top soil from a hydrological point of view has been "terra incognita" for a long time. Calculating groundwater movements in behalf of drainage or sub-irrigation problems for instance, the true hydrology of the top soil is left out of consideration generally and it is supposed that water transport through this soil layer happens vertically and quantitatively. This means that the hydrological buffering capacity is neglected. It is allowed to do so for long hydrological periods, e.g. years. The shorter the period, the more important factors as water storage in the top soil, habits of under- and overground parts of plants, weather, season a.s.o. are. After Worldwar II the interest in the Netherlands for water balances in short periods was growing quickly, due to the strongly increased need of fresh water in agriculture and horticulture, as well as in industry and housekeeping. Therefore a closer in-

vestigation of the water management of the top soil was becoming necessary.

The obvious way to handle this problem is to consider the top soil with its vegetation as a sort of reservoir, which has storage capacity and can take up and lose water as well on top as on its bottom, and to determine these intakes and losses in certain periods. This means that there must be made a study of the water balance.

## 2. The water balance of the top soil

The items of the water balance are:

a. as incomings the *precipitation* $R$ from above and the *infiltration* $I$ from below,
b. as outgoings the *evapotranspiration* $E$ upwards and the *run-off* $D$ downwards and
c. as gain the *increase* $S$ *of the moisture content* of the top soil.

Influences of dew, horizontal components of moisture movement and superficial run-off are neglected. The continuity equation is giving then the *water balance formula:*

$$(R + I) - (E + D) = S$$

The aim of the study of the water balance is to learn what these five items are as functions of time. It is sufficiently to measure four of them, after which the fifth can be derived from the balance formula. This method however meets two types of practical difficulties:

a. measurement of the different items with a sufficient accuracy is often not simply and
b. gathering results for the numberless possibilities of different soil profiles, vegetations, weather conditions and other circumstances takes a very long time.

Both will be discussed more in details, because they were determining the development of the research of the water balance of the top soil in the Netherlands to a great extent.

## 3. Measurements of the items of the water balance

*Precipitation*

At first sight the determination of the precipitation with the well-known rain gauge looks very simply. There are however some difficulties.

The so-called wind effect of rain gauges, which is greater when the top of the gauge is situated higher above the soil surface and which can be annoying, parti-

cularly in measuring snow-fall, causes the obtained value of $R$ to be too low. Further the effect is depending on the wind field in the neighbourhood of the rain gauge, which in turn is related to the local situation (vegetation, fences, hedges, grazing cattle, buildings a.s.o.), that alters often in the course of time.

Interception and run-off along trunks and stems make it difficult to determine what part of the precipitation enters the soil. Therefore in the study of the water balance of a top soil bearing a vegetation it is practical to consider soil and plant cover as a whole and to embody the direct evaporation of intercepted water in the evapotranspiration.

*Infiltration*

Generally it is difficult to determine infiltration from the ground water into the soil directly. In some cases it is possible to estimate infiltration from measurements of the vertical pressure gradient in the ground water. Usually however it is obtained indirectly from the balance formula. In fact ELINK STERK (1897/'98) has applied already this method estimating the infiltration of the Haarlemmermeerpolder by comparing the water balance of this polder with that of the drainage district Rijnland. With an artificial infiltration (e.g. sub-soil irrigation) the quantity of water supplied in that way can be measured usually.

*Run-off*

This item can be determined in the field in a rather simple way if only the groundwater level is so high that the drainage system is running or if there is a sub-soil run-off to the ditches with no infiltration. Therefore as early as the end of the 18th century one has had recourse to separating and walling in soil monoliths, that is to say in constructing lysimeters. These have to be so large that border effects are negligible. Then however it is nearly impossible to place a bottom under the monolith and one has to construct either large filled-in lysimeters like those in Castricum or lysimeters without bottom, but with a nearly constant groundwater level above a dense system of tile lines as in the Rottegatspolder (WIND, 1958). Smaller lysimeters can produce valuable data if containing a soil without a considerable swell and shrinkage.

*Evapotranspiration*

Direct measurement of the evapotranspiration is not possible. Usually it is calculated as closing entry from the water balance or estimated from meteorological data. For the last way different methods are available, from which that of PENMAN (1948) is well-known. It is based on measurements of the incoming radiation energy, the humidity and the temperature of the air and the wind

velocity. This method which comes up in chapters IV and V, determines in fact the evaporation from a free water surface. The evapotranspiration can be derived from this free water evaporation by multiplying it by an empirical factor, depending on the type of vegetation and on the time of year. In that way it is possible to obtain evapotranspiration results for a vegetation optimally provided with water (potential evapotranspiration), if the period under consideration is not too short (say two weeks or longer) and if the area in question is not too small (say a few times ten hectares). Conversion of the potential evapotranspiration into the actual one in case of sub-optimal water supply needs an empirical relation, in which density and growth of the root system are playing a part. Here again it appears how vegetation on the soil surface is complicating the evaporation problem, making it necessary to have recourse to the use of empirical conversion factors. A great advantage of PENMAN's method however is the possibility of utilizing available series of meteorological observations when the conversion factors are known.

The same drawback of moving the difficulties from the direct measurement of the evaporation to the determination of conversion relations and coefficients as arising with PENMAN's method, appears with the use of evaporimeters and evaporation pans (see chapter IV).

A determination of the evapotranspiration which is somewhat more directly, is the calculation of the vertical vapour transport, based on the theory of the humidity and wind velocity exchange in an eddying atmosphere and on measurements of vertical humidity and wind velocity profiles within a few meters above the soil surface or the vegetation. This method which is tested thoroughly by DEIJ (1960, *) in the Rottegatspolder, has the advantages of requiring an apparatus that can be moved fairly easy and yielding data for very short periods (e.g. hours). Disadvantages are the great deal of labour required by working out the records and especially the fact that the theoretical foundations hold only for stationary circumstances and an infinitely extensive area with a certain hydrodynamical roughness. A grassland area with short grass and without ditches, fences, live-stock and other obstacles meets these requirements to a certain degree, but they don't apply to waving wheat fields and other areas fluctuating in surface roughness.

It is also possible to derive the evapotranspiration from the energy balance of the soil surface. For that the net incoming radiation and the heat flux entering the soil are measured. The release of heat on the air is proportional to the heat required for evaporation. The ratio can be calculated from the temperatures and vapour pressures on two different altitudes. Then the energy balance is giving the evapotranspiration. Just as the method of the vertical vapour transport this energy balance method, with which there is only little

experience in the Netherlands till now, can give hourly values, but is also requiring much labour for working out the measurements. It is still unknown however if here the vegetation is also causing an irrelevancy of the theoretical foundations.

*Change of the moisture content of the soil profile*

At first sight the determination of this change seems to be very simple: take undisturbed samples from the different soil layers at the beginning and at the end of a balance period and determine their moisture percentages (in vol. %). The distribution in the soil of the moisture percentage however appears within short distances to be so heterogeneous, not only in a vertical but also in a horizontal direction, that a great number of samples is required to obtain the change of the moisture content correct to a few millimeters. The following example may illustrate this.

To determine change $S$ in moisture content of the top 80 cm of the profile on the drainage-lysimeter field in the Rottegatspolder (PEERLKAMP, 1955) with the degree of accuracy just mentioned, it appeared to be necessary that each time bore holes were made on 360 different places, from which 2160 samples were obtained. The great variability of a single determination of $S$ is caused partly by the fact that in sampling it is impossible to take a sample a second time from the same place. Using the modern neutron scattering method however, determinations at the beginning and the end of a balance period can be made on the very same places and a much smaller number of measuring spots suffices (on the drainage-lysimeter field just mentioned for instance 20 spots and 10 depths per spot). Although this method needs corrections for a certain inconstancy of the neutron source and eventually for the use of variant types of measuring tubes, it allows to work with short balance periods (e.g. two weeks). Just as with the methods of the vertical vapour transport and of the energy balance normal operation of the obtained data is requiring much work. It is necessary to consider the possibilities for operating an electronic computer.

Summarizing it can be said that generally simultaneous measurements of the different items of the water balance of the top soil, in so far as they can be done, are giving several difficulties and are expensive.

## 4. AIM AND SET-UP OF WATER BALANCE RESEARCH

Generally the study of the water balance results from the wish to obtain quantitative data about a certain item of that balance as a function of time, profile, vegetation, groundwater level a.s.o. Function of time means here in fact function of the development of weather and since this is showing a great variability, the demand for quantitative data leads to the problem of statistics of the

item in question. In the present hydrological problems a balance period of a year is no more sufficiently, but the evapotranspiration in say two weeks must be obtained. The most useful results would be frequency tables, from which could be read for instance the chance of a certain evapotranspiration in a certain period of two weeks, one thing and another differentiated according to type of soil, vegetation a.s.o.

The most obvious way to come to grips with the problem would be to take observations during some decennia, e.g. with lysimeters with different soil profiles and vegetations, in which the changes in moisture content are determined by means of neutron scattering. There are however different objections to this working-method: it takes too much time before the results are available, it will be rather expensive, and the question arises if and so yes how the lysimeter results can be extended to field circumstances.

Therefore water balance research has the task to discover ways and to develop methods for obtaining the data in question in a quick and relatively inexpensive manner. For that it must try to find relations between evapotranspiration and quantities of which long series of observations are already available or to be determined in a very simple way. Doing so it will be impossible generally to escape the use of empirical parameters. The results of the research with lysimeters may render good services in deriving and testing the necessary relations and in determining the required parameters.

## 5. Water balance research in the Netherlands

In the Netherlands about a quarter of the yearly precipitation disappears into the ground water and must be drained in an artificial or natural manner. The dune water companies near the west coast are managing large winning areas and for them it was a question what vegetation would give the biggest quantity of useful precipitation. This has made, that before Worldwar II as well in agriculture as in water management and in the supply of drinking-water the run-off through the top soil was the centre of interest. Some lysimeter installations were built to determine this run-off and drainage measurements of polders were carried out to study infiltration. The discharges of the catchment areas of some rivulets were measured too.

These run-off data were giving values of evapotranspiration as a "by-product", obtained from the balance formula with the use of precipitation data. Generally however it was only possible to get evapotranspiration values in this way for a period of a year, since the moisture content of the top soil in about February may be considered as constant by first approximation.

After Worldwar II the demand for fresh water was increasing strongly, especially in summer. It appeared that agriculture and horticulture could

increase their yields considerably by means of sprinkling or sub-soil irrigation, more drinking-water and industry-water was needed and the fight against high salt percentages in canals and ditches, especially in the western part of the country, was requiring an increasing quantity of fresh water. Storage and distribution appeared to be necessary and since in summer the greater part of the stored water disappears by evapotranspiration, the latter was becoming the centre of interest. It was, especially in view of the distribution, not only important to know the total quantity which was required, but also how it was divided among the different periods, e.g. weeks. This brought a need of evapotranspiration values of short periods and consequently of non-destructive methods for measuring the change of the moisture content of the soil profile. The lysimeters were changing from run-off meters into evaporation meters and the Commission for Hydrological Research T.N.O. founded (in 1949) the Working Group for Lysimeters, with the original task to develop the lysimeter technique. It is curious that the research done by the working group was leading just to the opposite, viz. the development of methods for the determination of evapotranspiration, which were making the lysimeters superfluous. The reasons (limited transferability, expensive apparatus, long series of measurements necessarily) are mentioned already.

The working group however does not agree with the statement of ELINK STERK (1897/'98) that "lysimeters should disappear from memory of men as quickly as possible". The group thinks the importance of the lysimeter can be based especially on the possibilities of producing and testing empirical parameters and relations between hydrologically important quantities, and not so much on the fact that evapotranspiration can be determined by means of it.

The investigations of the Working Commission for Evaporation Research, started in the Rottegatspolder after Worldwar II, also have had the aims to make the lysimeter superfluous. They were testing the method of the vertical vapour transport. Unfortunately practical application of the theoretical principles underlying this method appeared to be strongly limited as is mentioned already. Nevertheless, the work done by the Working Commission for Evaparation Research has given not only a storehouse of observation data, published in the reports (*) of the commission, but also valuable contributions to the investigations of the Working Group for Lysimeters T.N.O., with which the commission is co-operating closely.

Summarizing it can be stated:

- that the development of the investigation of the water balance of the top soil in the Netherlands was moving last twenty years towards the evapotranspiration research,

- that vegetation is complicating the problem of the evaporation highly,
- that direct measurement of the evaporation from an overgrown soil surface is not (yet) possible,
- that due to the multiplicity of weather, soil and vegetation circumstances indirect measurements with lysimeters are requiring several decennia of observation and are expensive,
- that at present evaporation research in the Netherlands is moving towards the search for relations with known meteorological quantities and/or factors that can be determined rather easily, from which evapotranspiration can be derived, if possible with the aid of an electronic computer.

In the following chapters results from this attempt will be treated.

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# II. PROGRESS IN THE KNOWLEDGE ABOUT THE EFFECT OF SOIL MOISTURE CONTENT ON PLANT PRODUCTION

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## 1. Purpose of the investigation

The study of the influence of soil moisture conditions on plant growth is gradually developing from a purely statistical sorting out of yield data with respect to the type of soil and the depth of the groundwater level, into an exact identification of growth relations and the expression of these relations in terms of physical or physiological laws.

Many scientists, all over the world, are investigating processes which determine in part the relations which link the soil moisture content and the dry matter synthesis together (Bierhuizen and Slatyer, 1964b; Kuiper, 1961; Gaastra, 1959). More and more the obvious question to what extent the chain of relations can be considered as known is intriguing practical people and researchers alike. It is understandable, that a comprehensive type of investigation has been developed to satisfy this, by integrating the result of studies, carried out by others. These investigations aim at linking together the relations governing the interactions between soil, plant and atmosphere, and deal with more and more steps the further this integration progresses. Radiation is linked in a certain number of steps to transpiration. The number of steps increases for the relation between soil moisture content and transpiration. A still larger gap has to be bridged between soil moisture content and plant production and the number of steps there increases still more. The question can be raised, to what extent each link in the chain of relations is known, and if the accuracy of description is sufficiently accurate to be used to predict their influence in the sequence of relations. Is it already possible to calculate the crop yield from soil moisture contents and a number of soil and crop parameters, which can be determined separately and independently from field experiments or other investigations of more restricted experimental duration? What might be the accuracy and what are the weekest links?

Much work is done by a large number of soil scientists, hydrologists, plant physiologists and climatologists to mention only Makkink and van Heemst, 1966 and Rijtema, 1965 here. It seemed worthwhile to bring all these contributions together to assess the parameters for a typical case of moisture – yield relation and check the progress that has up to now been made. Such a resume will be

important as a proof of the exactness of the various formulae involved and of the practical applicability of the theory behind these relations to water management problems.

As long as the major crops as grass, rye or potatoes were the main subject of study, the statistical method, though somewhat unwieldy, was not too cumbersome with respect to the size and significance of the problem. But if one considers the problem of establishing the reaction of the 50 to a 100 different horticultural crops on the moisture content of dozens of soil profiles, it will be clear that this requires an amount of work, not likely to be done for every crop and soil type with the care and precision that is needed. A more exact type of investigation, dealing separately with small groups of parameters, will be far more promising. This requires, however, that every part of the sequence of processes not only can be expressed verbally or graphically, but also mathematically, so that formulae are available in which to insert the parameters. Ten years ago this approach was tried out by WESSELING (1957). In the meantime new insight has been gained and in the following pages an example of calculation will be given to illustrate that nearly all the necessary links are available and sufficiently tried out to provide a basis for the construction of a comprehensive expression of the relation between soil moisture and dry matter production.

The theory to be described consists of four parts. First the description of the extraction of soil moisture by the plant according to VISSER (1964a) will be used. This conception is part of the theory of the behaviour of unsaturated soil moisture. The next part is the flow of the moisture and carbon-dioxide through the plant, where the results of the experiments of RASCHKE (1956, 1960) and BIERHUIZEN and SLATYER (1964a, 1964b) are used. The third part deals with the influence of radiation on transpiration, convection, temperature changes of plant leaves and the energy balance. Here the theory of RASCHKE (1956, 1960) has been used. Newer developments recently became available (RIJTEMA, 1965). The last part deals with the relation between the carbon-dioxide uptake and the plant response (BIERHUIZEN and SLATYER, 1964b). A recently developed growth function of VISSER (1964c) enables to tie in the growth factor $CO_2$ with the complex of the numerous other growth factors which govern plant growth and to quantify the significance of the soil moisture relations in this complex process.

## 2. THE SEQUENCE OF PROCESSES

The sequence of the processes to be discussed deals in general with the flow of moisture through soil, plant and stomata into the atmosphere, changing from

the fluid to the vapour phase at the plant-air interface just inside the stomata. In the same manner as the water vapour leaves the plant through the stomatal opening, carbon-dioxide can penetrate into the plant. The magnitude of the flow resistance for the water vapour in case of partly closed stomata also holds for the penetration of carbon-dioxide. And in this part of the pathway of the vapour flow, the transpiration and $CO_2$-transfer are linked together.

The penetrating $CO_2$ follows a flow path, partly identical, partly differing from that of the water vapour where it flows from the stomata through the mesophyll to the chloroplasts. There $CO_2$ is taken up in the assimilation process and disappears out of the system of gradients and resistances by fixation in the dry matter which is synthesized. In this respect there is again a similarity between the behaviour of water and carbon-dioxide, as water disappears also out of the system, in this case by transpiration.

The dry matter production is a problem of co-operating growth factors. These factors may support each other or may work antagonistically, they may limit the activity of other factors or may reinforce them. The significance of the availability of $CO_2$ depends on the availability of all the other factors. This co-operation of growth factors and their influence on the effects of soil moisture deserves the collective attention of those, engaged in pure and applied science alike.

With respect to moisture flow, two points in the pathway draw attention. The extraction of soil moisture by the roots tends to cause a dry soil cylinder around the root. This drier soil provides the moisture flow with a considerable resistance.

The transpiration of water at the plant-air interface is the second point which attracts much attention. Here the plant makes use of a remarkably useful relation between moisture stress and the vapour pressure which is in equilibrium with the stress in the fluid phase. Very large variations in the moisture stress correspond with very small variations in the vapour pressure. This change of large scale variations in the fluid phase into small scale variations in the adjacent vapour phase, demonstrated by the example of calculation in paragraph 9, shows how a stable vapour pressure gradient is coupled with large variations in transpiration. It is felt that this relation did as yet not attract the recognition which it deserves.

The carbon-dioxide transfer through the mesophyll has been subjected to investigations more recently than the transpiration (GAASTRA, 1959; BIERHUIZEN and SLATYER, 1964b). This part of the sequence of relations is still in a stage of investigation, the development of which is promising quick success in view of the large number of people working on this problem. The parameters used

in this type of work are important because they determine the difference in reaction of different crops.

## 3. The problem to be solved

The solution of the various unknowns will for some problems be easy and straightforward, for others, however, difficult and unwieldy, depending on the way in which the unknowns are present in the chain of formulae.

Further attention has to be given to the possibility of direct determination of the parameters, which is for some not readily attainable. In those cases the study will have to be directed to the determination of a parameter from known variables, instead of the determination of the variable from known parameters.

The assessment of the magnitude of the transpiration from given data on the soil moisture content for instance, will be far less accurate and easy than the calculation of the crop yield from soil moisture contents and transpiration data. The following discussion will deal with the latter problem. Data on soil moisture content and transpiration are given and the dry matter production is the unknown to be solved.

The moisture content $c$ of a certain light clay soil, expressed in per cent of the total soil volume or 100 $cm^3 . cm^{-3}$ and the real evapotranspiration $T$, given in $cal.cm^{-2} . min^{-1}$ of a certain area of grassland, changing with the soil moisture content, are given in table 1. The expression in calories is used in conformity with the calculations of Raschke (1956, 1960).

Table 1. Moisture content and real evapotranspiration

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$ | 42 | 32 | 28 | 25 | 22 | 20 | 16 | 13.5 | 11.5 |
| $T$ | 0.590 | 0.570 | 0.515 | 0.390 | 0.260 | 0.180 | 0.074 | 0.027 | 0.000 |
| $E_r$ | 5.0 | 4.85 | 4.4 | 3.3 | 2.2 | 1.53 | 0.63 | 0.23 | 0.00 |

To make the results more understandable the real evapotranspiration $E_r$, expressed in $mm . day^{-1}$, has been added, but will not be used. The uncertainty in $E_r$ is, that the evaporation over a day is heavily influenced by the more intense midday radiation. The value of $E_r$ should be obtained by integrating the radiation over a day of an average type, or an optimal type of radiation might be object of study, or what course of radiation intensity over the day may be desirable. The calculation of the effect of the radiation over a day is, however, somewhat laborious due to this necessity of integration. Here for the sake of simplicity the total daily radiation is assumed to be the equivalent of 500 minutes of optimal radiation. In that case the transposition of $T$ into $E_r$ is carried out by applying:

$$1 \text{ cal. cm}^{-2} . \text{min}^{-1} = 0.0017 \text{ gr } H_2O \text{ cm}^{-2} . \text{min}^{-1} * = 0.85 \text{ cm}^3 . \text{cm}^{-2} . \text{day}^{-1} = 8.5 \text{ mm} . \text{day}^{-1}$$

The quantity of evaporated water will be considered to be entirely passing through the plant.

The assumption that data, describing the relation between moisture content and transpiration may be considered to be available is founded upon a further part of the chain of processes not to be discussed here. As BLOEMEN (1966) shows, it is possible to find the moisture content as well as the evapotranspiration, if data on groundwater depth, rainfall and potential evaporation are available. In the Netherlands these data are obtainable for several thousands of observation wells. It is felt, that an important application of the catenary productivity formula might be the deducing of the yielding capacity of the various soil profiles with respect to their water status from these elaborations of the observations of groundwater tubes. One of the aims of this paper is to contribute to the realization of this purpose. It is visualized that from such easily obtainable data as rainfall and groundwater depth, a soil evaluation with respect to the moisture conditions, a productivity classification for different land use types or cropping patterns and a determination of curable shortcomings with respect to drainage or water application will at some time be derivable. It may be stressed here, that the knowledge of the beneficial influence of water for plant growth is far ahead of the knowledge of the injurious influence of water when present in excess. Efforts concerning the study of the effect of waterlogging and dense soil structure will be needed to obtain a full expression of the significance of soil moisture for crop growth. The influence of excess of water was discussed elsewhere (VISSER, 1964b) and will here be left out of the discussion.

## 4. SOIL MOISTURE CONTENT AND MOISTURE STRESS

The moisture conditions in the soil can be described in terms of the water balance and expressed as moisture contents. They may also be considered being part of the phenomenon of moisture flow and expressed in terms of moisture stresses. Though for practical use the moisture content is far easier understood and may seem more suitable, the soil moisture stress governs the flow and is therefore indispensable.

From the elaboration of BLOEMEN — or more reliably, by direct determination — data on soil moisture content are easier to obtain than data on soil moisture stress. Determination in the laboratory of the desorption- or pF-

* The enthalpie of water is taken to be 590 cal. cm$^{-3}$.

curve, however, provides the relation with which the moisture contents may be transposed into moisture stresses.

The determinations of the desorption curve in the way in which they are usually executed as routine analyses do not, however, easily show the desired accuracy. As this may cause difficulties in subsequent calculations, it is advisable to adjust the observations by fitting a sufficient number of pairs of observations for the moisture content $c$ and the moisture stress $\Psi$ to a suitable equation. A relatively simple and quick graphic method is available to determine the constants of the formula:

$$\Psi = A \frac{(P-c)^n}{c^m} \qquad \Psi = 1\,000\,000 \frac{50-c}{c^3} \qquad (1)$$

where $P$ = total pore space
$A$ = constant of scale
$m, n$ = exponents
$\Psi$ = mean soil moisture stress

The first formula shows its general shape, the second the constants to be used in the subsequent example of calculation.

The mathematical adjustment is somewhat laborious. The values chosen are a good example for a light clay soil. A simple general relation between the value of the parameters and the lithological characteristics of the soil, however, does not exist, safe for samples taken from a soil catena with a narrowly limited genetic origin.

The value of $A$ may vary over a very wide range of the order of $10^7$ to $10^{-4}$. The value of $m$ generally is somewhat below 3, but for heavy clays values near zero have been found. The exponent $n$ varies between zero and 10. The pore space has the well-known range between 0.40 and 0.60. In figure 1 the mutual relations between the parameters are depicted. A free estimation of the constants is not advisable, the resulting curve is seldomly realistic. Solution of the parameters from analysed data is indicated as the best method for procuring the constants.

As will appear from the example of calculation, there is no special need to determine certain fixed pF-values as 4.2 or 2.7. The moisture stress has to be calculated for any moisture content from an equation of the type of formula 1 and in the adjustment of the formula to the observations any pair of observations can be used. It is, however, advisable to determine pF-values of the order of 5 or 6 as well as the lower values, to facilitate the adjustment.

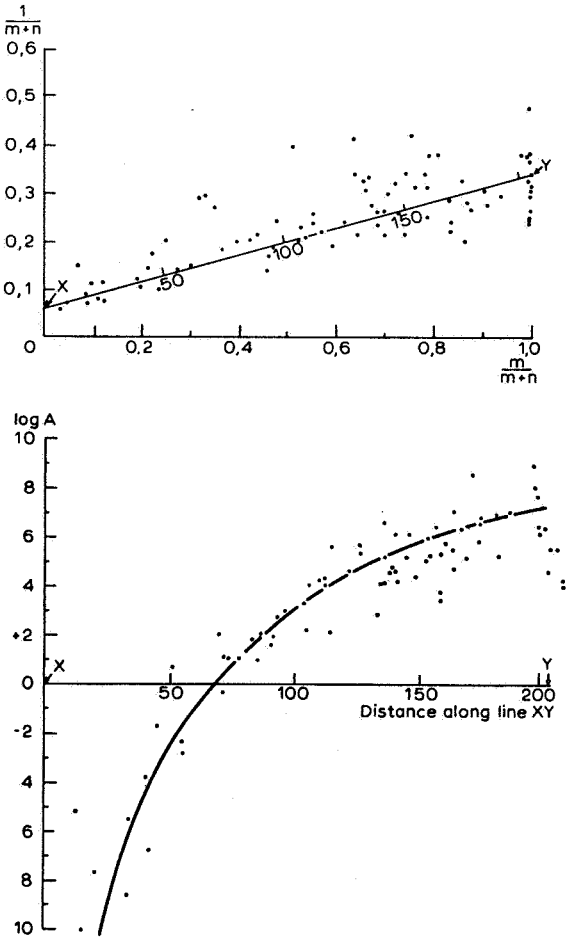


*Fig. 1.* A number of values of the exponents $m$ and $n$ in formula 1 are depicted in the combination $n/m+n = \mu$ and $1/m+n = \upsilon$, from which values follow: $m = \mu\,\upsilon$ and $n = (1-\mu)/\upsilon$. The value of $A$ is given as logarithm as projection in a vertical plane through the line $x\,y$ of the upper graph.

TABLE 2. Example of calculation with formula 1

| $c$ | $c^3$ | $P-c$ | $\Psi = 10^6 (P-c)/c^3$ |
|---|---|---|---|
| 42 | 74088 | 8 | 108 |
| 32 | 32768 | 18 | 549 |
| 28 | 21952 | 22 | 1002 |
| 25 | 15625 | 25 | 1600 |
| 22 | 10648 | 28 | 2630 |
| 20 | 8000 | 30 | 3750 |
| 16 | 4096 | 34 | 8301 |
| 13.5 | 2460 | 36.5 | 14837 |
| 11.5 | 1521 | 38.5 | 25312 |

## 5. THE FLOW OF SOIL MOISTURE TO THE ROOTS

The extraction of moisture from the soil by the plant is a problem of flow through the unsaturated zone. Elsewhere a formula for this flow was evolved (VISSER, 1964a). From this elaboration it was concluded that the gradient causing the moisture to flow must increase the soil moisture stress in the neighbourhood of the soil-root interface rather sharply. Due to this increase of the soil moisture stress with decreasing distance to the root, the soil moisture content must diminish progressively. The moisture content of the soil layer in contact with the root will be lowest. The conductivity of this relatively dry cylinder of soil around the root will considerably increase the resistance for liquid flow.

The elaboration is based on a hypothesis, not to be discussed here, that the unsaturated conductivity is proportional to $\Psi^{-(2-D)}$, where the factor 2 in the exponent is derived from the formula of Poisseuille for capillary flow and the factor $D$ accounts for the pore size distribution as described in formula 1. From recent investigations of WESSELING (1961) it appears that this expression for the unsaturated conductivity does not hold for the wetter range of soil conditions. In the range below pF 3, however, the available data on soil moisture content versus transpiration indicate that there the formula for the capillary conductivity is acceptable.

The formula to be used (VISSER, 1964a) is as follows:

$$T = B\left\{\frac{1}{\Psi^d} - \frac{1}{\Psi_r^{\,d}}\right\} \qquad T = 180\left\{\frac{1}{\Psi^{0.8}} - \frac{1}{\Psi_r^{\,0.8}}\right\} \qquad (2)$$

where $\Psi_r$ = moisture tension at soil-root interface

$B, d$ = constants

$d = (2-D) - 1$

The not explained symbols in the following pages have the same meaning as in the previous tables and formulae.

The values of $B$ and $d$ are not easily measured directly, but the technique discussed by BLOEMEN (1966) to adjust the calculated values of $E_r$ to the moisture contents, provides parameters from which, with the aid of the desorption curve given by formula 1, the parameters $B$ and $d$ may be derived.

The next step is now to calculate the moisture stress $\Psi_r$ at the plant-root interface from the given values of $T$ and the values of $\Psi$ as calculated in the previous paragraph.

TABLE 3. Example of calculation with formula 2

| $\Psi$ | $\log \Psi$ | $10\text{-}\log \Psi^{0.8}$ | $1/\Psi^{0.8}=\alpha$ | $T$ | $T/B=\beta$ | $1/\Psi_r^{0.8}=\alpha\text{-}\beta$ | $\log \Psi_r^{0.8}$ | $\log \Psi_r$ | $\Psi_r$ |
|---|---|---|---|---|---|---|---|---|---|
| 108 | 2.033 | 8.373 | 0.02362 | 0.590 | 0.00328 | 0.02034 | 1.692 | 2.114 | 130 |
| 549 | 2.740 | 7.808 | 0.00643 | 0.570 | 0.00317 | 0.00326 | 2.487 | 3.108 | 1283 |
| 1002 | 3.001 | 7.599 | 0.00397 | 0.515 | 0.00286 | 0.00111 | 2.955 | 3.693 | 4936 |
| 1600 | 3.204 | 7.437 | 0.00273 | 0.390 | 0.00217 | 0.00056 | 3.252 | 4.065 | 11608 |
| 2630 | 3.420 | 7.264 | 0.00184 | 0.260 | 0.00144 | 0.00040 | 3.398 | 4.247 | 17677 |
| 3750 | 3.574 | 7.141 | 0.00138 | 0.180 | 0.00100 | 0.00038 | 3.417 | 4.271 | 18664 |
| 8301 | 3.919 | 6.865 | 0.00073 | 0.074 | 0.00041 | 0.00032 | 3.495 | 4.369 | 23362 |
| 14837 | 4.171 | 6.663 | 0.00046 | 0.027 | 0.00015 | 0.00031 | 3.509 | 4.386 | 24311 |

## 6. THE FLOW OF MOISTURE THROUGH THE PLANT

The soil moisture, after having been taken up by the plant, flows through the vessels and the plant tissue from the soil-root interface to the leaf-air interface. The suction head $\Psi_r$ at the surface of the root was calculated in the preceding paragraph. When the conductivity of the plant vessels and tissue is known, it is possible to calculate the difference in tension and from this the tension $\Psi_i$ at the leaf-air interface as discussed elsewhere (VISSER, 1964a).

The formula and the constants used in the example are as follows:

$$T = C\,(\Psi_i - \Psi_r) \qquad\qquad T = 0.0025\,(\Psi_i - \Psi_r) \tag{3}$$

The value of $C$ can again be obtained from the elaboration of observations of the groundwater depth as described by BLOEMEN (1966). The accuracy with which the constant $C$ can be solved, however, will never be high. The parameter is situated at a very insensitive spot in the sequence of formulae for the different parts of the flow path. This means, however, that the influence of this parameter on the result of the calculations can only be small, for otherwise the

possibility of accurate determination would be better. The example of calculation shows that the difference in tension between $\Psi_r$ and $\Psi_i$ is small.

The data, contributing most to the accuracy of $C$, are those pertaining to the transition between the transpiration values mainly governed by the soil moisture content, and the transpiration mainly governed by the radiation. This transition point, however, shifts with changing radiation intensity and it is not always easy to predict what distribution of the observations over the range of $c$ and $T$ will give the best result. In the example the range of the transition points for different levels of radiation intensity appear at moisture contents between 28 and 32 %.

The correct assessment of $C$ has an undeniable importance, because it is the parameter accounting for the influence of the extensiveness and the activity of the root system.

In the following example, the values of $\Psi_r$ from table 3 are used to calculate the values of $\Psi_i$.

TABLE 4. Example of calculation with formula 3

| $\Psi_r$ | $T/0.0025$ | $\Psi_i$ |
|---|---|---|
| 130 | 236 | 366 |
| 1283 | 228 | 1511 |
| 4936 | 206 | 5142 |
| 11608 | 156 | 11764 |
| 17677 | 104 | 17781 |
| 18664 | 72 | 18736 |
| 23362 | 30 | 23392 |
| 24311 | 11 | 24322 |

## 7. FIRST APPROXIMATION OF THE STOMATAL RESISTANCE

The stomata are the sluice gates through which the moisture taken up by the plant is discharged and through which at the same time $CO_2$ is entering the plant. These sluice gates open if the moisture stress $\Psi_i$ is low and close gradually if the moisture stress increases. It is often assumed that they are closed entirely at a tension $\Psi_i = 16000$ ($\log \Psi_i = \text{pF} = 4.2$). The number of indications that a total closure is only to be expected at far higher tensions is increasing, in which case wilting would not so much be an indication of the effect of increased moisture tension as well as of a disarrangement of plant life as a whole for the part above the ground. This disarrangement will depend on the deficiency of water, but need not be correlated with the behaviour of the stomata.

The uptake of $CO_2$, however, will be correlated with the flow resistance in

the stomata and knowing the moisture stress $\Psi_i$, a first indication as to the stomatal resistance might be obtained if a mathematical description of the action of the stomatal mechanism was available. The action of the stomata is, however, hard to define quantitatively. The diffusion through a slit of varying aperture has been studied, but no indication has been found of a quantitative description of the elastic change in shape of the closing cells under the influence of the moisture stress in the tissue of the leaf.

The first purpose for which quantitative knowledge of the stomatal resistance is required, is to calculate the vapour pressure at the leaf-air interface, as described in paragraph 9. For this, however, the leaf temperature is needed. This temperature in its turn depends on the stomatal resistance, as described in paragraph 8. Now the correction of the vapour pressure due to the leaf temperature often is of limited importance and with an empirical formula as given in formula 4 the requirements can be met for a first approximation.

The empirical formula is selected for its simplicity in a calculation. Should the formula be incorrect, then this would not harm the further elaboration very much, because in paragraphs 10 and 11 the stomatal resistance is again calculated and the approximated values of this paragraph can be checked. There are indications that a more complicated description of the stomatal mechanism at some future moment may prove necessary. Some experiments, undertaken in this direction, show that improvement can only be achieved at the cost of much more laborious calculations.

As an empirical formula describing the stomatal resistance $b$ together with the constants to be inserted, the following equation was used:

$$b = \frac{\Psi_{wp}^{p} - \Psi_{i}^{p}}{D} \qquad b = \frac{25312 - \Psi_i}{1000000} \tag{4}$$

where $\Psi_{wp}$ = tension at wilting point
$p, D$ = empirical constants

The formula is constructed on the assumption that an increment in the flow resistance $db$ is dependent on an exponentional relation for a small increase in the moisture tension at the plant-air interface.

Expression 4 has a fair flexability in the magnitude of $p$ and can account for relations of different shape between $b$ and $\Psi_i$. The S-shape which was found indicated in other elaborations, however, cannot be represented by this formula. Because for $p$ there are insufficient indications as yet, $p$ is assumed to be unity. In paragraph 11 this assumption will be checked. The value of $D$ is

obtained by taking $\Psi_i$ zero and chosing $b$ in such a way that an acceptable flow resistance is obtained. According to RASCHKE (1956, 1960) this resistance $r_a + r_b$, equal to $0.035/b$ should have a value of 1 to 3 sec. $cm^{-1}$. In paragraph 12, formula 14, this is described in more detail. For $D$ a value of 1 000 000 seems a fair provisional estimate. In paragraph 11, it will be shown, however, that the value of $D$, assumed here, proves to be too large. This means that the flow resistance in and around the leaf is rather small. The values of $b$, found as an approximation, are given in table 5.

TABLE 5. Example of calculation with formula 4

| $\Psi_i$ | $25312-\Psi_i$ | $b$ |
|---|---|---|
| 366 | 24946 | 0.0250 |
| 1511 | 23801 | 0.0238 |
| 5142 | 20170 | 0.0202 |
| 11764 | 13548 | 0.0135 |
| 17781 | 7531 | 0.0075 |
| 18736 | 6576 | 0.0066 |
| 23392 | 1920 | 0.0019 |
| 24322 | 990 | 0.0010 |

## 8. RADIATION AND RISE OF LEAF TEMPERATURE

When the solar radiation hits the plant this tends to increase the temperature of the leaf. That this does not cause an inadmissable rise in temperature is due to losses in energy in three different ways. The loss of energy by the leaf is brought about by convection, by the vaporization of water and by back radiation. In a humid atmosphere the gradient of vapour pressure may be too small to allow transpiration of a sufficient magnitude. The increase in temperature of the leaf, however, will bring transpiration nearer to the required level. In that case a limited rise of leaf temperature increases the vapour pressure at the leaf-air interface and this enables the transpiration to continue as a consequence of the increase in the vapour pressure gradient.

In the following discussion the concept of RASCHKE is followed. However, the addition is made not only to relate the transpiration to the leaf temperature rise, but also to the decrease resulting from increasing moisture tension at the leaf-air interface. For details the reader is referred to the work of RASCHKE (1956, 1960). This theory holds for one single leaf. Here it is assumed that an equivalent leaf may be substituted for the canopy of the vegetation, having the same properties as the canopy with respect to transpiration. The parameters of this equivalent leaf are inserted in the formula.

The approach is – in conformity with the way in which other scientists have treated this problem – that for the three relevant terms of the energy balance – convection, latent heat and back radiation – the sum must be equal to the incoming radiation. In these formulae the temperature rise $\theta$ is separated from the temperature $t$. The value of $\theta$ can be solved and expressed in terms of the incoming radiation $S_a$ and the difference in vapour pressure $P_{it}$ at the leaf-air interface and the vapour pressure $P_a$ in the air. The index $i$ refers to the moisture tension $\Psi_i$. For saturated situations this is indicated by a first index $o$. The second index for $P$ and the only index for $S$ refer to the temperature. An $a$ stands for the temperature of the air, an $L$ for the temperature of the leaf. The convection term depends on the windspeed. This proves to be one of the most difficult parameters in the whole elaboration. The convection constant $h$ is here based on the statement of RASCHKE that the conductivity of the boundary layer can be described by:

$$h_{sat} = 0.1\ u^{0.5}$$

where $h_{sat}$ = conductivity of boundary layer in cm. min$^{-1}$ at saturated soil moisture content

$u$ = windspeed in m. sec$^{-1}$

For the windspeed 50 cm. sec$^{-1}$ is chosen, from which the value of $h$ of 0.071 is calculated.

The windspeed has more influence on transpiration and dry matter production than is often realized. The theory on turbulent flow, as worked out for hydraulics, as that of COLEBROOK-WHITE (1937), will clarify a part of the problems. KUIPER (1961) gives interesting observations in this respect. A few furthers constants have to be chosen. For air temperature 25° C is taken, for $S_a$ 0.7 cal. cm$^{-2}$ . min$^{-1}$ and for $P_a$ 16.8 mm Hg.

RASCHKE obtains the following equations:

$$\Theta = \frac{S_a - b\ (P_{ia} - P_a)}{2\ (h+h_s) + b\ dP/dt} \qquad \Theta = \frac{0.7 - b(24.0 + 10^{\frac{-\Psi_i}{3.18.10^6}} - 16.80)}{0.158 + 1.45b} \qquad (6)$$

where $h$ = conductivity of boundary layer

$h_s$ = radiation transfer coefficient

$dP/dt$ = vapour pressure-temperature derivative

He allows for the temperature rise by inserting $P_{iL} = P_{ia} + \Theta \frac{dP}{dt}$. The influence of the tension $\Psi_i$ is accounted for by applying formula 7. In the next paragraph this formula will be dealt with in more detail. The vapour pressure $P_a$ describing the conditions in the air – where the moisture stress has no meaning and the vapour flow is determined by the vapour pressure gradient – can be measured directly or can be calculated with formula 8 as is discussed by RASCHKE (1956, 1960). The formulae are:

$$P_{ia} = P_{oa}\, 10^{\frac{-\Psi_i}{3.18.10^6}} \tag{7}$$

$$\log P_a = \frac{7.5 t_a}{t_a + 237.3} + 0.6609 \tag{8}$$

$t$ = expressed in ° C

The relative vapour pressure $P_a/P_{oa} = 0.70$ is taken as the parameter for the humidity of the atmosphere. The values for $h_s$ and $dP/dt$ depend on the temperature. RASCHKE gives the following relation.

TABLE 6. Relation of temperature with radiation transfer coefficient and vapour pressure-temperature derivative

| $t$° C | –20 | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|---|
| $h_s$ cal. cm$^{-2}$ . mm$^{-1}$ | 0.0050 | 0.0064 | 0.0072 | 0.0080 | 0.0088 | 0.0096 | 0.0105 |
| $dP/dt$ mm Hg °C$^{-1}$ | 0.10 | 0.33 | 0.63 | 1.09 | 1.81 | 2.89 | 4.46 |

By inserting these data in formula 6 and using the first approximation for $b$, an approximate value for $\theta$ which suffices for the use in the next paragraph can be calculated.

TABLE 7. Example of calculation with formula 6

| $\Psi_i$ | $\frac{\Psi_i}{3.18.10^6} = \alpha$ | $10^\alpha$ | $P_{ia}$ | $P_{ia}$-$P_a$ | $b$ | $0.7$-$b(P_{ia}$-$P_a)$ | $0.158 + 1.45b$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| 366 | 0.000115 | 1.00027 | 23.99 | 7.19 | 0.0250 | 0.520 | 0.194 | 2.68 |
| 1511 | 0.000475 | 1.00110 | 23.97 | 7.17 | 0.0238 | 0.529 | 0.193 | 2.75 |
| 5142 | 0.001617 | 1.00373 | 23.91 | 7.11 | 0.0202 | 0.557 | 0.187 | 2.97 |
| 11764 | 0.003699 | 1.00856 | 23.79 | 6.99 | 0.0136 | 0.605 | 0.178 | 3.41 |
| 17781 | 0.005592 | 1.01296 | 23.69 | 6.89 | 0.0075 | 0.648 | 0.169 | 3.84 |
| 18736 | 0.005892 | 1.01366 | 23.67 | 6.88 | 0.0065 | 0.655 | 0.167 | 3.91 |
| 23392 | 0.007356 | 1.01708 | 23.60 | 6.80 | 0.0019 | 0.687 | 0.161 | 4.27 |
| 24322 | 0.007648 | 1.01776 | 23.58 | 6.78 | 0.0010 | 0.693 | 0.159 | 4.35 |

## 9. The vapour pressure — moisture stress equilibrium

At the plant-air interface the liquid and vapour phase of water are in equilibrium with each other. This equilibrium is influenced by temperature. When the moisture stress $\Psi_i$ and the temperature $t$ are known, then the vapour pressure $P_{iL}$ at the interface can be calculated.

The physical formula describing the relation between the three variables reads:

$$\Psi_i = \frac{-R_t^k}{gM} \ln \frac{P_{iL}}{P_{oL}} \qquad \Psi_i = 3.18 \,.\, 10^6 \left( \frac{t_a^k + \Theta}{t_a^k} \right) \log \left\{ \frac{P_{oa} + \Theta \, d\,P/dt}{P_{ia} + \Theta \, d\,P/dt} \right\}$$

$$\Psi_i = 3.18 \,.\, 10^6 \left( \frac{298+\Theta}{298} \right) \log \left\{ \frac{24.0+1.45\Theta}{P_{ia}+1.45\Theta} \right\} \qquad (9)$$

where $R$ = gas constant
$t_a^k$ = temperature of air in degrees Kelvin
$g$ = gravitational constant
$M$ = molecular weight of water

The multiplication of $Rt_a^k / gM$ with $(t_a^k + \theta) / t_a^k$ has the purpose to account for the temperature rise $\theta$ of the leaf, in so far as it influences the multiplication constant. The formula makes it possible to calculate the magnitude of the decrease of the vapour pressure $P_{ia}$ at the leaf boundary for any chosen rise of the moisture stress $\Psi_i$, or for any lowering $\theta$ of the temperature. With this relation it is possible to determine the vapour pressure gradient with respect to the humidity $P_a$ of the air. This gradient is the driving force with respect to the transpiration. The influence of the rise in temperature causes the largest increase in transpiration when high air humidities exist. For those circumstances a more elaborate calculation than the one applied in formula 6 in the previous chapter — where the temperature rise, not yet being known, was left out of the formula — is desirable.

Table 8. Example of calculation with formula 9

| $\Psi_i$ | $\theta$ | $\alpha = 3.18\ 10^6 \cdot \frac{298+\theta}{298}$ | $\Psi_i/\alpha$ | $10 - \log \Psi_i/\alpha$ | $10^{-\Psi_i/\alpha} = \beta$ | $P_{oL} = \gamma = 24+1.45\theta$ | $P_{iL} = P_{ia} + 1.45\theta$ | $P_{ia}$ |
|---|---|---|---|---|---|---|---|---|
| 366 | 2.68 | 3209x10³ | 0.000114 | 9.999886 | 0.999640 | 27.89 | 27.88 | 23.96 |
| 1511 | 2.75 | 3209 | 0.000471 | 9.999529 | 0.998916 | 27.99 | 27.96 | 23.97 |
| 5142 | 2.97 | 3212 | 0.001601 | 9.998399 | 0.996320 | 28.31 | 28.21 | 23.90 |
| 11764 | 3.41 | 3216 | 0.003658 | 9.996342 | 0.991605 | 28.94 | 28.70 | 23.76 |
| 17781 | 3.84 | 3221 | 0.005521 | 9.994479 | 0.987372 | 29.56 | 29.19 | 23.63 |
| 18736 | 3.91 | 3222 | 0.005816 | 9.994184 | 0.986697 | 29.67 | 29.27 | 23.61 |
| 23392 | 4.27 | 3226 | 0.007252 | 9.992748 | 0.983445 | 30.20 | 29.70 | 23.50 |
| 24322 | 4.35 | 3226 | 0.007538 | 9.992462 | 0.982784 | 30.31 | 29.78 | 23.48 |

## 10. The improved calculation of the stomatal resistance

The central theme of the papers of Raschke is the construction of the equation which – based on the energy balance – describes the relation between transpiration, radiation and vapour pressure gradient. While he aims at the assessment of the transpiration, the purpose of the elaboration here is the determination of the stomatal resistance $b$ in a more reliable way than was followed in paragraph 7, to obtain the first approximation. The rather simple means of calculation of the evapotranspiration from groundwater observations makes the calculation of the transpiration from the energy balance less pressing.

The derivation of the formula by solving $\theta$ and $T$ from the constituting formulae will not be discussed here. The reader is referred to the already mentioned papers of Raschke. The formula takes the shape:

$$T = \mu S_a + \nu\,(1-\mu)\,(P_{ia}-P_a)$$

$$T = \left\{ \frac{b}{\dfrac{2\,(h+h_s)}{dP/dt} + b} \right\} \left\{ S_a + \frac{2\,(h+h_s)}{dP/dt}\,(P_{ia}-P_a) \right. \tag{10a}$$

In formula 10a has been substituted:

$$\mu = \frac{b}{\dfrac{2\,(h+h_s)}{dP/dt} + b} \qquad \nu\,(1-\mu) = \frac{2\,(h+h_s)}{dP/dt}\,b \tag{10b}$$

In view of the purpose of the elaboration given here to solve the magnitude of the stomatal resistance $b$ instead of the value of the transpiration $T$, as was the aim of Raschke, the resulting formula is used:

$$b = \frac{\dfrac{2\,(h+h_s)}{dP/dt}\,T}{\left\{ S_a + \dfrac{2\,(h+h_s)}{dP/dt}\,(P_{ia}-P_a) \right\} - T}$$

$$b = \frac{0.109\,T}{\{\,0.7 + 0.109\,(P_{ia}-P_a)\,\} - T} \tag{10c}$$

In formula 10c the same parameters are substituted as in paragraph 8. For $2(h + h_s)$ is taken 0.158, for $dP/dt$ at 25 ° C the value of 1.45 is calculated from table 6. $P_a$ is again assumed to be 16.80 and for $S_a$ is chosen 0.7 cal. cm$^{-1}$ as in formula 6.

No contradiction should exist between the results of the empirical equation 4 and the more fundamental equation 10c. It will be understood that this is basically impossible, because with the two equations the problem is over-identified. Only one of the formulae can contribute to the ultimate solution. The result for $b$ obtained is given in table 9.

TABLE 9. Example of calculation with formula 10c

| $P_{ia}$ | $P_{ia}-P_a$ | $T$ | $0.7\text{-}T=\beta$ | $\beta+0.109\alpha=\gamma$ | $0.109T=\delta$ | $b=\delta/\gamma$ |
|---|---|---|---|---|---|---|
| 23.96 | 7.16 | 0.590 | 0.110 | 0.891 | 0.0643 | 0.072 |
| 23.97 | 7.17 | 0.570 | 0.130 | 0.912 | 0.0621 | 0.068 |
| 23.90 | 7.10 | 0.515 | 0.185 | 0.958 | 0.0561 | 0.059 |
| 23.76 | 6.96 | 0.390 | 0.310 | 1.068 | 0.0425 | 0.040 |
| 23.63 | 6.83 | 0.260 | 0.440 | 1.184 | 0.0283 | 0.024 |
| 23.61 | 6.81 | 0.180 | 0.520 | 1.262 | 0.0196 | 0.016 |
| 23.50 | 6.70 | 0.074 | 0.626 | 1.356 | 0.0081 | 0.006 |
| 23.48 | 6.68 | 0.027 | 0.673 | 1.401 | 0.0029 | 0.0021 |

## 11. THE CHECK OF THE EMPIRICAL FORMULA FOR THE STOMATAL RESISTANCE

The data for $\Psi_i$ and for $b$ from table 9 can be related to each other by plotting them in a way which takes into consideration the mathematical descrip-

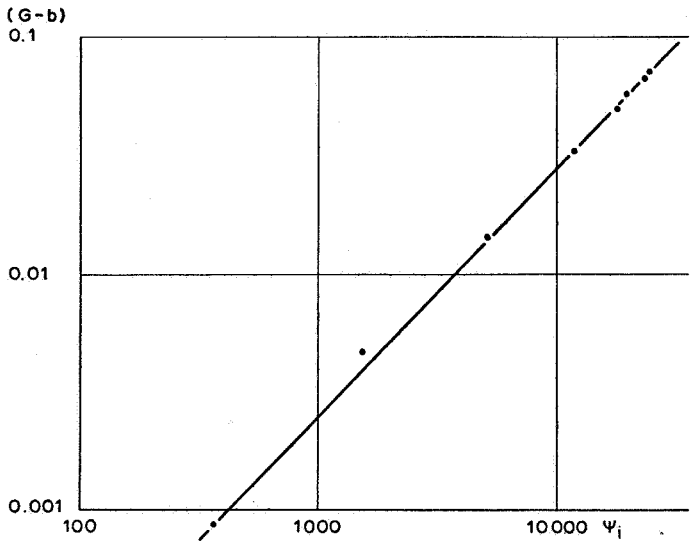


*Fig. 2.* The values of $b$, calculated with formula 10c and plotted against the moisture stress $\psi_i$ in the leaf, conform to formula 4 for a value of $\psi_{wp}$ and $p$, slightly differing from the tentative values, used in paragraph 7.

tion of the action of the stomatal mechanism. To fit formula 4 from paragraph 7 to the values for $\Psi_i$ and $b$ the equation is rewritten:

$$Db = \Psi_{wp}^{p} - \Psi_i^{p} \qquad \Psi_i^{p} = D\left(\frac{\Psi_{wp}^{p}}{D} - b\right) \qquad p \log \Psi_i = \log (G-b) + \log D \quad (11)$$

The data of table 8 and 9 for $\Psi_i$ and $b$ were adjusted to formula 11 (fig. 2).

The constant value of $G$ should be chosen in such a way that $\log \Psi_i$ and $\log (G-b)$ show a linear relation with tangent $p$. The value of $G$ of 0.0728 appears to be the most satisfactory in producing a straight line. For the slope $p$ of the line, 1.04 is obtained and for the value of $D$ 515000. For $\Psi_{wp}$ 25008 is found. Comparing with table 5, the value of $D$ in formula 4 has been chosen too high. In case accurate solutions are desired, a recalculation of $\theta$ and $P_{ia}$ might be done with the better value of $D$. Here this is omitted to keep the example simple. The equation for the stomatal resistance becomes with the new parameters:

$$b = \frac{25008^{1.04} - \Psi_i^{1.04}}{515000} \qquad (12)$$

For the adjusted values of $b$ the values given in table 10 are found.

TABLE 10. Example of calculation with formula 12

| $\Psi_i$ | $\log \Psi_i$ | $\log \Psi_i^{1.04}$ | $\Psi_i^{1.04}$ | $\Psi_{wp}^{1.04}-\Psi_i^{1.04}$ | $b$ |
|---|---|---|---|---|---|
| 366 | 2.5635 | 2.6660 | 463 | 37034 | 0.0719 |
| 1511 | 3.1793 | 3.3065 | 2025 | 35472 | 0.0689 |
| 5142 | 3.7111 | 3.8595 | 7236 | 30162 | 0.0586 |
| 11764 | 4.0706 | 4.2334 | 17116 | 20381 | 0.0396 |
| 17781 | 4.2500 | 4.4200 | 26303 | 11194 | 0.0217 |
| 18736 | 4.2727 | 4.4436 | 27771 | 9726 | 0.0189 |
| 23322 | 4.3691 | 4.5439 | 34986 | 2511 | 0.0049 |
| 24322 | 4.3860 | 4.5614 | 36425 | 1072 | 0.0021 |
| $\Psi_{wp}$ 25008 | 4.3981 | 4.5740 | 37497 | | |

## 12. THE UPTAKE OF CARBON-DIOXIDE

Water and $CO_2$, when flowing through the atmosphere and the stomatal cavern, follow the same path, though in opposite directions. From the plant-air interface into the plant, the pathway, but also the flow phenomena become

different. Moisture flow in the liquid state is governed by the pressure gradient. Water vapour and $CO_2$ diffuse as gas and this flow is governed by the concentration gradient. As formula 9 shows, the relation between stress and concentration is not a simple one. Further, for liquid flow the flow resistance depends on the shape and ramifications of the soil pores. For vapour flow the conductivity depends only on the cross-sectional area of flow. There is less similarity than the identicity of the mathematical expression for liquid and vapour transfer seem to indicate. The flow through the plant in the liquid phase has already been accounted for by formula 3 in paragraph 6. The flow of $CO_2$ through the mesophyll to the chloroplast is governed by the same diffusion laws as the flow through the atmosphere and stomatal area and can be inserted in the same formula. Here we follow the approach of BIERHUIZEN and SLATYER (1964b) and express the relations with the same symbols and with the same units as in their paper.

The partial identity of the flow path for the transpiration $T$ and the $CO_2$ uptake $P$ is embodied in the formulae:

$$T = \frac{\triangle c}{r_a + r_l} \qquad P = \frac{\triangle c'}{r'_a + r'_l + r'_m} \qquad r'_a + r'_l = 1.7\ (r_a + r_l) \tag{13}$$

$\triangle c = P_{iL} - P_a$

$\triangle c' = c_i - c_a$

$c_i, c_a =$ $CO_2$-concentration in $cm^3 \cdot cm^{-3}$ at chloroplast and in atmosphere respectively

$r_a, r_l =$ diffusion resistance in $sec \cdot cm^{-1}$ for water vapour in atmosphere respectively stomata

$r'_a, r'_l, r'_m =$ same for $CO_2$ in atmosphere, stomata respectively mesophyll

The factor 1.7 accounts for the difference in diffusion resistance of water vapour and carbon-dioxide.

Uniting the concepts of RASCHKE and BIERHUIZEN-SLATYER is only possible when presupposing that both systems are consistent and expressed in the same units. This is, however, not the case. RASCHKE expresses his resistance values in $cm^{-1} \cdot min$, BIERHUIZEN and SLATYER in $cm^{-1} \cdot sec$. To this should be added that in the calculations in the previous paragraphs the values of $P_a$ and $P_{iL}$ are expressed in $cal \cdot cm^{-2} \cdot min^{-1} \cdot mm\ Hg^{-1}$. This has first to be rearranged. The change from one scale to an other easily leads to errors. It is therefore given in detail.

The transposition of $b$ to $(r_a + r_l)$ is done as follows:
For RASCHKE's formula holds:

$$b \text{ cal. cm}^{-2}\text{. min}^{-1}\text{. mm Hg}^{-1} =$$

$$= \frac{0.00074 \times 584 \times 60}{760} \, b \text{ g. cm}^{-2}\text{. sec}^{-1}\text{. cm}^{3} =$$

$$= \quad 0.035 \, b \text{ g. cm}^{-2}\text{. sec}^{-1} \tag{14}$$

0.00074 = specific volume weight water vapour
760 = atmospheric pressure in mm Hg
584 = calory per gram evaporated water
60 = sec per min

Now we assume from the study of BIERHUIZEN and SLATYER (1964b) that at high light intensity $r'_m$ will be of the order of 3 sec . cm$^{-1}$. Further it can be taken from relations 13 and 14 that:

$$b\,(r_a + r_l) = 0.035$$

$$b\,(r'_a + r'_l) = 1.7 \times 0.035 = 0.06$$

$$(r'_a + r'_l) = \frac{0.06}{b}$$

$$P = \frac{0.0003}{\frac{0.06}{b} + 3} \tag{15}$$

0.0003 = $CO_2$ concentration of the air in cm$^3$ . cm$^{-3}$

Now from the values for $b$ from table 10, the amount $P$ of $CO_2$ that is taken up by the plant can be calculated. The value of $P$ is expressed in cm$^3$ . cm$^{-2}$ . sec$^{-1}$.

TABLE 11. Example of calculation with formula 15

| $b$ | $3 + 0.006/b$ | $10^{-7}P$ |
|---|---|---|
| 0.0719 | 3.83 | 783 |
| 0.0689 | 3.87 | 775 |
| 0.0586 | 4.02 | 746 |
| 0.0396 | 4.52 | 664 |
| 0.0217 | 5.76 | 521 |
| 0.0189 | 6.18 | 485 |
| 0.0049 | 15.30 | 196 |
| 0.0021 | 31.85 | 94 |

## 13. The dry matter production

As long as the $CO_2$ transfer is the limiting factor for growth, the uptake of carbon-dioxyde is directly – and in principle linearly – related to the dry matter production. For every 44 gram of $CO_2$, 30 grams of carbohydrate are synthesized. Respiration is not accounted for because it seems more advisable to apply this phenomenon as a separate process. It is better not included in the equation for assimilation though it should be included in the formula for the yield. The magnitude of the yield $q$ may therefore be expressed by:

$$q_{\text{kg ha}^{-1}\text{ year}^{-1}} = \frac{30}{44} P_{\text{kg ha}^{-1}\text{ year}^{-1}}$$

The intensity of growth factors other than $CO_2$ transfer as for instance the nutrient uptake or the respiration have also to be taken into account. For this purpose a formula is available which states how in general growth factors work in combination (Visser, 1964c). This formula reads:

$$(1-\frac{q}{Q})\,(1-\frac{q}{x_1})\,(1-\frac{q}{x_2})\,\ldots\ldots = F \qquad (16)$$

In this formula each $x$ represents a growth factor and $F$ is a constant. The factor should quantitatively be expressed with a magnitude which is equal to the availability of the factor to the plant. This availability may be influenced by environmental factors as soil moisture content, temperature and so on. The formula should comprise all growth factors in existence as follows from the construction of the equation. Is a number of terms $x_n$ left out then this will lower the number of the exponents with which the yield $q$ is represented in the formula. The shape of the curve for $q$ changes. As long as the value of the omitted $x_n$ is sufficiently high above the limiting level and $q/x_n$ has – because of the value of $x_n$ – a small value, the influence of leaving such a factor out will not be appreciable. The fact that the formula cannot be split up in additive parts may be considered to be the mathematical expression that life is indivisible.

In the formula, $Q$ represents the maximum capacity of plant growth determined by the genetic properties of the plant. Here for instance the growth rhythm of the plant as a function of time can be accounted for. In this example of calculation $Q$ will, however, for the sake of simplicity be used to describe the

limiting level of any factor, be it genetic, climatic or pedologic with the lowest availability, safe the $CO_2$ uptake. This reduces formula 16 to two terms as shown in formula 17.

At least the variables, encountered in the previous calculation and affecting plant growth, as the radiation $S_a$ or the moisture tension $\Psi_i$ in the plant should be accounted for in the formula. One and the same variable may quite well show up in different growth terms. In the example of calculation, however, the formula will, for the sake of convenience, be limited to two growth terms, one for the $CO_2$ uptake and the other for the arbitrary growth factor with the lowest limiting level.

Using the scale values of BIERHUIZEN and SLATYER, the value of $Q$ for the lowest limiting level, assumed here at 10 000 kg dry matter in a growth season, can be transposed in $2.7 . 10^{-5}$ $cm^3 . cm^{-2} . sec^{-1}$ of $CO_2$ uptake. Here again the daily radiation sum has been assumed to correspond with 500 minutes of optimal $CO_2$ uptake and the growing season with 100 days of this radiation intensity. The uptake as function of the soil moisture content is taken from table 11 in the previous paragraph. The constant $F$ is assumed to have the value of 0.04, as is found in yield experiments.

The formula to be used now can be written:

$$(Q-q)\ (P-q) = F\,P\,Q \qquad (2.7 . 10^{-5} - q)(P-q) = 1.08 . 10^{-6}\,P \qquad (17)$$

By inserting $P$, the value of $q$ can be solved.

TABLE 12. Example of calculation with formula 17

| $P \times 10^{-7}$ | $0.04\,PQ\,10^{-12}$ $=\alpha\beta$ | $P-q$ $=\alpha$ | $2.7-q$ $=\beta$ | $q . 10^{-6}$ | $q/Q$ |
|---|---|---|---|---|---|
| 783 | 84.56 | 52.90 | 1.60 | 25.40 | 94.1 |
| 775 | 83.70 | 52.11 | 1.61 | 25.39 | 94.0 |
| 746 | 80.57 | 49.24 | 1.64 | 25.36 | 93.9 |
| 664 | 71.71 | 41.14 | 1.74 | 25.26 | 93.5 |
| 521 | 56.27 | 31.34 | 2.07 | 24.93 | 92.3 |
| 485 | 52.38 | 23.71 | 2.21 | 24.79 | 91.8 |
| 196 | 21.17 | 2.20 | 9.60 | 17.40 | 64.4 |
| 94 | 10.15 | 0.56 | 18.16 | 8.84 | 32.7 |

## 14. DISCUSSION OF THE RESULTS

The example shows that for the successive steps from the moisture content

of the soil to the calculation of the crop yield, formulae are available and that potentials and resistances can be calculated (fig. 3). It is possible to account for the influence of the soil-, plant- and climate properties in case a result for a certain environment has to be transposed to another set of environmental

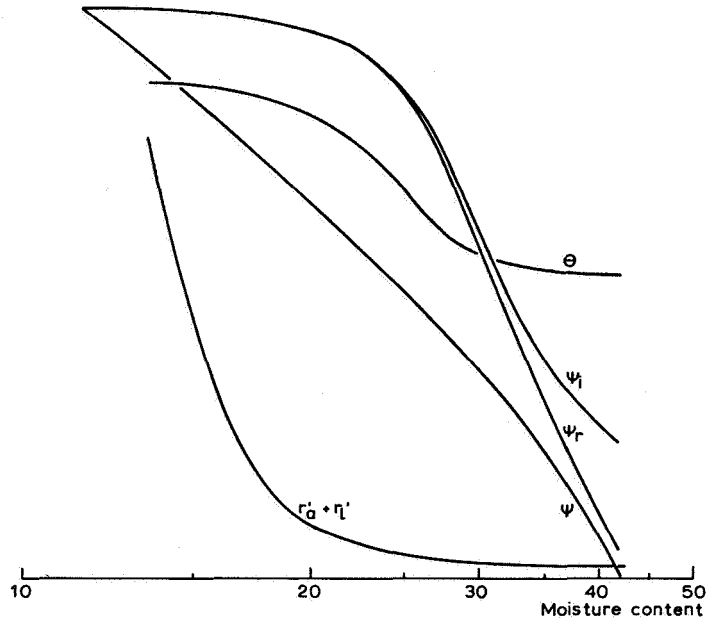


*Fig. 3.* Results of calculation for:
the moisture stress $\psi$ in the soil (table 2)
the moisture stress $\psi_r$ at the soil-root interface (table 3)
the moisture stress $\psi_i$ at the plant-air interface (table 4)
the temperature rise $\theta$ in the leaf (table 7)
the $CO_2$ flow resistance $r'_a + r'_l$ in air and leaf (table 10)
plotted against the moisture content $c$. The vertical scale should be taken from the tables mentioned.

parameters, provided that the necessary constants are known. All kinds of improvements in the description of the constituting equations and parameters are subject of study in many parts of the world. It will be of importance to concentrate the attention first on those points where improvement is most effective.

It may be surmised that the determinations of diffusion resistances from measured transpiration or carbon-dioxyde uptake is sufficiently reliable. Here the application will be aided mostly by obtaining a larger number of resistance values for different crops and climatological conditions.

In the example the theory has been found deficient on one certain point,

which was overcome by devising an empirical formula for the description of the stomatal mechanism. What has been aimed at with formula 4 is to express the value of $b$ in the following way:

$$b = \frac{1}{r_a + r_{li}} = \frac{1}{r_a + \frac{r_{lo}\Psi^p_{wp}}{\Psi^p_{wp} - \Psi^p_i}} = \frac{\Psi^p_{wp} - \Psi^p_i}{(r_a + r_{lo})\ \Psi^p_{wp} - r_a \Psi^p_i} \tag{18}$$

In this representation of the flow resistance, the influence of the moisture tension in the leaf on the resistance $r_l$ in the stomata is separately accounted for as a function of $\Psi_i$, leaving variations due to other factors as for instance light intensity, to be accounted for by variations in the value of $r_l$. In the range of differences between $\Psi_{wp}$ and $\Psi_i$, the value of $(r_a + r_{lo})\ \Psi_{wp}$ will generally be so much larger than the value of $r_a \Psi_i^p$ that the latter value may be neglected. Where this is not possible the range of $\Psi_i$ will be small and of restricted interest because it coincides with small values of the transpiration. Therefore, the use of equation 4, in which $D = (r_a + r_{lo})\Psi^p_{wp}$, seemed sufficiently accurate, also taking into account that the function of $\Psi_i$ —which describes the stomatal mechanism — is still a poin of investigation. Any attempt to construct an improved model of calculation is not very realistic if this is not supported by a sufficient number of observations. It is felt that this extension of the RASCHKE-concept is a consequence of the attempt to use this theory on transpiration for practical soil moisure - plant - weather problems and that it will be a primary task to get a deeper insight in the stomatal mechanism as a link in the chain of relations.

Another point in need of further investigation is the effect of windspeed on the diffusion resistance. As mentioned in paragraph 8, the concept used in the mechanics of fluid flow, as summarized by COLEBROOK-WHITE, may prove to be an important contribution from the field of theoretical hydraulics.

The set of formulae may be used to calculate the actual evapotranspiration from data of the soil moisture content $c$, the air temperature $t$, the vapour pressure at that temperature $P_{oa}$ and the actual humidity of the air $P_a / P_{oa}$. The difficulties in assessing the necessary constants appears to be much more formidable than the determination of the evapotranspiration by direct or statistical means. Therefore it seems advisable to use the available theory to solve the formula for the stomatal mechanism. These stomatal characteristics then can be used to obtain a quantitative expression for the yield. It is felt that the attainment of this object has come within reach of the applied research (fig. 4).

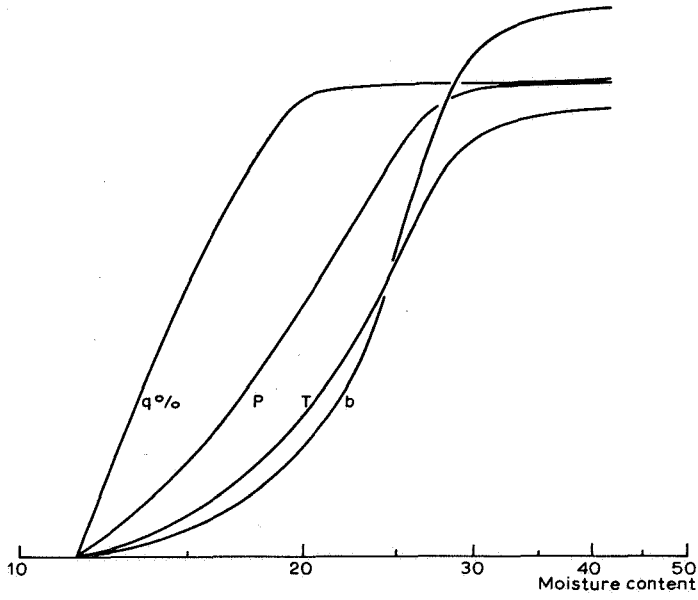


*Fig. 4.* Results of calculation for:
the vapour conductivity $b$ of the stomata (table 10)
the $CO_2$ consumption $P$ (table 11)
the relative yield $q$ % (table 12)
plotted against the soil moisture content $c$.
The transpiration $T$ is the known value, given in table 1. It should be noticed that the transpiration $T$ still increases at moisture contents, where the yield $q$ % no longer is influenced by an increase in soil moisture content.

## 15. Summary

The need to know more in detail what the effect of water management practices is on the yield of crops, can – after a certain level of knowledge has been attained – no longer satisfactorily be met with field experiments or other general solutions. These experiments cannot be done for the full range of crops and environmental conditions because of the amount of work that would be involved. The transposition of results from one set of conditions to another will not be possible if the rules of such a transposition are not known. Vast problems are linked with the solution of the question how the results obtained under well-defined conditions can be made applicable for other situations.

This paper has the purpose to show how the moisture transfer through soil and plant, and the transfer of water vapour and carbon-dioxyde through the atmosphere – the latter also through a part of the plant – can be calculated. This carbon-dioxyde transfer ends in the synthesis of carbo-hydrates and the theory enables one to determine the yielding capacity of a crop at a given moisture status.

The formulae for each step of this calculation are given, the way to obtain the relevant constants is discussed and an example of calculation is given for each step of the elaboration. In carrying out this calculation, it is striking that already without much experience in selecting the constants, acceptable results for the relation between plant yield and soil moisture content are obtained. This supports the widely acknowledged fact that the plant can easily establish an equilibrium between soil moisture content, radiation and transpiration — otherwise the environment would be hostile to plant growth. It also means that the mathematical simulation of this process will lead in a straightforward way to physically possible solutions. The calculation may seem laborious, the results that are obtained, however, easily make sense.

Not the least important point that this paper attempts to make clear is, that theoretical work of biologists, climatologists, hydrologists, soil physicists and agriculturalists is available that may be very useful for project engineering.

A large amount of work has been done on the study of transpiration and assimilation of plants with the aid of the determination of mass transfer or energy balances. This work seems applicable to problems of water management after some extension of the theory in the direction of soil moisture flow as well as in the direction of plant productivity as the result of co-operating growth factors. For both these extensions, the theory as well as the methods to determine the soil and plant parameters are available.

The value of this theory is that it enables the transposition of experience derived from careful investigations under well known soil- and climatological conditions, to conditions for which these investigations are lacking. Taking into consideration the large number of soil profiles and the great many crops grown for agricultural or horticultural purposes, the assessment of their hydrological properties and requirements would be too vast a task to be solved by investigations along non-functional statistical lines. The study of the chain of relations provides the rules of transposition, which will simplify the study of productive capabilities of soils and the moisture requirements of separate crops.

The paper gives an extensive example of calculation to elucidate what fundamental studies have taught about the relations between soil, plant and climate with respect to water consumption. Often these basic studies are expressed in such a way that it provides no easy reading for the project engineer. Still these studies may materially assist in making better designs, speed up the preceding field survey and clear up the interrelation between environmental properties and the effects of land management practices.

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# III. THE CALCULATION OF EVAPOTRANSPIRATION FROM GROUNDWATER DEPTH OBSERVATIONS

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## 1. Introduction

It has been explained repeatedly how with data on groundwater depth, on atmospheric evaporation and on rainfall, it is possible to produce figures for the magnitude of run-off, storage and evapotranspiration (Visser en Bloemen, 1958, 1959, 1960; Bloemen, 1959; Visser, 1960).

The results of a graphical technique, developed for the elaboration of the soundings of observation wells, are satisfactory when applied to periods of a month or longer. The technique appears not to meet reasonable accuracy demands when applied to periods into which successive years are divided when the groundwater level is sounded with intervals of ten or fifteen days. When these figures are then to be used in connection with problems as the calculation of water supply in agriculture or the probability of water shortage or of seepage, errors will be expected to occur (Valken, 1960).

When variations in evapotranspiration in successive short periods are needed as a basis for further calculations, they have to be determined with a larger accuracy than the technique in its simplest form gives. This larger accuracy may be obtained by eliminating systematic errors which normally may occur in the estimations of run-off and storage.

The main problem, however, is that in evapotranspiration, which is determined as a balancing term in the water balance calculations, all the errors are collected. This inaccuracy may be overcome if parameters are available which are correlated with the balancing term, in this case the evapotranspiration. Parameters for evapotranspiration are the soil moisture content and the atmospheric evaporation. It will be shown that the data on evapotranspiration can be submitted to an elaboration by which accidental deviations, due to inaccuracy, are separated from variations which originate from causes of plant-physiological, soil-physical or climatological nature. The magnitude of these latter influences is not affected by this elaboration.

## 2. Deduction of items on the water balance from variations in groundwater depth

### a. *A systematic error in estimated evapotranspiration*

The use of variations in groundwater depth is based on the fact that the equation for the water balance can be written as a total of items which is equal to zero. Of these items only rainfall is used as a primary observation. The magnitudes of run-off, of changes in storage and of evapotranspiration, however, are secundary items related respectively to pressure-head, changes in groundwater level and evaporative capacity of the atmosphere. Thanks to this relation between the balance terms and easily measurable reference values, the principle of the water balance can be used for the evaluation of the various terms.

The technique to effect this evaluation is based on comparing groups of data, groups in which one of the balance terms is zero or in which two terms are within sufficiently narrow limits equal to each other.

Amounts of rain in periods in winter, without an appreciable evaporative capacity of the atmosphere, may for instance be compared with rainfall in periods in which there has been a certain evaporative capacity and in which run-off and storage, according to the pressure-head and the changes in groundwater level, equalled those in the periods in winter. Run-off and storage being equal, the differences between the mean amounts of rain in the two groups indicate the magnitude of the real evapotranspiration in periods with an evaporative capacity of the atmosphere.

In this way a mean value of evapotranspiration for a group of periods can be established. These periods have to be selected in such a manner that the climatological conditions should — as far as evapotranspiration is concerned — be as homogeneous as possible. Generally the possibilities for selection of homogeneous groups of data are not large. Next to selecting according to groundwater level, the periods may be selected according to the evaporative capacity of the atmosphere. This might mean, however, that periods in spring and in autumn are taken together and this implies that the factor for the conversion of changes in groundwater level into amounts of stored water is assumed to be the same in both seasons. There are indications, however, that in spring real evapotranspiration, at equal values of groundwater depth and evaporative capacity of the atmosphere, is larger than in autumn. This is due to the depletion of the soil moisture content and to differences in the density of the crop. When these differences are not taken into account and in spring and autumn a mean conversion factor is applied, it appears that evapotranspiration is underestimated in spring and overestimated in autumn. The data must therefore be split up

in homogeneous groups, not only with respect to run-off and storage, but also to the time of the year.

b. *The importance of an accurate estimation of run-off*

When the data have to be classified into groups by groundwater level and the month of the year, then the total amount of available information will have to be fairly large to be able to establish within each group a relation between rainfall and the differences in successive groundwater levels. These two variables determine the storage, which is a relation which may change with the season. This means that the data of a certain month of the year have to be kept apart from those of other months. The data cannot be lumped together, larger groups which contain more observations can not be made. With run-off this is different. Run-off depends on pressure head and permeability and is, as hydrological relation, not affected by the month of the year. The groundwater depth and the run-off may change, but the drainage curve giving the relation between the two remains the same. The data on run-off may be lumped together, provided that it is done in a correct way. Because of this invariability it is preferable to start a study of groundwater movement with the determination of the run-off in separate periods as accurately as is possible. Now it is known that the curves obtained in this way for successive periods have the same shape and that only rainfall may have to be corrected for constant differences per group of storage and evaporation values. In the graphic elaboration this means that the curves will coincide after a shift parallel to the rainfall axis. In this way far larger numbers of observations can be used to determine the discharge curve. This improves the accuracy of the estimate of the run-off considerably. By subtracting these amounts of run-off from rainfall, the first variable quantity is eliminated from the water balance. Subsequently the periods classified in homogeneous groups as was indicated before, are used for the elaboration of the rainfall data, corrected for run-off, as will be explained in the paragraphs 4 and 5, to obtain values for storage and real evaporation.

## 3. Graphical technique for the deduction of run-off from changes in groundwater depth

a. *Effects of rainfall on groundwater movement*

The method, advocated in paragraph 2b, renders the determination of a function for run-off a most important and essential part of studies on evapotranspiration. The graphical treatment of data on rainfall and groundwater

depth is basically the same for all lengths of periods. It serves the purpose of coping with difficulties which arise because in short periods the similarity between run-off and rainfall is interfered with by the buffering capacity of the storage in the soil.

The method of elaborating the data, as explained later, with daily observations as a basis, can be applied with only small changes on series of observations with larger intervals. Daily means are then substituted by mean values for *k*-day periods. Data from periods in winter, without evaporative capacity of any importance, are giving the least trouble and consequently have been used.

In figure 1 a number of successive daily observations of a testwell in the province of Zeeland are depicted. The daily amount of rain is plotted against the mean of the pressure-heads. This mean value is calculated from the water depth sounded on the day itself and on the previous day. The chronological sequence of the days is indicated for each dot with the first number. The difference in the groundwater depth between the two soundings is indicated with the second.

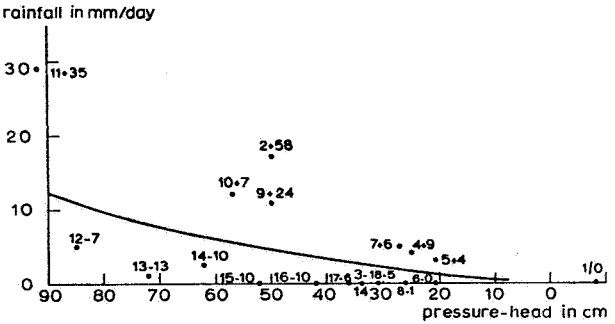



*Fig. 1.* Daily measured amounts of rain plotted against the mean pressure-head of the ground water sounded in a testwell on the same day. At each datum the first figure gives the chronological sequence of the days, the second figure the positive or negative difference in cm of two groundwater soundings.

Figure 1 shows how in a rainy period the ground water rises quickly, while after the rain has fallen the ground water slowly falls again. At first, run-off lags behind rainfall because the pressure-head must increase to get run-off going. This involves a rise of the groundwater table. Then a moment comes when run-off is surpassing rainfall and previously stored water also begins to discharge. This involves a fall of the groundwater level and a decrease of run-off. The changes in groundwater level during the day, which are indicated in figure 1, are reference values for the changes in storage. To relate run-off to pressure-head these changes in storage must be eliminated.

b. *Eliminating the changes in storage*

The determination of the changes in storage, which is necessary to calculate run-off from rainfall, is achieved by taking together days with approximately equal pressure-heads. This selection means that these days will have approximately equal, though still unknown, amounts of run-off. The variations in rainfall occurring in such a group of days, with the changes in groundwater level, represent changes in storage.

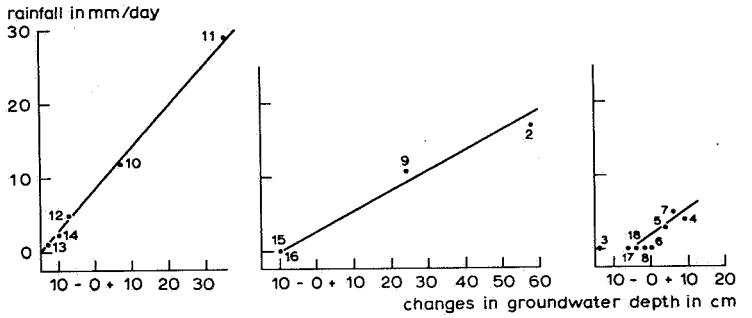



*Fig. 2.* After splitting up the data of fig. 1 into three groups with approximately equal pressure-heads, the daily amounts of rain were plotted against the change in groundwater level on the same day.

In figure 2 the rainfall data for three such groups of days are plotted against the changes in groundwater level. The regression-lines in this diagram show which intensity of rainfall is needed to keep the groundwater level constant. This amount is equal to the run-off that occurs when the pressure-head has the mean value for which the group of days under consideration was selected. So figure 2 will give three points of the curve representing the relation between pressure-head and run-off. The curve constructed through these three points has been drawn in figure 1, but it is to be considered as a first approximation. That is because the days in one group have a certain divergence in the values for the pressure-head. The smaller number of data is the reason that the range of pressure-head values cannot be chosen as small as might be desired. These residual variations in run-off give part of the scatter in figure 2.

It is possible to get a better approximation of the function for run-off. Changes in groundwater level are again plotted against 14-day rainfall-totals after eliminating – with the provisional curve of figure 1 – the variations of run-off within the separate groups. With these rainfall-totals adjusted for run-off, the curves for the storage are again constructed.

In figure 3, the data already given in figures 1 and 2 are plotted after adjustment. The regression-lines in figure 3 are used to get a second approximation of the run-off curve. The difference between the amount of rain for a zero and

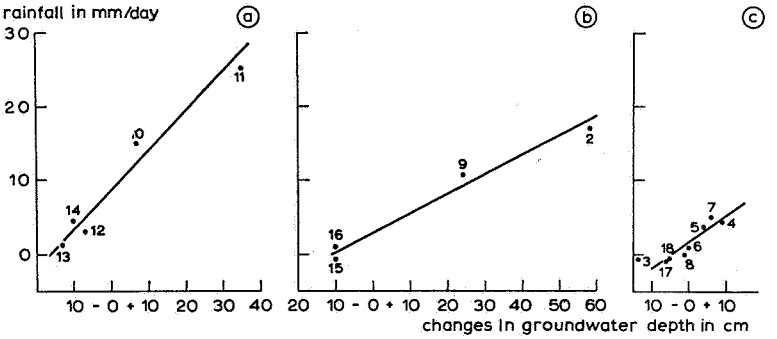



*Fig. 3.* The same as fig. 2, but the pressure-heads per group adjusted to the same value. For a: 63 cm; for b: 41.5 and for c: 165 cm.

a given change in groundwater level are now determined, using the regression-lines of figure 3. These differences are again used to adjust the rainfall data for storage and are plotted against the pressure-heads again. Figure 4 shows the results.

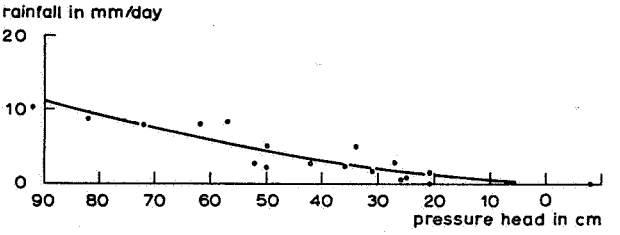



*Fig. 4.* The same data as in fig. 1 after shifting them parallel with regression-lines of fig. 3 to a vertical line through the zero change in groundwater level. The correct course of the curve has been determined with the aid of fig. 5.

c. *Determining the function for run-off*

The real importance of this graphical elaboration of the data is, that the discharge conditions are split up into a steady and a non-steady part. The merit of this is, that every observation is converted into an estimation of a point on the curve which represents the relation between pressure-head and run-off when the flow is steady. In this way the steady-state drainage-curve is based on all observations and a minimum amount of data will suffice to give enough information on its shape.

Another merit is, that it becomes possible to compare the curve in figure 4 with a formula for steady flow and adjust the curve to a well-founded hydro-

logical formula. This is done by determining the parameters which give the best fit. For this hydrological formula, the one of Hooghoudt is used. This formula is written:

$$S = \frac{8\ K_0\ D}{l^2}\ h + \frac{4\ K_b}{l^2}\ h^2$$

$S$ is the symbol for run-off and $h$ for pressure-head. The other symbols are for a particular site constants so

$$\frac{8\ K_0\ D}{l^2} = a \quad \text{and} \quad \frac{4\ K_b}{l^2} = b$$

The Hooghoudt-equation is of the second degree, but easily converted into an expression for a straight line:

$$S = a\ h + b\ h^2 \quad \text{or}$$
$$\frac{S}{h} = a + b\ h$$

From this property good use can be made in fitting curve and equation to each other.

In figure 5 the straight line is depicted which fits the curvi-linear relation of figure 4 best. For three points on the curve of figure 4, the ratio of run-off over pressure-head has been calculated and plotted in figure 5 against pressure-head. In this diagram the function is known to be a straight line. The correct shape of the curve in figure 4 can easily be deduced from the straight line in figure 5.

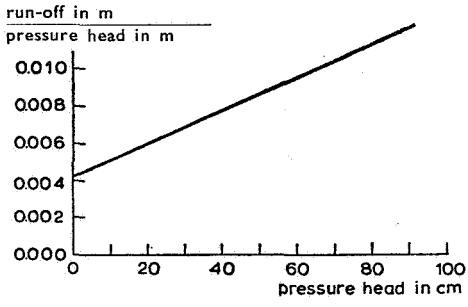



*Fig. 5.* The function of run-off under conditions of steady-state flow as given in fig. 4, presented as a linear regression.

d. *Most important sources of inaccuracy*

That the groundwater level is in most cases sounded with rather long intervals, has always been considered to be one of the most important sources of inaccuracy in the technique of elaborating the data on groundwater depth. The deduction of evapotranspiration is based on the mean depth of the groundwater level in a period between two soundings. This mean depth is a measure for the magnitude of the mean run-off. Only when the pressure-head, which is to be related with run-off, is the mean of a sufficient number of soundings, for instance one for every day of a $k$-day interval, it will be a fair representation of the mean discharge. The characterization of run-off in this manner becomes less satisfactory in case the period is longer, the rainfall in it has been distributed more irregularly and the groundwater depth observations are only available for the beginning and the end of the period. The mean pressure-head should account for all variations in groundwater level during the period. These variations are, by absence of observations, not known, however. Reconstruction of the course of the groundwater depth is necessary, should a high accuracy be needed.

Another source of inaccuracy is, that rain that falls close before the soundings of the groundwater level may not yet have penetrated to the groundwater level at the moment of the sounding. In case daily soundings are made, this may show up in a rise of the groundwater level together with a decreasing intensity of rainfall. The rise has to be attributed to the rain of the previous day. In case of soundings with longer intervals, pressure-heads may not be proportional to preceding rainfall because the full effect of heavy rain at the end of the period escapes notice. A satisfactory way of calculating, which part of the rain is affecting groundwater level on the day of precipitation and which part arrives at the groundwater table on the following days has to be worked out. This paper, however, is focussed on the determination of evapotranspiration. This mainly concerns the months in summer in which discharge is unimportant and errors in the estimation have a still smaller influence. The correct assessment of the percolating water arriving at the groundwater table and the way to find the mean pressure-head will be left out of the discussion here.

4. Fitting a formula to data on evapotranspiration

a. *Estimation of mean monthly values of evapotranspiration*

The preceding paragraph did show, how for a testwell a function for run-off

can be established which is valid for any moment of the year. This run-off can be subtracted from the rainfall. The then remaining rainfall excess is equal to evapotranspiration except for storage. The changes in storage are again correlated to the changes in groundwater level. When there would be a sufficient number of periods in which no changes in groundwater level occurred, evapotranspiration could be known for these periods.

The relation between changes in groundwater depth and storage are in summer, however, less simple than in winter. A deep water table does not react as strongly on the evaporation of water as a shallow one. The deeper the water table, the more the evaporated water is extracted from the unsaturated zone of the profile and the less is supplied out of the ground water by capillary rise. Still, the mean extraction of ground water by capillary rise will be a measure for the change in storage, and a zero change in the total storage in the saturated and unsaturated zone taken together will be indicated by a zero change in the groundwater level.

The changes in groundwater depth resulting from capillary rise will be small and for the purpose of estimating the amount of capillary rise the measurements of the groundwater depth may prove to be rather inaccurate. The elaboration has to be based on this supposition.

The evapotranspiration depends on the evaporative capacity of the atmosphere, but also on two other factors: the soil moisture content and the density of the canopy. For the soil moisture content no indication other than a measurement is as yet available, but for the cover of the soil by vegetation the month of the year can be used as an indication. For the soil moisture content in some way a value has to be deduced.

The observations for a number of successive years are therefore first grouped

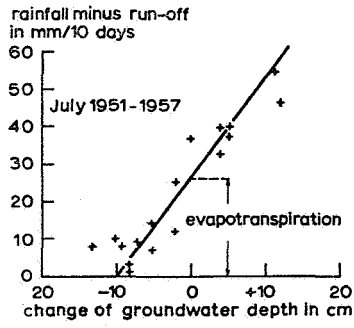



*Fig. 6.* The mean real evapotranspiration in separate months estimated from the relation of rainfall which did not run off and the changes in groundwater level in the same period.

together according to their places on the calendar, as was already suggested in paragraph 2b. The excesses of rainfall over run-off are then plotted against the changes in groundwater level. Figure 6 gives an example. This refers to the same testwell in the province of Zeeland, as has been discussed in paragraph 3a. In figure 6 ten-day periods in July in the years 1951 to 1957 inclusive, are included. Diagrams like this allow a regression-line to be calculated, relating the two quantities. This relation shows how the groundwater level lowers proportionally to the excess of run-off plus evapotranspiration over rainfall, or how it rises when there is an excess of rain over evapotranspiration and run-off. In the same way as in paragraph 3b, it indicates the magnitude of the storage. The angle of inclination of the regression-line provides the rate of conversion or storage-coefficient, with which can be calculated what the amount of rain would have been when not only run-off, but also changes in storage were eliminated. Evapotranspiration is equal to these amounts of rain because all the other items of the water balance are reduced to zero.

The conversion factors which are determined in this way differ for the successive months. They can be applied in two ways:

1. The changes in groundwater level in the separate periods can be converted into an amount of water with which the amounts found for evapotranspiration have to be corrected. Since the data in figure 6 are scattered, however, the amounts of evapotranspiration calculated in this way will have an accidental error and will consequently have to be adjusted to some theoretical relation in the same way as with the Hooghoudt-formula was done for the run-off data.

2. In figure 6 and similar diagrams, evapotranspiration as a mean value for all periods included in the diagram can be read directly. This value is read where a vertical coinciding with a change in groundwater depth equal to zero, intersects with the regression-lines.

The advantage of the second method is that the errors of the separate rainfall totals, of the estimations of run-off and of the changes in storage are not accumulated in the amounts of evapotranspiration that are read. No systematic error is introduced resulting from a false differentiation of storage coefficients. It is a disadvantage that the monthly totals of evapotranspiration which are read are means over a number of years and do not allow for yearly variation. In this way final figures become very few in number. Still, the second method gives a valuable first approximation of the evapotranspiration and a check on data obtained in another way.

b. *Graphical representation of a theory on evapotranspiration*

Adjusted figures on evapotranspiration may be obtained by fitting the mean monthly values of evapotranspiration ($E_r$) to a theoretical relation with mean monthly values of evaporative capacity of the atmosphere ($E_o$) and of the moisture condition of the soil ($M$). A theory on evapotranspiration, of which the development is given elsewhere (VISSER, 1964), states that evapotranspiration is governed by the two other quantities mentioned according to a relation which is depending on the value of some constants, representing well-defined properties of soil and crop, and can be expressed in a simple formula:

$$(gE_o - E_r)\,(aM^p - E_r) = B$$

The value of $B$ is small and the formula indicates that the closer $B$ approaches to zero, the more clearly evapotranspiration is governed exclusively by either the moisture condition of the soil or the evaporative capacity of the atmosphere. The relation of evapotranspiration with one of those two quantities is then almost linear, but when one of these quantities exceeds the critical value of $gE_o = aM^p$, the effect of the other disappears. The higher the value of the limiting factor – be it $E_o$ or $M$ – the higher are the values of the other factor at which the limitation occurs. Evapotranspiration cannot exceed $gE_o$ or $aM^p$ and when the value of $B$ is assumed to be zero, it will equal the lowest of the two values.

A graphical representation of the formula shows that the curves relating real evapotranspiration ($E_r$) to atmospheric evaporation ($E_o$) are identical in shape, though different in level, for different moisture conditions of the soil ($M^p$). The same holds when plotting $E_r$ against $M^p$ for different values of $E_o$. These curves can be schematically represented by a system of one oblique and a number of horizontal asymptotes. This is shown in fig. 7.

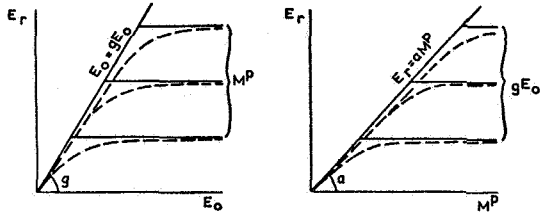



*Fig. 7.* Graphical representation of the formula relating real evapotranspiration ($E_r$) with atmospheric evaporation ($E_o$) and moisture content of the soil ($M^p$).

The oblique asymptotes are coinciding for all values of the third variable and form one single sloping line, the horizontal asymptotes are on a level of $gE_o = E_r$ or $aM^p = E_r$. The similarity in shape of the curves is an important property because it greatly simplifies the determination of the constants present in the formulae for the curves.

Because the oblique asymptotes are approximating one and the same line, all curves can be brought to coincide by shifting the data belonging to the same horizontal asymptote in the direction of the oblique asymptote till the horizontal asymptotes cover each other.

Because only the data for a small number of periods with ample variation of the value for the third variable are available, the corrections towards an arbitrarily chosen mean value $M$ or $E_o$ for the third variable should be carried out by calculation.

c. *Graphical elaboration of a formula on evapotranspiration*

To every pair of observations of $M$ and $E_r$ belongs a potential evapotranspiration $gE_o$ or to every $E_r$ and $E_o$ an asymptotical level of $aM^p$. These properties are graphically represented by the horizontal asymptote.

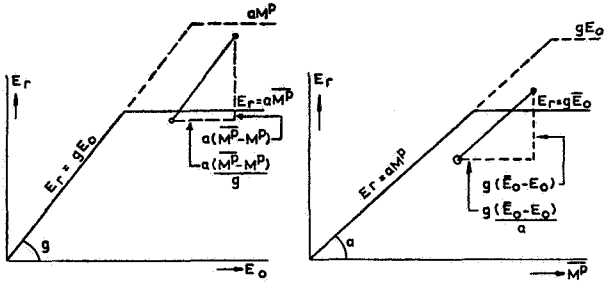
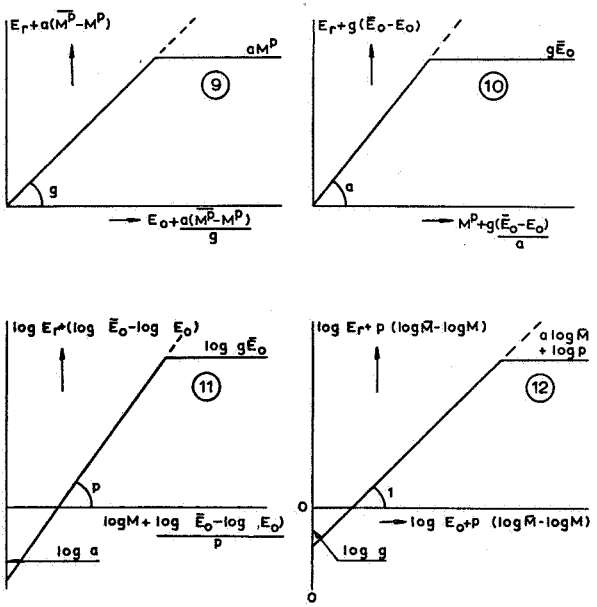



*Fig. 8.* Example of the calculation of corrections to reduce all available data to the mean value of $gE_o$ or $aM^p$.

It follows from figure 8 that the corrections in question are composed of a vertical and a horizontal shift of every point of observation in such a way that the horizontal asymptote shifts from a level $E_o$ to a level $\overline{E}_o$ or from $aM^p$ to $a\overline{M}^p$. This results in a shift of the points representing the observations in the direction of the oblique asymptote. The vertical correction is a change in the value of $E_r$. The horizontal correction is a change in the value of $E_o$ or $aM^p$. It has proved to be valuable to repeat this correction for the variations in the level of the horizontal asymptote in the diagram in which log $E_r$ is plotted against log $E_o$ or log $M$. This is possible because the value of $B$ is very small

and because the oblique asymptotes are straight lines through the origin. Plotted on logarithmic scales they remain straight lines through the origin and also the horizontal branch does not change. This double elaboration is useful because then the curvilinear relation $E_r = aM^p$ becomes a straight line which can be adjusted graphically. The corrections transpose, by eliminating the third variable, a three-dimensional relation between $E_r$, $E_o$ and $M^p$ into two two-dimensional ones: between $E_r$ and $E_o$ as well as between $E_r$ and $M^p$. The constants $a$, $g$ or $p$, linked with the third variable, are still unknown at the stage of the analysis. A crude approximation must first be made and used in the first elaboration. The resulting graphs show what the value of $g$, $a$ and $p$ should have been. With the improved estimate, the elaboration can be repeated if that is desired.




*Figs. 9—12.* Four different ways of representing the relation between $E_o$, $E_r$ and $M^p$. It is indicated where the constants are read.

It follows in figure 9 from the formula $E_r = g\overline{E}_o$ that plotting $E_r$ against $E_o$ changes into plotting of corrected values of $E_r$ against corrected ones of $E_o$. The constant obtained as tangent in this way is the $g$. The intercept is in the origin. The corrections are obtained according to the equations:

$$\text{correction } E_r = a(\overline{M}^p - M^p)$$

$$\text{correction } E_o = \frac{a(\overline{M}^p - M^p)}{g}$$

When plotting $E_r$ against $M^p$ as in figure 10, the constant $a$ shows as a tangent. The intercept again is in the origin. Here the corrections to be applied are calculated with:

$$\text{correction } E_r = g(\overline{E}_o - E_o)$$

$$\text{correction } M^p = \frac{g(\overline{E}_o - E_o)}{a}$$

When plotting the logarithm of $E_r$ against $M$, the constant $p$ shows as a tangent because $\log E_r = p \log M + \log a$. The correction should depend on $\log g\overline{E}_o - \log gE_o$, but $\log g$ cancels out. The diagram is independent of the value of $g$. The intercept on the vertical axis is $\log a$. So in figure 11 it is obvious that:

$$\text{correction } \log E_r = \log \overline{E}_o - \log E_o$$

$$\text{correction } \log M = \frac{\log \overline{E}_o - \log E_o}{p}$$

When $\log E_r$ is plotted against $\log E_o$, as in figure 12 the absence of an exponent shows up as a tangent $= 1$, because of the relation $\log E_r = \log E_o + \log g$. This is a check on a correct drawing of the oblique line. The intercept on the vertical axis is $\log g$. To make the data for different values of $M^p$ comparable, it is clear that the following corrections can be used:

$$\text{correction } \log E_r = p(\log \overline{M} - \log M)$$

$$\text{correction } \log E_o = p(\log \overline{M} - \log M)$$

The angle of inclination of 45° is reflected in the identical corrections.

The four graphs mutually give a good check. The logarithmic graphs should show tangents $p$ and 1 and intercepts of $\log a$ and $\log g$. The metric graphs give tangents $a$ and $g$ and intercepts in the origin. The tangent 1 and the intercepts in the origin are strict directives when fitting lines through the scatter diagram. Further, $a$ and $g$ have to conform to $\log a$ and $\log g$. All these checks are made with the oblique lines. The horizontal lines also provide four checks, namely for their level, for which $E_r$ is equal to $aM^p$ or $gE_o$, in the metric as well as in the logarithmic diagrams. Here again, values of $a$, $g$ and $p$ can be deduced. The four graphs provide 12 values for 3 parameters so that there is ample room for checking and adjusting. In case the horizontal and the oblique lines fit with these requirements as well as with the scatter of the corrected data, then the value of $p$ for which the check is least dependable may be estimated correctly.

d. *The assessment of the soil moisture content*

In appendix 1, the calculations preceding the graphical determination of the constants *a, g* and *p* for the earlier discussed testwell in the province of Zeeland are given. The mean monthly values of atmospheric evapotranspiration are those of the meteorological station Vlissingen (KRAMER, 1957).

The soil moisture contents, shown in the previous paragraph to be the factor which governs the actual evapotranspiration in all those cases where the potential evapotranspiration does not represent the limiting factor, are not determined by direct determination in this type of investigation. The knowledge of the water balance, however, provides a means to determine these moisture contents in an indirect manner.

The desorption curve shows what the moisture content of a certain layer is under conditions of moisture equilibrium between the successive soil layers above the groundwater table. With a mean groundwater depth of 95 cm goes a moisture content of 36 volume percents over a layer of 80 cm below soil surface.

The equilibrium situation will occur at the moment that real evapotranspiration and rainfall are equal, what generally may be expected at the change from April to May. From this amount of moisture, the mean values of real evapotranspiration determined for the successive months are subtracted, while the mean monthly amounts of rainfall are added to find the moisture content at the beginning of the next month. It is further assumed that in this case the evaporating water is extracted from a zone 80 cm deep and that rainfall is absorbed by the same layer until the equilibrium moisture content is reached. Additional rain is assumed to drain to the ground water. In this way from month to month the moisture contents can be calculated.

In cannot be claimed that these moisture contents will accurately agree with the real moisture content, but a certain procentual deviation will only mean that the constant with which the moisture content has to be multiplied in the equation for evapotranspiration would get a different value. The actual evapotranspiration remains in such a case unaffected, because at known values of rainfall, run-off and storage it is not possible to insert into the water balance values for the evapotranspiration which as a mean are too high or too low. On the other hand, the relation with the soil moisture content does not allow that part of the data, which is too high, are balanced by another part which is too low. Any contradiction between the four values and their zero sum can only appear as a wide random scatter of the observations. A few iterative calculations, in which the constants of the evaporation formula are changed or the improved values of the adjusted evaporation data are used, can show

what the best fitting relation between soil moisture content and actual evapotranspiration is.

e. *The values of the constants for the calculation of evapotranspiration*

The technique of shifting the data in the direction of the oblique asymptote till the horizontal asymptotes coincide was used to determine the constants of the evapotranspiration formula.

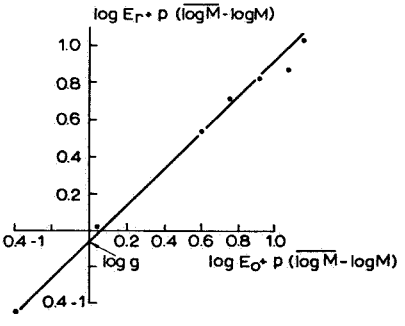



*Fig. 13.* The determination of $g$ equalling 0. 9.

It follows from figure 13 that $g = 0.9$. This result is not affected by the estimation of $p$ in this stage of the elaboration. This exponent does not influence the possibility of coincidence because the corrections are identical. From figure 14, set up with $g = 0.9$ and $m = 3$, which is used as first approximation, it appears that, judged on mean values, only in a single month, July, the soil concerned did desiccate so much that evapotranspiration was unmistakingly

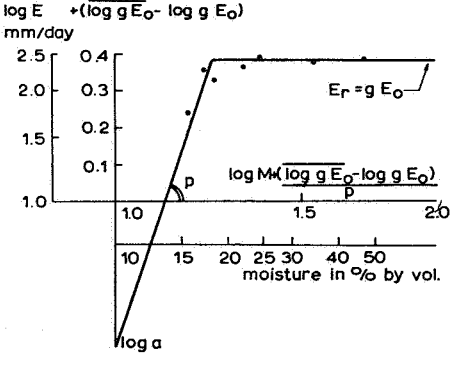



*Fig. 14.* The small number of data does not allow an accurate determination of the tangent $p$.

limited. When the evaporative capacity of the atmosphere is at a level of approximately 2.4 millimeters per day, this limitation occurs when moisture content drops below 18 or 19 % of volume, as a mean for 0 to 80 centimeters below soil surface. The small number of data does not allow in this case to determine accurately the tangent $p$ of the oblique asymptote. When $p$ is taken to be 3, then the point of intersection of the oblique line with the vertical axis through log $M = 0$ shows that $a$ is approximately 300, when moisture content is expressed in parts per unit volume. Is moisture content expressed in per cents, then $a$ is in the neighbourhood of 0.0003. Figures 14 and 15 confirm that $g = 0.9$. The level of the horizontal asymptote in figures 14 and 15 must correspond with the slope of the line in figure 13 and with each other. Furthermore it follows again from figure 15 that the moisture content can have a limiting effect on evapotranspiration. The combination of $p = 3.1$ and $a = 0.0003$ (for percentage by volume) or $a = 475$ (for parts per unit volume)

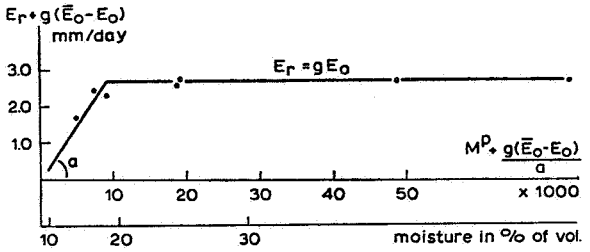



*Fig. 15.* Combination of $p = 3.1$ and $a = 0.0003$ (for volume per cent) gives an oblique asymptote which fits the data best.

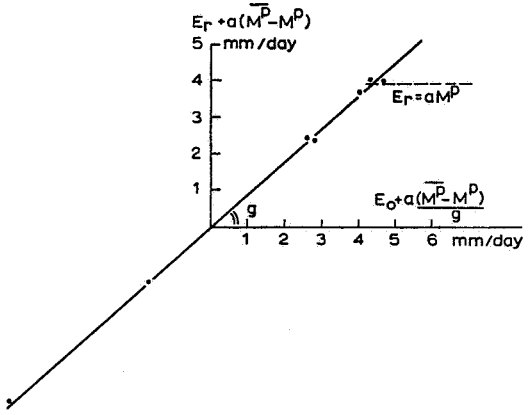



*Fig. 16.* Confirmation that $g = 0.9$.

gives an oblique asymptote which fits the data best. In figure 16 it appears again that $g = 0.9$. The mutual dependency of the constants, determined in the four diagrams of figures 9 to 12 shows up in figures 14 and 15. In figure 15, the oblique line must go through the origin and through the mean value of the three points along that line. In figure 16, the oblique line must intersect the vertical axis at log $a$ and go again through the mean value of the three points along the oblique line. This does not leave much room for variation for the value of $p$. Still it should be kept in mind that a small variation in the value of $p$ requires a considerable change in $a$, to keep the curve within the range of the relevant observations at the same place.

The constants to calculate evapotranspiration for the testwell are now known. According to the formula in paragraph 4 b, as the actual evapotranspiration the lowest of the two values, calculated as $E_r = 0.9\ E_o$ or $E_r = 0.0003\ (M\ \%)^{3.1}$ should be taken.

With these constants the monthly values for evapotranspiration in appendix 1, can be reproduced with an accuracy of 0.21 millimeters per day, calculated as:

$$S = \frac{\sqrt{(\triangle E_r)^2}}{n\text{-}1}$$

## 5. Calculation of evapotranspiration in 1958 and following years

### a. *Application of the constants*

The practical importance of the determination of the constants, referred to in paragraph 4e, is that for any short period evapotranspiration can be calculated when the values of the evaporative capacity of the atmosphere and of the moisture condition of the soil which existed during that period are known. This holds also for periods in other years than those during which the data on groundwater depth were collected. As an example, values of evapotranspiration of ten-day periods in the years 1958 up to and inclusive 1964 were calculated for the soil for which the testwell is representative.

First, the required values of atmospheric evaporation are determined by interpolation on a fluent curve through the mean values for the successive months of the meteorological station in the neighbourhood.

The moisture contents from 0 to 80 cm below surface are again calculated from desorption curves of different layers of the soil profile, with as a starting point the mean moisture content occurring at the equilibrium situation with a groundwater level of 95 cm, which moisture content was found to be 36 per cent by volume. It is assumed that when in some year the groundwater level,

at the moment that evapotranspiration is exceeding rainfall, is higher than 95 cm below surface, the amount of water over 36 per cent by volume is rapidly discharged.

The course of evapotranspiration during the years can be determined for the months with moisture contents higher than $M = (gE_o/a)^{1/p}$ by calculating the values of evapotranspiration from the atmospheric evaporation. Where the moisture content is lower, the actual evapotranspiration follows from the moisture content. It is instructive to calculate the evapotranspiration in both ways. Actual evapotranspiration always equals the lowest of the two values.

b. *An example of calculation*

The calculation that will be given in this study has been simplified as much as possible, since the purpose is to give an example, and not to give a comprehensive review of the evapotranspiration over a number of years for the area in question. On many points the accuracy of the elaboration might have been increased by devoting more time to it than now seemed necessary. It is furthermore felt that in many cases a less time consuming method will be appreciated even at the loss of part of the attainable accuracy.

The calculation is carried out as follows. In column 3 and 4 of appendix 2 the daily and 10-day evapotranspiration, calculated from meteorological data are listed. The data for the third 10-day periods of March, May, July, August and October are multiplied by 1.1 to allow for the 31st day of those months. Here the calculation is carried out for an 11-day interval.

In column 5 and 6 the real evapotranspiration and the rainfall is given. The value of $E_r$ over 10 or 11 days is calculated as $gE_o$ or $aM^p$, depending on which has the lowest value.

In column 7 and 8 the changes in moisture content in the profile and the changes in percentage of moisture are indicated. The first value is equal to $N - E_r$, the second is obtained by dividing $N - E_r$ by 8, the number of decimeters from which it is assumed that the soil moisture is extracted.

In column 9 the soil moisture contents for the successive 10-day periods are listed, beginning with 36 %, as deduced from the desorption curves at the second 10-day period of April, when evapotranspiration is surpassing rainfall. The soil moisture content, however, is during 2 months still sufficiently high to make potential evapotranspiration possible.

In column 10 to 13 the calculation of $E_r = aM^p$ is detailed, from the second period in June onward and as long as the soil moisture content is limiting evapotranspiration. For part of the other months the value of $aM^p$ has been calculated to show how the two limiting levels change during the summer.

In column 14 the value of $E_r = aM^p$ is given as a mean over the 10-day periods. The $E_r$-values of column 13 hold, as the moisture contents of column 9, for the first and last moment of the interval. These values are averaged in column 14. In both columns attention has to be given to the 5 periods of 11 days, which have to be recalculated to 10 days.

Column 15 gives the value of $E_r = gE_o$, column 16 the reduction factor $E_r/E_o$.

The simplification of the elaboration is situated in the calculation over 10-day intervals. Calculation over days would have been possible, though 10 times more work. The point, however, is that in column 7 a value for indicating that first rainfall and then evapotranspiration starts, now will apply to the full 10-day period. Part of this time the rain, however, had not yet fallen and therefore evapotranspiration is calculated of soil moisture that is not yet present. The rain is shifted to the first day of the interval when calculating with 10-day intervals, which increases evapotranspiration. Calculation on a daily basis would have decreased the error arising from this untruth.

The second simplification is that for the calculation of the loss of soil moisture the value of $10\,E_r = 10\,aM^p$ in column 5 is taken from column 13 instead of from column 14. This means that the $E_r$-value of the beginning of the interval is used for the mean value of the whole interval. For the first period of July in column 5, the value of 10 x 1.31 is taken from the end of the third interval of June in column 13, where 1.31 is listed. The value of 1.11 of column 14 on the next line should have been taken. But at the moment that the values for the first interval of July were calculated, the value of 1.11 was not yet known since it is the mean of 1.31 and 0.92, and this value of 0.92 still had to be calculated. Though it was possible to get the value, that has to take the place of the 0.92, this was considered more laborious than was acceptable for the purpose of this study. The technique to calculate these unknown quantities is, however, available.

The shortcut to use the data of column 13 has as an effect that in the first part of the summer $E_r$ is assumed at too high a value, but after the minimum of the soil moisture content has been passed, the value of $E_r$ which is used is lower than it in reality is. This means that the shift of the rainfall to the beginning of the interval works as a shift of the evaporation curve to a moment 5 days earlier than in reality and the $E_r$-value of column 13 shifts the evaporation curve to a moment 5 days later. Though the two actions are not identical, they partly cancel each other.

In August the first interval shows a value $E_r = gE_o = 3.01$ and $E_r = aM^p = 3.20$. Both calculations must be carried out to establish which process is

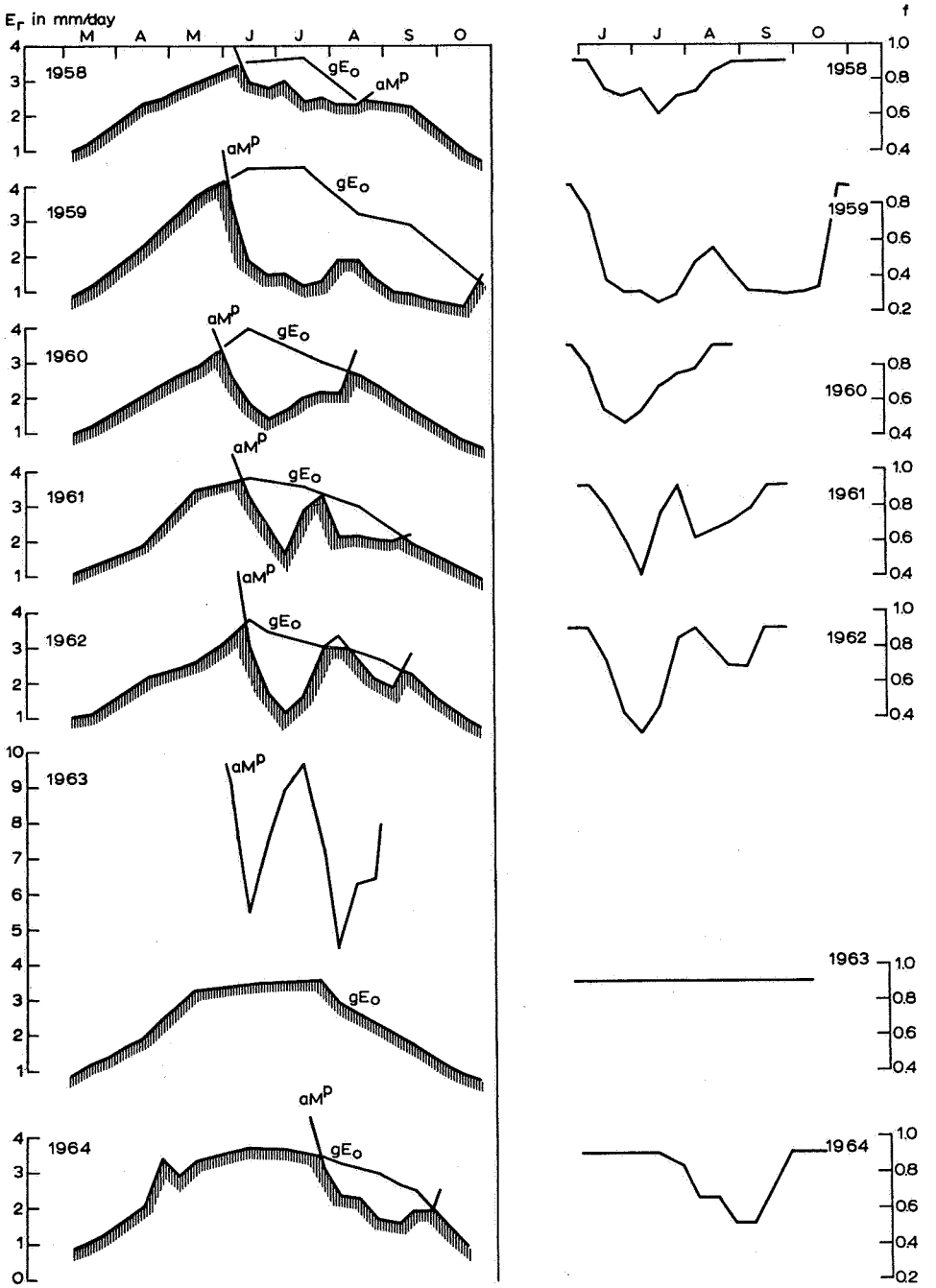



*Fig. 17.* Calculated course of actual evapotranspiration (with either $gE_o$ or $aM^p$ limiting) and of the reduction factor of Penman.

valid for this period and what the moisture content at the end of the period will be. The columns which had to be calculated, but appear not to be valid, have been put in a box.

In figure 17, for 1958 and following years the course of the values of $aM^p$ and $gE_o$ is shown. The course of actual evapotranspiration follows from this. The course of the reduction factor of Penman is also shown.

It appears that with regard to evapotranspiration there are quite some differences in the successive years. There is little variation in the moment when the moisture content of the soil is beginning to act as a limiting factor for evapotranspiration. As a rule this occurs in the first half of June. The course of evapotranspiration from that moment on is different every year, however. Not only the dry year of 1959, but also the wet year of 1963 attracts attention.

c. *Checking the results*

Some direct check on the values which constitute figure 17 was not possible. Data on evapotranspiration from lysimeters are not very suited for comparison, because of the different conditions – for instance the steady groundwater table – under which evapotranspiration takes place. In the area of this study, moreover, no lysimeter exists. Though in the experimental drainage and lysimeter field the 'Rottegatspolder' it is tried to keep a constant groundwater level, the

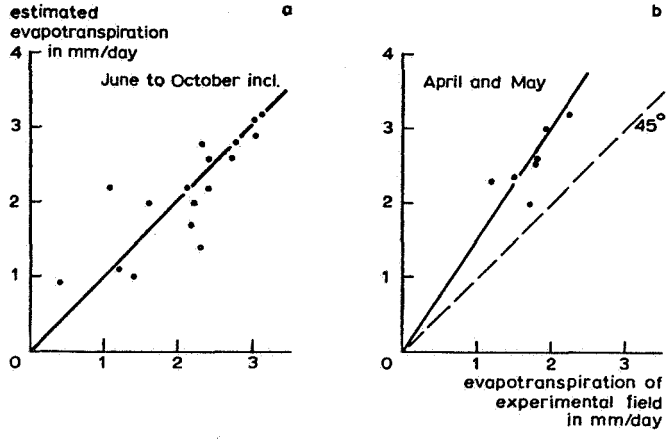



*Fig. 18.* Comparison of the data for actual evapotranspiration given in fig. 17 ('estimated evapotranspiration'), with data from the experimental drainage and lysimeter field the 'Rottegatspolder' in the province of Groningen (some 250 km from the testwell used in the figs. 1–17).
a. June to October inclusive; b. April and May.

figures from this large experimental field are as a comparison probably the best suitable of all the lysimeter data that are available (PEERLKAMP, 1959 and later). These data are directly calculated from measured precipitation, run-off and soil moisture content. Further, they are valid for a reasonably large area with an undisturbed profile. Apart from that, the groundwater level does still show a fairly large natural fluctuation. That this experimental field is arable land instead of grassland, that water is let in and that it is situated in the province of Groningen at some 250 km from the testwell in the province of Zeeland, will explain the differences in evapotranspiration that show up when the data calculated from the observation well and from the Rottegatspolder are compared with each other in figure 18. It can be seen that in April and May evapotranspiration in the Rottegatspolder was considerably lower than in Zeeland. Here the difference shows between evapotranspiration of arable land and of grassland which – as is well known – occurs in spring as a consequence of crop development and density of the canopy as has been pointed out before (VISSER and BLOEMEN, 1965). It also shows that in June up to September inclusive, evapotranspiration calculated for the testwell in Zeeland in general agrees with that in Groningen. The separate values ($E$) have a mean deviation which, calculated as

$$S = \frac{\sqrt{(\triangle E)^2}}{n\text{-}1}$$

is 0.452 mm per day in June/September and when the systematic difference is eliminated, of 0.387 mm per day in April and May.

When all the differences which exist between the two cases compared here are considered in relation with the comparatively small mean errors in figure 18, the evapotranspiration as calculated for the testwell in Zeeland can be accepted as a fairly accurate estimation. It may be expected to be valid for a more extended area than the close surroundings of the studied testwell. The result of the elaboration holds for grassland. On the basis of figure 18 it may be assumed that in case the results have to be applied on arable land, the estimated values have to be reduced to 2/3 in April and May.

## 6. SUMMARY AND CONCLUSION

Methods exist of which it is expected that they are suitable to calculate accurate values for evapotranspiration from soil-physical, plant-physiological and meteorological properties. These methods are, however, perhaps of more interest for the explanation of the phenomenon of evapotranspiration than for their practical applicability. As a rule the calculation of evapotranspiration of

a random spot will probably be hampered by the fact that many properties which should be known are unknown.

For that reason a calculation method has been evolved which is based on a statistical elaboration of testwell readings and generally available climatological data, which can readily be used in practice and which requires only a minimum of effort in the field or in the laboratory. That the results will have only a tolerable accuracy when given for periods of ten days or two weeks is not a large disadvantage when viewing the hydrological problems where the method is to be applied.

The significance of the method described here is, that basically every testwell turns out to be the equivalent of a lysimeter, whether the soundings have long since been stopped or still are in progress. Since figures on rainfall or on atmospheric evaporation are generally available, the only supplement on the soundings of the groundwater level will have to be a sampling to determine the desorption curves of the profile involved.

The elaboration given in this article was set up to show the simplest techniques fit for practical use. Along the same lines, studies of a higher accuracy can be carried out if more labour can be put into it. The advantage of statistical methods with adjustment to scientific formulae afterwards is, that the results of the study are of necessity in agreement with the actual condition over the full range of situations, because the result presents the mean trend over this actual range. Transposing the results from scientific experiments to practical environments is then not necessary. It is felt that the technique has not only its use for application in practice, however, but is also of theoretical interest.

In regions where, as in the Netherlands, the groundwater depth is generally heavily affected by weather conditions, the discussed method and the techniques which were used, particularly deserve attention.

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*Appendix 1.* Calculations necessary for the graphical representation of the relation between the factors listed in columns 1, 2 and 3

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 4 + 8 | 5 + 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | diagram 13 | |
| month | $E_o$ | $E_r$ | $M$ | $\log E_o$ | $\log E_r$ | $\log M$ | $\log\overline{M}-\log M$ | $p(\overline{\log M}-\log M)$ | $X$ | $Y$ |
| March . . . | 1.16 | 1.06 | 36 | 0.0645 | 0.0253 | 1.5563 | — 0.1563 | — 0.4689 | — 0.4044 | — 0.4436 |
| April . . . . | 2.2 | 2.0 | 32 | 0.3424 | 0.3010 | 1.5051 | — 0.1051 | — 0.3153 | + 0.0271 | + 0.0143 |
| May . . . . | 3.9 | 3.4 | 25 | 0.5911 | 0.5315 | 1.3979 | + 0.0021 | + 0.0063 | + 0.5974 | + 0.5378 |
| June . . . . | 4.1 | 3.4 | 20 | 0.6128 | 0.5315 | 1.3010 | + 0.0990 | + 0.2970 | + 0.9098 | + 0.8285 |
| July . . . . | 4.0 | 2.6 | 18 | 0.6021 | 0.4150 | 1.2553 | + 0.1547 | + 0.4641 | + 1.0662 | + 0.8791 |
| August . . . | 3.5 | 2.8 | 26 | 6.5441 | 0.4472 | 1.3010 | + 0.1990 | + 0.5970 | + 1.1411 | + 1.0442 |
| September . . | 2.2 | 2.0 | 23 | 0.3424 | 0.3010 | 1.3617 | + 0.1383 | + 0.4149 | + 0.7573 | + 0.7159 |

| 9 | 10 | 11 | 12 | 5 + 11 | 6 + 12 |
|---|---|---|---|---|---|
| | | | | diagram 14 | |
| $gE_o$ | $\log gE_o$ | $\overline{\log gE_o}-\log gE_o$ | $\frac{\overline{\log gE_o}-\log gE_o}{p}$ | $Y$ | $X$ |
| 1.04 | 1.0170 | + 0.3630 | + 0.1210 | + 0.3883 | + 1.6773 |
| 1.98 | 0.2967 | + 0.0833 | + 0.0281 | + 0.3843 | + 1.5332 |
| 3.5 | 0.5441 | — 0.1641 | — 0.0547 | + 0.3674 | + 1.3432 |
| 3.7 | 0.5682 | — 0.1882 | — 0.0627 | + 0.3559 | + 1.2383 |
| 3.6 | 0.5563 | — 0.1763 | — 0.0589 | + 0.2387 | + 1.1964 |
| 3.14 | 0.4969 | — 0.1169 | — 0.0393 | + 0.3303 | + 1.2617 |
| 1.98 | 0.2967 | + 0.0833 | + 0.0281 | + 0.3843 | + 1.3898 |

| 13 | 14 | 15 | 16 | 2 + 15 | 13 + 16 | 17 | 18 | 1 + 18 | 2 + 18 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | diagram 15 | | | | diagram 16 | |
| $Mp$ | $\overline{E}_o-E_o$ | $g(\overline{E}_o-E_o)$ | $\frac{g(\overline{E}_o-E_o)}{a}$ | $Y$ | $X$ | $\overline{M}^p-M^p$ | $a(\overline{M}^p-M^p)$ | $X$ | $Y$ |
| 66,700 | + 1.84 | + 1.66 | + 5530 | + 2.72 | + 72,230 | — 40,700 | — 2.21 | — 11.05 | — 11.15 |
| 46,300 | + 0.8 | + 0.72 | + 2400 | + 2.72 | + 48,700 | — 20,300 | — 6.09 | — 3.89 | — 4.09 |
| 21,600 | — 0.9 | — 0.81 | — 2700 | + 2.59 | + 18,900 | + 4,400 | + 1.32 | + 5.22 | + 4.72 |
| 10,800 | — 1.1 | — 0.9 | — 3300 | + 2.41 | + 7,500 | + 15,200 | + 4.56 | + 8.66 | + 7.96 |
| 8,000 | — 1.0 | — 0.9 | — 3000 | + 1.7 | + 5,000 | + 18,000 | + 5.4 | + 9.4 | + 8.0 |
| 10,800 | — 0.5 | — 0.45 | — 1500 | + 2.35 | + 9,300 | + 15,200 | + 4.56 | + 8.06 | + 7.36 |
| 16,700 | + 0.8 | + 0.72 | + 2400 | + 2.72 | + 19,200 | + 9,300 | + 2.79 | + 4.99 | + 4.79 |

*Appendix 2.* Calculation of actual evapotranspiration for 10 day-periods in 1962 for a sandy clay in the province of Zeeland

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| month | decade | $E_o$ | $E_o$ | $E_r$ | rainfall | $R-E_r$ | $\frac{R-E_r}{8}$ | $M80$ | log $M$ | log $M^{3,1}$ | $M^{3,1}$ | $0.0003M^{3,1}$ | mean of 13 | $0.9\,E_o$ | $f$ |
| March | 1 | 1.11 | 11.10 | 9.99 | 7.1 | | | | | | | | | 1.00 | 0.90 |
| | 2 | 1.19 | 11.90 | 10.71 | 11.2 | + 0.49 | | | | | | | | 1.07 | 0.90 |
| | 3 | 1.59 | 17.49* | 15.74 | 19.7 | + 3.96 | | | | | | | | 1.43 | 0.90 |
| April | 1 | 1.99 | 19.90 | 17.91 | 43.1 | + 25.19 | | 36.00 | | | | | | 1.79 | 0.90 |
| | 2 | 2.38 | 23.80 | 21.42 | 6.9 | — 14.52 | 1.81 | 34.19 | | | | | | 2.14 | 0.90 |
| | 3 | 2.55 | 25.50 | 22.95 | 1.7 | — 21.25 | 2.66 | 31.53 | | | | | | 2.30 | 0.90 |
| May | 1 | 2.72 | 27.20 | 24.48 | 11.6 | — 12.88 | 1.61 | 29.92 | | | | 11.29 | | 2.45 | 0.90 |
| | 2 | 2.90 | 29.00 | 26.10 | 10.6 | — 15.50 | 1.94 | 27.98 | | | | 9.17 | 10.19 | 2.61 | 0.90 |
| | 3 | 3.30 | 36.30* | 32.67 | 13.0 | — 19.67 | 2.46 | 25.52 | | | | 6.89** | 7.72 | 2.97 | 0.90 |
| June | 1 | 3.70 | 37.00 | 33,30 | 0.5 | — 32.80 | 4.10 | 21.45 | 1.3308 | 4.1255 | 13.351 | 4.01 | 5.13 | 3.33 | 0.90 |
| | 2 | 4.10 | 41.00 | 36.90 | 0.5 | — 36.40 | 4.55 | 16.87 | 1.2271 | 3.8040 | 6.368 | 1.91 | 2.96 | 3.69 | 0.72 |
| | 3 | 3.87 | 38.70 | 19.10 | 3.6 | — 15.50 | 1.94 | 14.93 | 1.1741 | 3.6397 | 4.362 | 1.31 | 1.61 | 3.49 | 0.41 |
| July | 1 | 3.65 | 36.50 | 13.10 | 0.4 | — 12.70 | 1.59 | 13.34 | 1.1252 | 3.4881 | 3.077 | 0.92 | 1.11 | 3.29 | 0.30 |
| | 2 | 3.42 | 34.20 | 9.20 | 42.3 | + 33.10 | 4.14 | 17.48 | 1.2425 | 3.8518 | 7.109 | 2.13 | 1.52 | 3.08 | 0.44 |
| | 3 | 3.38 | 37.20* | 2343* | 48.0 | + 24.57 | 3.07 | 20.55 | 1.3128 | 4.0697 | 11.741 | 3.52** | 2.66 | 3.04 | 0.79 |
| | | | | | | | | | | | | 3.20 | | | |
| August | 1 | | | 32.00 | 26.9 | — 5.10 | 0.64 | 19.91 | 1.2991 | 4.0272 | 10.646 | 3.19 | 3.20 | | |
| | | 3.34 | 33.40 | 30.06 | 26.9 | — 3.16 | 0.39 | 20.16 | | | | 3.32 | | 3.01 | 0.90 |
| | 2 | 3.32 | 33.20 | 33.20 | 20.8 | — 12.40 | 1.55 | 18.61 | 1.2697 | 3.9361 | 8.632 | 2.59 | 2.45 | 2.97 | 0.74 |
| | 3 | 3.00 | 33.00* | 28.49* | 6.7 | — 21.79 | 2.72 | 15.89 | 1.2011 | 3.7234 | 5.289 | 1.59** | 2.02 | 2.70 | 0.67 |
| | | | | | | | | | | | | 1.45 | | | |
| September | 1 | 2.70 | 27.00 | 20.20 | 24.5 | + 4.30 | 0.54 | 16.43 | 1.2156 | 3.7684 | 4.769 | 1.43 | 1.44 | 2.43 | 0.53 |
| | 2 | | | 14.40 | 48.8 | + 34.40 | 4.30 | 20.73 | 1.3166 | 4.0815 | 12.064 | 3.62 | 2.52 | | |
| | | 2.40 | 24.00 | 21.60 | 48.8 | + 27.20 | 3.40 | 19.83 | | | | 3.15 | 2.29 | 2.16 | 0.90 |
| | 3 | 1.93 | 19.30 | 17.37 | 3.8 | — 13.57 | 1.70 | 18.13 | | | | 2.39 | 2.77 | 1.74 | 0.90 |
| October | 1 | 1.47 | 14.70 | 13.23 | 11.6 | — 1.63 | 0.20 | 17.93 | | | | 2.31 | 2.35 | 1.32 | 0.90 |
| | 2 | 1.00 | 10.00 | 9.00 | 0.0 | — 9.00 | 1.12 | 16.81 | | | | | | 0.90 | 0.90 |
| | 3 | 0.80 | 8.80* | 7.92 | 52.6 | + 44.68 | 5.58 | 22.39 | | | | | | 0.72 | 0.90 |

* converted from 10 to 11 days

** converted from 11 to 10 days

$E_o$ = evaporating capacity of the atmosphere according to the Penman-formula

$R$ = rainfall

$M80$ = moisture content in the layer 0 to 80 cm below surface

$0.9\,E_o$ = actual evapotranspiration limited by evaporating capacity of the atmosphere

$a\,M^p$ = actual evapotranspiration limited by moisture content of the soil

$f$ = reduction factor $E_r/E_o$

# IV. EVAPOTRANSPIRATION

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## 1. Introduction

Evaporation from free water surfaces depends only on meteorological factors as radiation, temperature, relative humidity and wind velocity. Evapotranspiration from a cropped soil is not only dependent on these meteorological factors, but also on factors related to the crop and to the available amount of moisture in the soil. For this reason the concept of potential evapotranspiration has been introduced; it is defined as the rate of evapotranspiration, dependent on weather conditions, from an extensive surface of a short green crop of uniform height, completely shading the soil actively growing under conditions of optimum water supply (Penman, 1948, 1956). In order to calculate the evapotranspiration from a certain crop with other specifics, the potential one must be multiplied by a crop factor (Makkink, 1960; Makkink and van Heemst, 1962).

In the present paper the term potential evapotranspiration will be used for the loss of water in vapour state, in its dependence on weather conditions, from any cropped soil, under conditions of optimum water supply. This water loss includes the vaporization of water directly from the upper soil layers (evaporation) and the vaporization of water taken from the soil by plants (transpiration). For the vaporization of intercepted precipitation the term evaporation is also applied.

Due to the development of the crop and to a possible lack of water, the evapotranspiration has under many conditions no direct relation to the evaporation from a free water surface. An analysis of the effects of climate, crop development and available soil moisture on evapotranspiration can be made using data obtained from lysimeters. The results obtained from this type of study can be practically applied in hydrological investigations under field conditions.

## 2. Climatological factors

The main climatological factors determining the rate of evapotranspiration are the amount of energy available for the vaporization of water and the transport of water vapour from the air layers close to the evaporating surface to higher layers.

a. *Energy balance*

Evapotranspiration is a change in state, demanding a supply of energy to be used as latent heat of vaporization. The problem in the energy balance is to measure all other sources and sinks for energy, leaving evapotranspiration as the only unknown.

When neglecting the amount of energy used for photosynthesis, the energy balance can be written as follows:

$$H_{nt} = (1-r)H_{sh} - {}^{nt}H_{lo} = LE + K + S + G \qquad (1)$$

where $H_{nt}$ is the net radiation, $H_{sh}$ the global shortwave radiation, ${}^{nt}H_{lo}$ the net longwave radiation, $LE$ the latent heat flux density of evapotranspiration, $K$ the sensible heat flux density to the atmosphere, $S$ the sensible heat flux density to the soil, $G$ the storage of heat in the crop, $r$ the reflection coefficient of the evaporating surface and $L$ the latent heat of vaporization.

The shortwave radiation can be measured with reasonable ease and accuracy using solarimeters or it can be calculated with empirical relations of the form:

$$H_{sh} = (a + b\, n/N)H_a \qquad (2)$$

where $H_a$ is the theoretical maximum incoming shortwave radiation if there was no atmosphere, $n/N$ the actual duration of bright sunshine as a fraction of the maximum possible duration for a cloudless sky, $a$ and $b$ are empirically determined constants depending on the place on earth.

Data of $H_a$ for various latitudes are given in the Smithonian Meteorological Tables. For latitudes $\varphi < 60°$ the constant $a$ can be approximated with $a = 0.29 \cos \varphi$, while a good mean value of $b$ is 0.54.

Because the reflection coefficient depends on the angle of incidence, which in particular for the direct solar radiation varies with latitude, with season and with the time of day, as well as with the type of surface, the amount of total incident radiation that is reflected by a given surface is by no means constant. Mainly, only mean values of the reflection coefficients of various land surfaces are known. Data from literature (Scholte Ubing, 1959; Monteith, 1959; Rijtema, 1965) for various crops give values varying from 0.20 to 0.26. For free water surfaces a mean value of 0.05 is generally accepted.

The magnitude of net longwave radiation depends on surface temperature, air temperature, atmospheric water vapour content and cloudiness. Equality of surface temperature and air temperature at 2 m height is usually assumed, when calculating net longwave radiation for periods of more than one day. Moreover a good correlation between relative duration of bright sunshine and cloudiness is assumed for longer periods. This proposition offers the possibility to use data of $n/N$ in the calculation of net longwave radiation. Using the tables given by Wesseling (1960), net longwave radiation can be calculated with the following empirical relation:

$$^{nt}H_{lo} = \sigma\ (273+T_a)^4\ (0.56-0.092\sqrt{e_a})\ (0.10+0.90\ n/N) \tag{3}$$

where $\sigma$ is the constant of BOLTZMAN ($118.10^{-9}$ cal. $cm^{-2}$ . $day^{-1}$ $°K^4$), $T_a$ air temperature in °C, $e_a$ the vapour pressure at 2 m height in mm Hg and $n/N$ the relative duration of bright sunshine.

The main problem in the approach of the energy balance is the division of net energy between the other sinks expending energy. The height and the density of the crop and the water supply to the crop affect the distribution of energy used for evapotranspiration, for sensible heat transfer to the air and for the transfer of heat to the soil. The storage of heat in the crop is generally a very small component of the balance and it may be neglected over periods longer than one day.

The amount of heat transfer to or from the soil is a small component of the energy balance only for periods of some days. During short periods, however, this short term transfer may become more important (RIJTEMA, 1965). MONTEITH (1958) measuring the storage of heat in the soil beneath various crops (wheat, potatoes and grass) concludes that the neglect of this term in balance periods of some days does not lead to serious errors. The errors resulting from the neglect of a heat storage in the soil of 10 % of the net radiation, when calculating evapotranspiration under conditions of clear summer weather, are in the order of 7 % of the potential evapotranspiration.

Mostly, the storage of heat in the soil is neglected, when calculating evapotranspiration for periods of 10 days or longer. Net radiation has to be divided in that case only between evapotranspiration and sensible heat transfer to the atmosphere. Both terms can be separated using the Bowen-ratio, which requires the time averaged measurements of the vertical temperature and vapour pressure gradients. This ratio is written as follows:

$$\beta = \frac{K}{LE} = \gamma\ (\frac{T_s-T_a}{e_s-e_a}) \tag{4}$$

where $\gamma$ is the psychrometer constant (0.485 mm Hg. $°C^{-1}$) to keep units consistent; $T_s$ and $T_a$ are respectively the surface and air temperature, and $e_s$ and $e_a$ respectively the vapour pressure at the surface and in the air.

Combining the energy balance equation and the Bowen-ratio gives the expression of the energy used for evapotranspiration:

$$LE = \frac{H_{nt}-S}{1+\beta} \tag{5}$$

As long as $\beta$ is not less (more negative) than $-0.5$, an error in $E$ is significantly less than an error in $\beta$.

b. *Turbulent vapour transport*

In air without movement or under conditions of laminar flow, the vertical transport of water vapour is determined by molecular diffusion. The layer of laminar flow is in the open under normal conditions very thin. The thickness of this layer is determined by the wind velocity and the roughness of the evaporating surface. Above this boundary layer the vapour transport does not only take place by molecular diffusion, but is mainly governed by the irregular turbulent flow of air, which causes an increased vertical transport of water vapour. Under neutral conditions of the atmosphere the transfer of water vapour can be given by the following equation:

$$E = \frac{\rho\, k^2 (u_2 - u_1)\,(q_1 - q_2)}{\left[\ln\left(\frac{z_2+z_0-d}{z_1+z_0-d}\right)\right]^2} \tag{6}$$

where $\rho$ is the air density in g, cm$^{-3}$, $k$ ($\approx 0.4$) a turbulence constant, originally introduced by VON KARMAN, $u_2$ and $u_1$ are respectively the wind velocity in cm. sec$^{-1}$ at height $z_2$ and $z_1$, $q_1$ and $q_2$ are respectively the specific humidity in gram water vapour per gram of moist air at $z_1$ and $z_2$, $z_0$ is the roughness length of the evaporating surface and $d$ the displacement of the zero plane of wind velocity in relation to the earth surface.

Formally, equation (6) only holds under neutral conditions of the atmosphere. Under other atmospheric stratifications a correction will have to be applied. It appears, however, from data taken from literature (RIJTEMA, 1965) that a maximum deviation of approximately 15 % is present when calculating evapotranspiration with equation (6) under other atmospheric conditions.

Assuming an air pressure of 76 cm Hg and an air temperature of 20° C, subject to the boundary conditions:

$$z_2 = 200 \qquad u_2 = u \qquad e_2 = e_a$$
$$z_1 = d \qquad u_1 = 0 \qquad e_1 = e_s$$

equation (6) transforms into:

$$E = \frac{13.65}{\left[\ln\frac{200+z_0-d}{z_0}\right]^2}\, u(e_s - e_a) = \mathrm{f}\,(z_0,\, d) u(e_s - e_a) \tag{6a}$$

where $u$ is the wind velocity at 2 m height in m. sec$^{-1}$, $e_s$ and $e_a$ the vapour pressure in mm Hg respectively at the surface and at 2 m height.

Both the value of the zero plane displacement $d$ and the value of the rough-

ness length $z_0$ are dependent on the wind velocity. The effect of wind on the roughness function $f(z_0, d)$ gives an increase in roughness at low wind velocities due to the flutter of the leaves. At high wind velocities, however, the leaves adapt positions parallel to the direction of flow which results in a streamlining effect making the surface aerodynamically more smooth. The influence of wind velocity is likely to be higher for tall crops than for short ones. In the analysis of this effect for grass it was not possible, however, to study the effect of wind velocity on the roughness of the evaporating surface in its interrelationship with crop height. For this reason it is assumed that the combined effect of crop height and wind velocity on the roughness of the grass cover can be expressed as:

$$f(z_0, d) = g(l) \cdot h(u) \tag{7}$$

where $g(l)$ is a function of crop height with the same dimensions as $f(z_0, d)$ and $h(u)$ is a dimensionless factor depending on wind velocity. It is assumed that $h(u)$ equals unity at a mean wind velocity of 1.75 m. sec$^{-1}$ at 2 meter

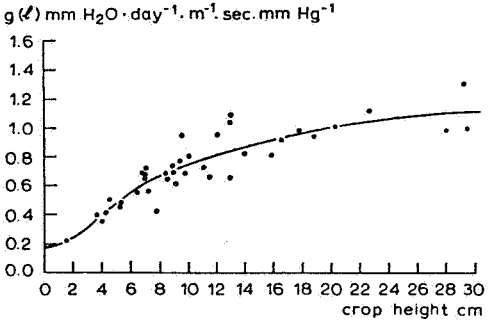


*Fig. 1.* The relation between $g(l)$ and height of grass for a mean wind velocity at 2 m height of 1.75 m.sec$^{-1}$.

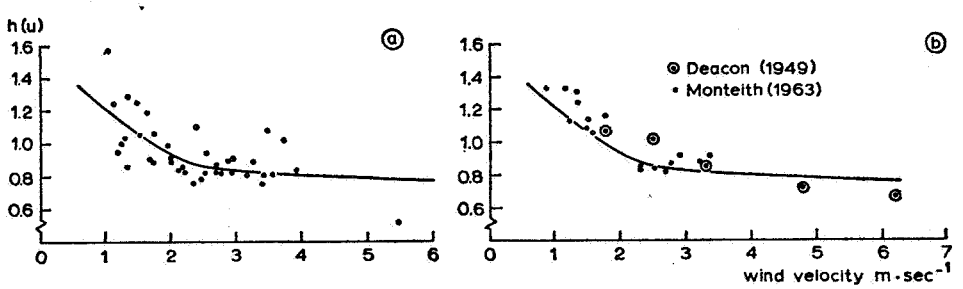


*Fig. 2.* The relation between $h(u)$ and wind velocity at 2 m height.
a. data obtained from an analysis of grass covered lysimeters at Wageningen; b. calculated from data for grass given by DEACON (1949) and by MONTEITH (1963). The curve originates from figure 2a.

height. The relation between grass length and $g(l)$ is presented in figure 1, for data reduced to a mean wind velocity of 1.75 m. sec$^{-1}$. The calculated values of $h(u)$ are presented in figure 2a in relation to mean wind velocity at 2 meter height. In figure 2b the same relation is presented for data, given by DEACON (1949) and by MONTEITH (1963), of grass with a mean crop height of 70 cm. The curve drawn in this figure is taken from figure 2a.

## 3. COMBINED AERODYNAMIC AND ENERGY BALANCE METHODS

The success of the energy balance method as well as the vertical vapour transport equation depends on the accuracy by which the vertical gradients of temperature, vapour pressure and wind velocity can be measured. In many hydrological investigations mainly only standard meteorological data are available, so both methods can hardly be used on a routine basis.

PENMAN (1948, 1956), neglecting the storage of heat below the evaporating surface combined the simultaneous equations:

$$LE = Lf(u)\ (e_s - e_a) \tag{8}$$

and

$$H_{nt} = LE + K \tag{9}$$

in order to obtain an equation for the calculation of the evaporation from a free water surface. Introduction a new variable $\Delta = \delta\varepsilon/\delta T$, the slope of the temperature-vapour pressure curve and assuming a saturated vapour pressure at the surface, PENMAN derived his well-known equation for the evaporation from a free water surface:

$$E_o = \frac{\Delta H_{nt}/L + \gamma\ E_a}{\Delta + \gamma} \tag{10}$$

where $E_a$ in the equation is given by:

$$E_a = 0.35\ (0.50 + 0.54\ u_{200})\ (\varepsilon_a - e_a) \tag{11}$$

A good agreement exists between measured evaporation from sunken pans and the data calculated from meteorological data using the Penman-equation. The calculated data are generally somewhat higher than the measured ones. The main source causing the systematic deviation seems to be the wind function used in the Penman-equation. The constants in this function were determined from a combination of his own experiments and the results of the Lake Heffner study (see PENMAN, 1956). Because of both the size and the exposure of the pans used in the lysimeter experiments at Wageningen, as well as the influence of

the immediate surroundings, the effect of the wind velocity may have been smaller than in the conditions under which the experiments reported by Penman were performed. A back solution with data obtained in 1957 and 1958 was applied in order to obtain a corrected function for the effect of wind velocity on pan evaporation. It appeared from this analysis that the function:

$$E_a = 0.182\ \gamma u_{200}\ (\varepsilon_a - e_a) \tag{11a}$$

gave the best fit of the data. A comparison between measured pan evaporation and the calculated data, using the modified wind function is presented in figure 3. The data give mean values for periods of 10 days, measured from April 1 to November 1 during the years 1960, 1961 and 1962.

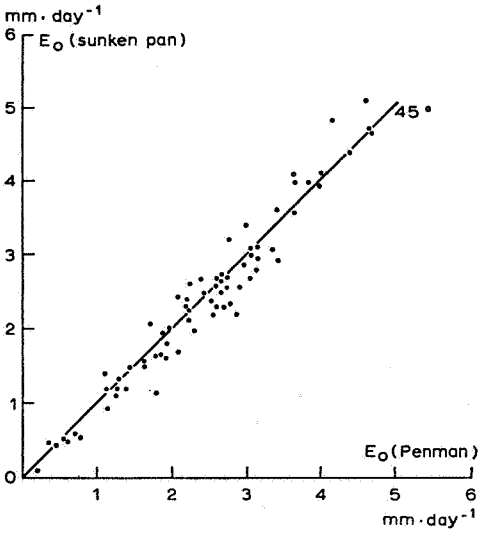


*Fig. 3.* Relation between the evaporation calculated after PENMAN with a modified wind function and data measured with sunken pans. Mean values for periods of 10 days from April to November in 1960, 1961 and 1962.

In order to calculate the potential evapotranspiration from a crop, PENMAN introduced an experimentally determined reduction factor $f$ by which the calculated evaporation from a free water surface is to be multiplied. However, a number of factors are giving the equation proposed by him a limited applicability in calculating the evapotranspiration from crops. These factors are neglect of the heat storage below the evaporating surface, as well as neglect of the vapour pressure deficit at non-saturated evaporating surfaces. Moreover, fixed constants have been assumed in the function of wind velocity, irrespective

of the type of surface and the surface roughness. Further, the difference in reflection of a crop and of a free water surface is not taken into account in this type of calculations. The values of the reduction factor $f$ only hold for a grass cover, cut very short like a lawn, under the climatological conditions of his experiments.

PENMAN and SCHOFIELD (1951) have tried to give an equation for this reduction factor $f$, based on plant physiological and physical arguments. Based on this theory, PENMAN (1956) concludes that the values of $f$ are mainly determined by the factor day-length, because of the stomatal closure during night-time. It is shown, however, in another study (RIJTEMA, 1965) that the factor day-length is not an important factor in determining the values of $f$, but that these values have to be explained by the difference in reflection between both surfaces. The very short grass cover used in the experiments performed by PENMAN, has a surface roughness which is nearly equal to the corresponding value of a free water surface, so for both surfaces the same wind function can be used. In order to get an impression of the significance of the difference in reflection, the ratio $E'_p/E_o$ was calculated for monthly values, using the Penman-equation (10) for the calculation of the evaporation from a free water surface ($E_o$), as well as for the calculation of the corresponding value ($E'_p$) from a freely evaporating surface with the same reflection as grass. The ratio $E'_p/E_o$ in relation to $E_o$ is presented in figure 4. The result leads to the conclusion that the reduction factor $f$ is mainly determined by the difference in reflection

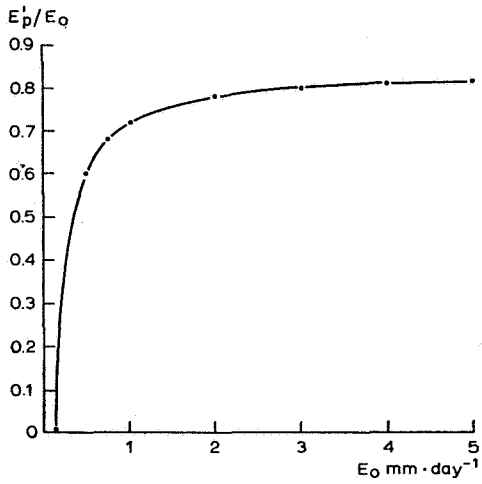


*Fig. 4.* Relation between the ratio $E'_p/E_o$ and the calculated evaporation from a free water surface ($E_o$).

of the two surfaces. That the factor day-length has little influence on the value of the reduction factor, agrees also with the small amount of energy available for vaporization during the night. This means that stomatal closure during the night has only a small influence on evapotranspiration.

The procedure used by PENMAN for the combination of the equations (8) and (9) holds only when the vapour pressure at the evaporating surface is saturated. Moreover, crops may not be considered as a horizontal flat evaporating surface, when surface properties have to be taken into account. Under these conditions a deviation of the real geometry of the diffusion process appears, depending on the type of crop. Due to the modification in the real geometry of the evaporating surface, it is necessary to introduce an apparent diffusion resistance $R_c$ of the crop. This factor is dependent on both stomatal opening and the number of stomata, as well as on the type of crop and on the covering of the soil. There is some indication, however, presented by MARLATT (1961) that row crops covering more than 50 % of the soil act as a full cover crop in relation to evapotranspiration, this being due to the interactions of the micro-climate within the rows. The evaluation of this apparent diffusion resistance has to be performed experimentally. An equation for evapotranspiration, taking into account the introduction of the apparent diffusion resistance, the reflection coefficient and the roughness of the evaporating crop was given in a previous study (RIJTEMA, 1965). The equation for evapotranspiration can be written as:

$$E = \frac{\Delta H_{nt}/L + \gamma E'_a}{\Delta + \gamma \{1 + f(z_0, d)\, u R_c\}} \tag{12}$$

where $H_{nt}$ has to be calculated using the reflection coefficient of the crop considered and $E'_a$ is given by the expression:

$$E'_a = f(z_0, d)\, u\, (\varepsilon_a - e_a) \tag{13}$$

## 4. INTERCEPTION AND EVAPOTRANSPIRATION

Part of the precipitation remains on the leaf surface and does not reach the soil. Particularly under conditions when, due both to low light intensity and to a shortness of available moisture in the root zone, a strong reduction in transpiration is present, the interception increases evapotranspiration in comparison with a dry crop under the same conditions. The amount of water remaining at the leaf surface depends highly on the amount of precipitation

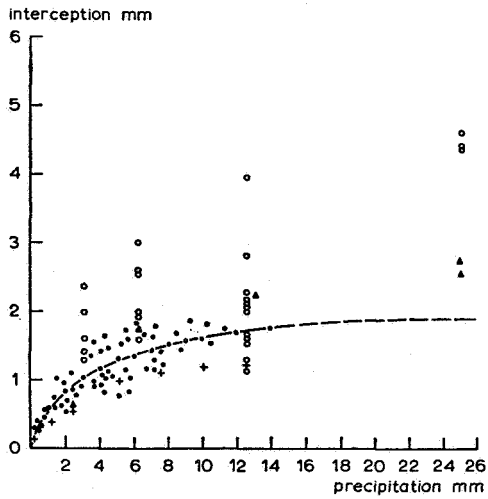


*Fig. 5.* Relation between interception and precipitation for grass o CLARK (1940); Δ BEARD (1962); + BURGY and POMEROY (1958); • RIJTEMA (1965).

as well as on the type of the crop. The relation between interception and precipitation for a grass crop is given in figure 5. Data reported by CLARK (1940), BURGY and POMEROY (1958) and BEARD (1962) are also given in this figure. The generally higher values given by CLARK concern field experiments

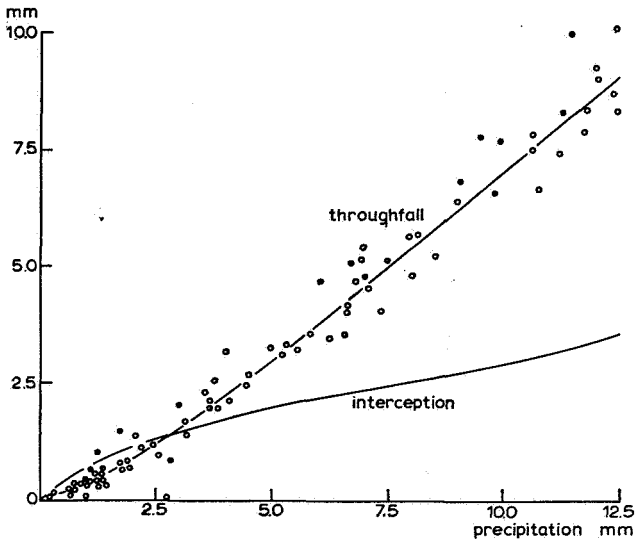


*Fig. 6.* Relation between throughfall, interception and precipitation for Pinus ponderosa after ROWE and HENDRIX (1951).

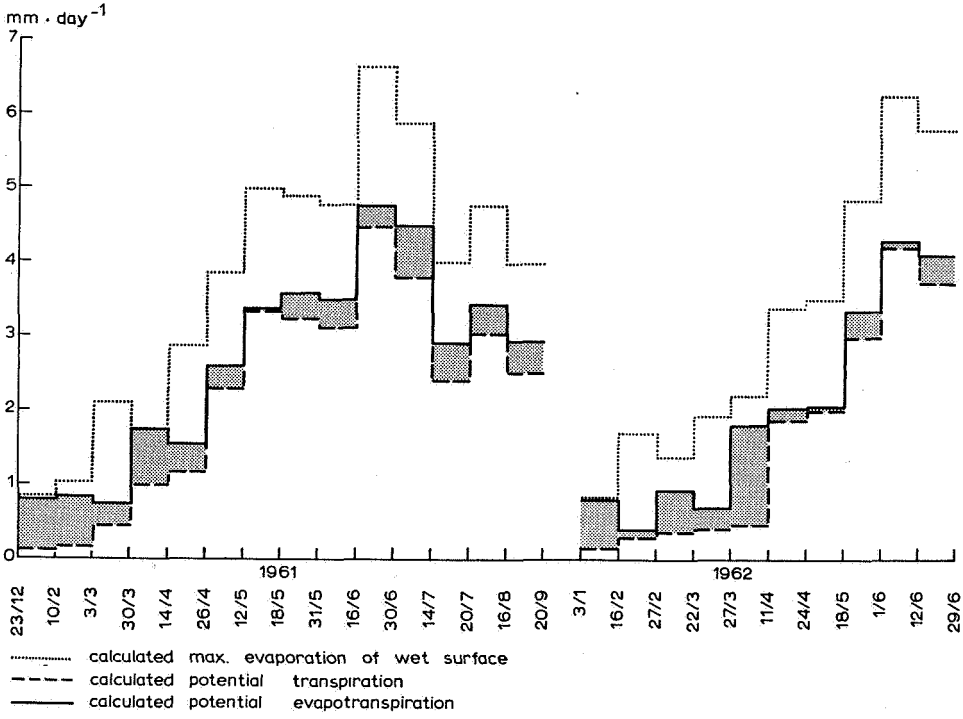


*Fig. 7.* The course of the maximum evaporation of a wet surface, the potential transpiration and the potential evapotranspiration for Pinus nigra austriaca L.

with prairy grasses. Possibly the type of the crop as well as the experimental conditions may have affected the data obtained in these experiments. The relation between precipitation and interception for Pinus ponderosa, according to ROWE and HENDRIX (1951) is presented in figure 6.

The transpiration from wet leaves is, under all conditions of evapotranspiration, smaller than the corresponding value from dry leaves. The energy available for vaporization is partly used for evaporation of the intercepted precipitation, which reduces the amount of energy available for transpiration. Moreover, transpiration from wet leaves is reduced by the increased water vapour content in the micro-climate of the crop causing a reduction in the vapour pressure gradient between the sub-stomatal cavities and the leaf surface.

The assumption, that under conditions of a complete soil cover the evaporation coming directly from the soil surface can be ignored, offers a possibility to take the effect of interception on evapotranspiration into account (RIJTEMA,

1965). The expression for evapotranspiration can under these conditions be written as:

$$E = \frac{\Delta H_{nt}/L + \gamma \{E'_a + f(z_0, d)\, u\, R_c E_I\}}{\Delta + \gamma\{1 + f(z_0, d)\, u\, R_c\}} \tag{14}$$

where $E_I$ is the amount of intercepted precipitation given in mm. day$^{-1}$.

The distribution of the rain-showers has to be taken into account in the calculation of the mean interception during a balance period. Particularly in periods with much precipitation, when the crop does not become dry between the successive showers, the evaporation term $E_I$ is difficult to determine exactly.

Since equation (14), assuming $R_c$ equals zero, also holds for the evaporation from a wet surface with the same aerodynamic and reflective properties as the crop considered, the evapotranspiration calculated with equation (14) may not exceed the evaporation from the wet surface.

Data given by RYHINER and RIJTEMA (1963), concerning the potential evapotranspiration from Pinus nigra, obtained from the lysimeters at Castricum, are given in figure 7. The dotted line in this figure gives the evaporation of a continuously wetted surface with the same properties as the Pinus nigra. The broken line gives the potential transpiration of the crop when the leaf surface is dry. The increase in potential evapotranspiration, due to the intercepted precipitation is also given.

## 5. THE APPARENT DIFFUSION RESISTANCE OF CROPS

In paragraph 3 the apparent diffusion resistance of a crop was introduced in relation to both the geometry of the evaporating surface and the stomatal

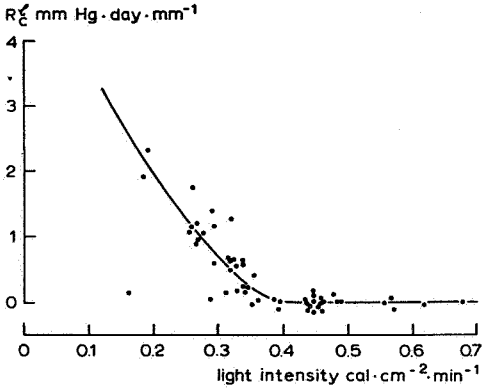


*Fig. 8.* The relation between the apparent diffusion resistance of grass ($R_c^l$) and mean light intensity.

opening. It appeared from the data given by RYHINER and RIJTEMA (1963) that the apparent diffusion resistance of Pinus nigra is under conditions of optimum water supply and high light intensity, not equal to zero. For this crop both the geometry of the evaporating surface and the number of stomata, as well as the possible internal resistances, reduce the transpiration of the crop under optimum conditions of water supply.

Lysimeter experiments with grass under conditions of high light intensity and optimum water supply, performed by BURGY and POMEROY (1958), in which the crop surface of one of the lysimeters was wetted continuously, do not show a systematic deviation in evapotranspiration from the data obtained from a lysimeter with a dry crop. These data indicate that

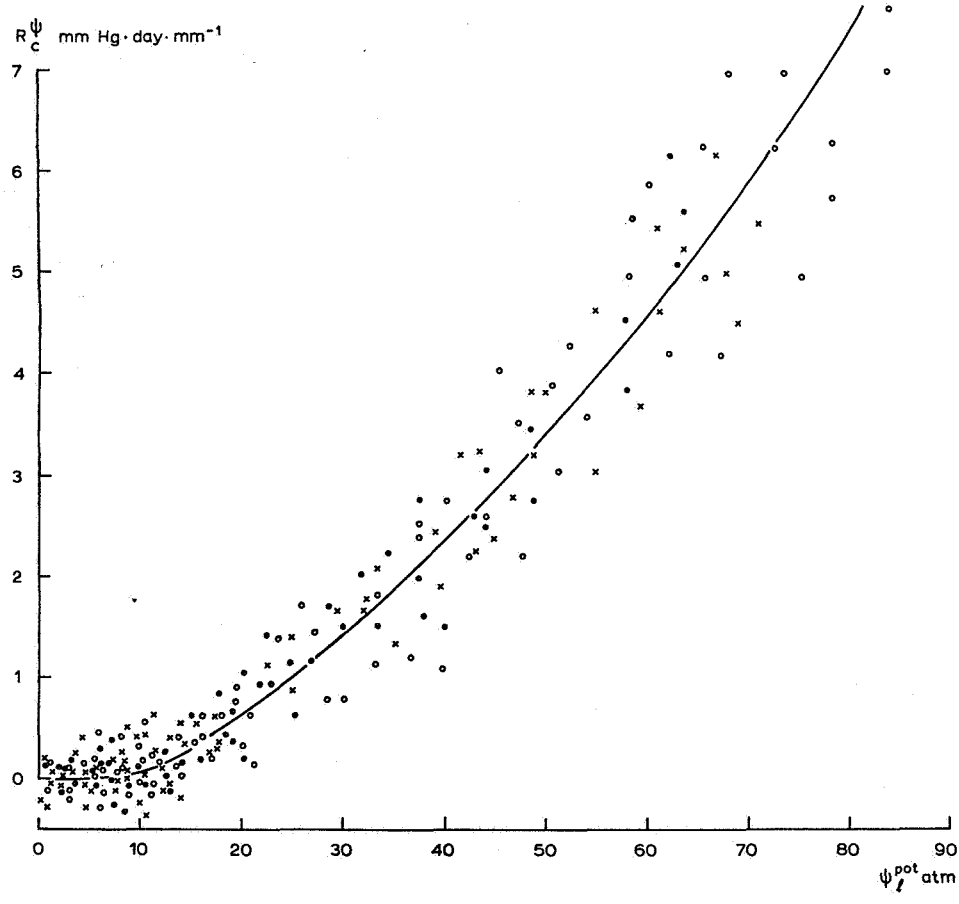


*Fig. 9.* The relation between the apparent diffusion resistance of grass and the potential suction in the leaf tissue ($\psi_l^{pot}$). The data are calculated for grass on loamy sand (•), sticky clay (o) and peat (x).

the apparent diffusion resistance of the grass equals zero. An analysis of the data obtained from the grass covered lysimeters at Wageningen shows a good relation between the apparent diffusion resistance $R^l_c$ (see (16)) and the mean light intensity during the balance period, as is presented in figure 8. The resistance reaches its minimum value, equal to zero, at a mean light intensity of 0.38 cal. cm$^{-2}$ . min.$^{-1}$. This light sensitive reaction, under conditions of optimum water supply, seems to be related essentially to photosynthesis.

The presence of too small a water supply to the roots for potential transpiration of the crop, results in an increase of the apparent diffusion resistance $R^{\Psi}_c$, which is related to the increase in suction in the leaf tissue. It is shown (RIJTEMA, 1965) that the value of $R^{\Psi}_c$ (see (16)) is dependent on the potential transpiration ($E_T^{pot}$), the suction ($\Psi$) and the capillary conductivity ($k$) in the root zone, the geometry of the root system ($b$) and the resistance for liquid flow in the plant ($R_{pl}$). The relation can be given by the following expression:

$$R^{\Psi}_c = g(\Psi_l^{pot}) = g\left\{E_T^{pot}\,(R_{pl} + b/k) + \Psi\right\} \tag{15}$$

Mean values of $R_{pl}$ and $b$ for the grass on the lysimeters were respectively 1042 cm . day . mm$^{-1}$ and 0.47 cm. The relation between $R_c^{\Psi}$ and $\Psi_l^{pot}$ is given in figure 9.

It is assumed that the combined effect of the light sensitive reaction and the suction reaction on the value of $R_c$ can be expressed as:

$$R_c = R^l_c + R^{\Psi}_c \tag{16}$$

where $R^l_c$ is the factor depending on light intensity and $R_c^{\Psi}$ the factor giving the effect of the suction in the leaf tissue on the value of $R_c$. Under these conditions the relation between real transpiration and potential transpiration can be given as:

$$E_T^{re} = \frac{\Delta + \gamma\left\{1 + f(z_0, d)\, u\, R^l_c\right\}}{\Delta + \gamma\left\{1 + f(z_0, d)\, u\, (R^l_c + R^{\Psi}_c)\right\}} E_T^{pot} \tag{17}$$

It appears from this expression that the influence of the suction on the reduction of transpiration, at a given value of both potential transpiration and the suction in the root zone, also depends on temperature, on wind velocity, on light intensity and on the roughness of the crop. The reduction in transpiration due to a shortness of water increases with increasing surface roughness

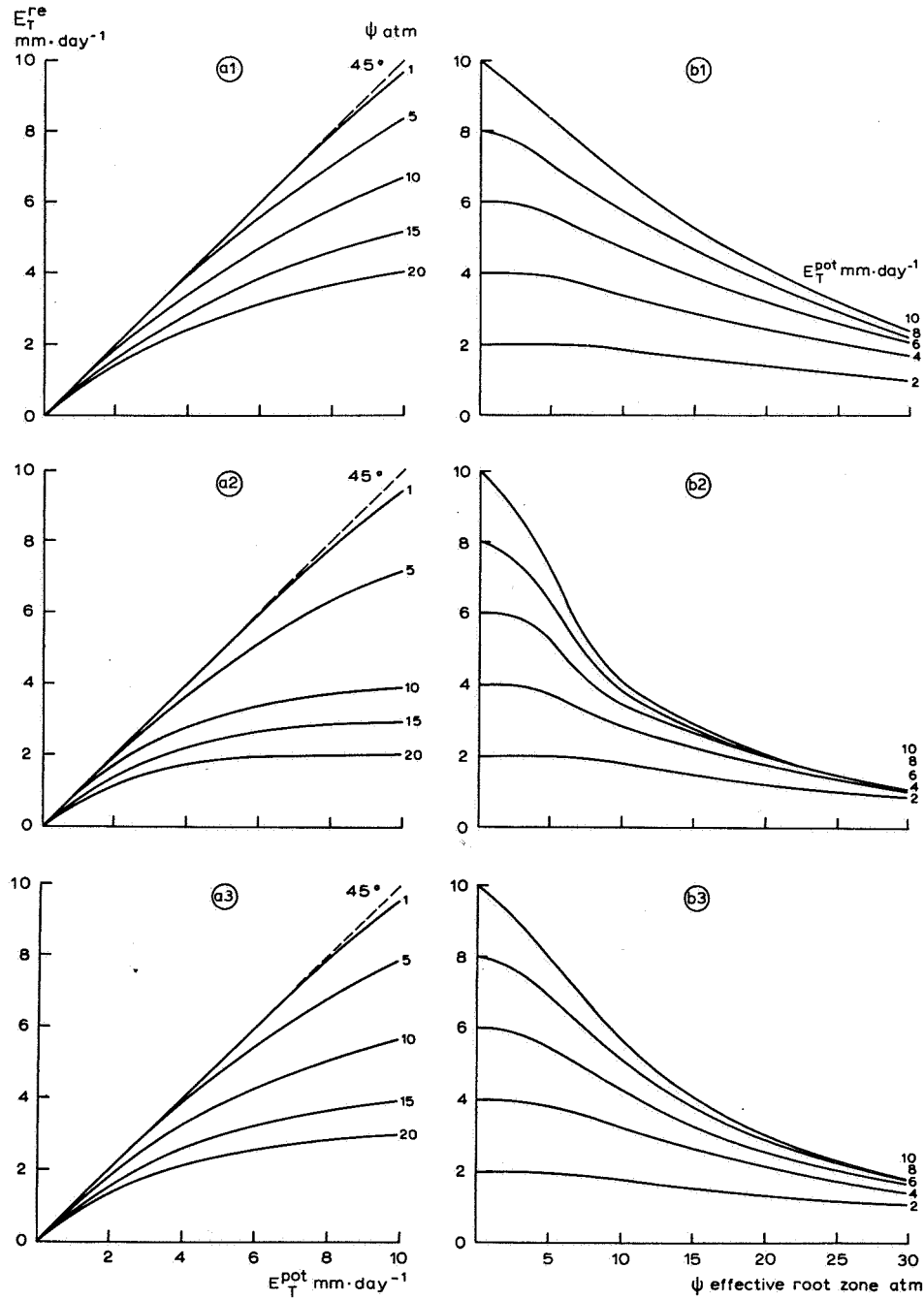


*Fig. 10.* a. The relation between real transpiration and potential transpiration for various values of the suction in the effective root zone. 1. loamy sand; 2. sticky clay; 3. peat
b. The relation between real transpiration and mean suction in the effective root zone for various values of potential transpiration. 1. loamy sand; 2. sticky clay; 3. peat

and increasing wind velocity, whereas it decreases with increasing temperature.

The relation between real transpiration, potential transpiration and mean suction in the effective root zone of the grass is given in figure 10 for the loamy sand, the sticky clay and the peat soil present in the lysimeters. These relations are calculated for an air temperature of 20° C, while $f(z_0, d)\,u$ and $R^l_c$ have assumed values of respectively unity and zero. Figure 10 shows that the reduction in transpiration at a given value of both potential transpiration and mean suction, also depends on the capillary conductivity of the soil.

The course of the potential evapotranspiration of Pinus nigra, according to data given by RYHINER and RIJTEMA (1963) is presented in figure 11. In the same figure also the data of calculated real evapotranspiration, as well as the real evapotranspiration determined with the water balance equation are given. The figure shows that a reasonable agreement exists between both methods. It appears from figure 11 that the potential evapotranspiration of Pinus nigra exceeds the evaporation of a free water surface, calculated with the Penman-formula.

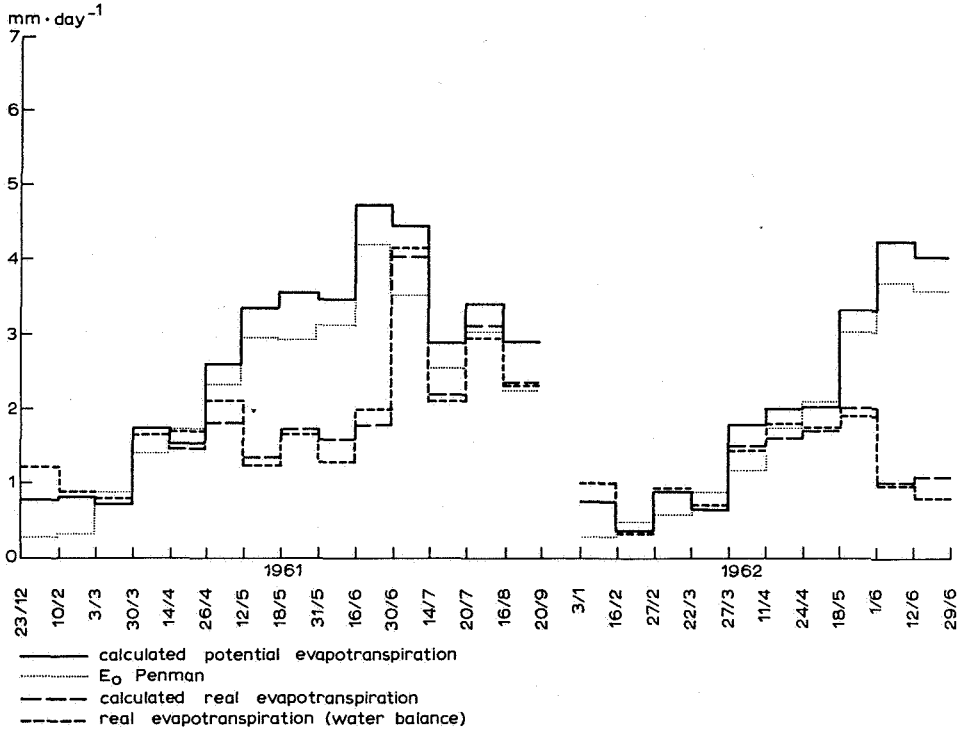


*Fig. 11.* The course of potential evapotranspiration and real evapotranspiration from Pinus nigra austriaca L.

## 6. Summary

A discussion is given of the main factors determining the actual evapotranspiration from crops in their interrelationships with each other.

The empirical reduction factor $f$ for short grass is mainly determined by the difference in reflection between a short grass surface and an open water surface. The factor day-length has little influence on the value of the reduction factor.

For the calculation of the transpiration of crops a new factor was introduced: the apparent diffusion resistance of the crop. This factor takes into account the geometry of the evaporating surface, as well as the variation in stomatal resistance under influence of light intensity and of the suction in the leaf tissue.

Actual evapotranspiration is calculated with a combined aerodynamic and energy balance approach, taking into account the reflection coefficient of the crop, the surface roughness in relation to crop height and to wind velocity, and the apparent diffusion resistance of the crop.

It is shown that the amount of precipitation intercepted by the crop increases evapotranspiration when the apparent diffusion resistance is not equal to zero. An approach to the evaluation of this effect is given.

The value of the apparent diffusion resistance depends on both light intensity and on suction in the leaf tissue. It appeared that the reduction in transpiration, due to a shortness of water also depends on temperature, on wind velocity and on the roughness of the crop.

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# V. WATER BALANCE AND WATER BOOKKEEPING OF REGIONS

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## 1. Introduction

The Ministry of Public Works is carrying out an investigation on evaporation in the polder Rottegat (province of Groningen) since the autumn of 1951. Several institutes have joined in a Working Party on this and similar research and contribute their part in a yearly report, edited by the ministry.

The elaborate and detailed material collected in the course of so many years is valuable in checking theories and models on evapotranspiration. In 1957 the calculation was started of actual evapotranspiration of the experimental area in the polder (25 x 25 $m^2$ in quadruplicate) and of the whole polder as well (86 ha) (Makkink, 1958). The intention was to use only values of local precipitation, free water evaporation ($E_0$), depth of water table, information on crops in space and time (species, height, maturity a.s.o.) and constants on soil and crops for the computation.

All lacking terms of the water balance-sheet can be found from these values, viz. the actual evapotranspiration, run-off, infiltration from the ground water, and changes in the water content of the soil. Since actual evapotranspiration is also calculated from observed values of run-off, infiltration and changes in water content of the soil in monthly or half-monthly periods, the calculated values can be checked, enabling the improvement of the model.

The computations for the years 1961 and 1962 are not reported, because they were rather rainy and for a number of processes in our digital model not critical; therefore 1959 and 1960 were used, the first being very dry in summer.

## 2. Approach

Our aims concern values of the following magnitude: actual evapotranspiration in period $i$ ($E_i$), run-off to the ground water during a short period $i$ ($A_i$), infiltration into the soil from the ground water in period $i$ ($I_i$) and the moisture content of the soil at the moment $i$ ($V_i$). These 4 magnitudes will be called aim-magnitudes. They are calculated according to the following equations:

$$E_i = f_1 (bv_{i-1}, E_{Pi}, B_i) \quad (1)$$

$$A_i = f_2 (V_{i-1}, N_i, \Delta P_i) \quad (2)$$

$$I_i = f_3 (V_{i-1}, \overline{P}_i) \quad (3)$$

$$V_i = V_{i-1} + N_i + I_i - A_i - E_i \quad (4)$$

where $i$–1 is the preceding short period, $N$ precipitation, $\triangle P$ change of water table, $P$ average water table, $B$ maximum water content within reach of the evaporative potential of the atmosphere with or without mediation of vegetation and without water risen from the ground water, $E_p$ potential evapotranspiration of soil and vegetation, $b_V$ actual quantity of water available for evapotranspiration within the zone of $B$, $f_1$, $f_2$ and $f_3$ functions which will be described in more detail below.

The 4 aim-magnitudes occur in 4 equations in which all other magnitudes

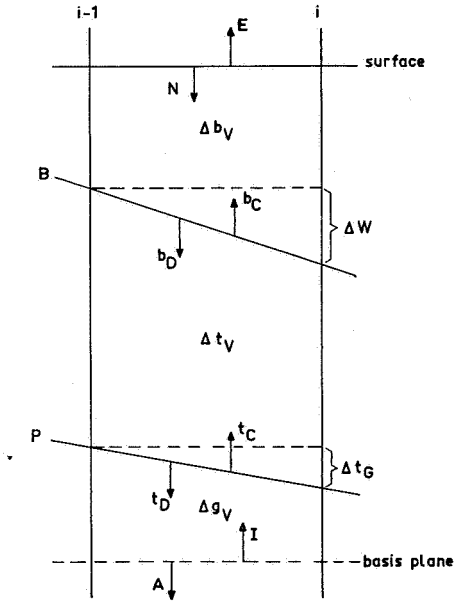


*Fig. 1.* Scheme of the water balance in period i in a soil block of constant depth, with chaning root extension and changing water table. $B$ = level under the maximum available water, $P$ = phreatic level, $E$ evapotranspiration, $N$ precipitation, $\Delta b_v$ change in water content within reach of evaporation with or without mediation of roots, $\Delta W$ correction for water due to root growth, $b_D$ water percolating out of the evapotranspiration zone, $\Delta t_V$ changes in water content in the transito zone, $t_C$ capillary infiltration into transito zone, $\Delta t_G$ correction for water due to changes in water table, $t_D$ water percolating into ground water, $\Delta g_V$ change in water content in groundwater zone, $I$ infiltration into balance volume, $A$ run-off from balance volume.

are either measured ($N_i$, $\triangle P_i$, $\overline{P}_i$) or easy to calculate ($E_{p_i}$), easy to estimate ($B_i$) or known from the preceding period ($V_{i-1}$, $b_{V_{i-1}}$). The initial value $V_0$ is easy to estimate if the first period is chosen at a convenient moment.

The top soil is considered a storage and defined as the layer from the surface up to a depth where no more variations in water content take place, i.e. the deepest level ever reached by the water table.

This storage is emptied by evaporation with or without mediation of the vegetation, but only in so far the roots or the atmosphere can achieve the required potential gradient for capillary depletion, without completion by the ground water. This means that evapotranspiration is limited by the water content of the zone of root activity or the zone of evaporation if living roots are absent (fig. 1).

The storage is refilled by precipitation from above and by infiltration of ground water from below, and by ground water in the zone in which water table varies.

The capacity of the unit of soil volume of storage is capable of changing by dehydration and rehydration, since the soil is a heavy clay soil with 56 % of particles smaller than 16 micron. Because percolation and refilling are dependent on the capacity of the soil, dehydration and rehydration are taken into account on the basis of a special study (MAKKINK and VAN HEEMST, 1965).

Since actual evapotranspiration under field conditions is a non-stationary process, time is divided into short periods of 1 to 5 or 6 days, 2 periods of 1 day located at the end of each balance sheet period of the measurements. The water economy of the soil is a complicated phenomenon in which several processes occur, for a great part simultaneously. Since the results can only be calculated sequentially, the whole phenomenon is treated as a succession of separate elementary processes, while it is decided beforehand which is the most natural and/or the most convenient order of sequence.

## 3. ELEMENTARY PROCESSES AND THE ADOPTED SEQUENCE

### a. *Percolation*

Distinction is made between rapid percolation ($D_r$), occurring through cracks and root holes, and slow or normal percolation ($D_n$), gradually filling the soil in downward direction upto field capacity.

Although a soil with cracks and root holes is not homogeneous, it may be assumed that their distribution is homogeneous on 4 areas of 25 m² or on an area of the whole polder, so that percolation consists of the two kinds of percolation mentioned. Rapid percolation is treated before the slow one. A selection of suitable days with rain – no rainy days immediately before and

after it – showed a relationship between precipitation and drainage measured on the experimental fields (fig. 2), being approximately rectilinear according to the equation:

$$D_r = 0.1\,N - 0.5 \text{ mm} \tag{5}$$

In this study the relationship between rate of percolation or drainage and rainfall, tile distance and height of the water table is not used, because the bookkeeping method makes its use superfluous.

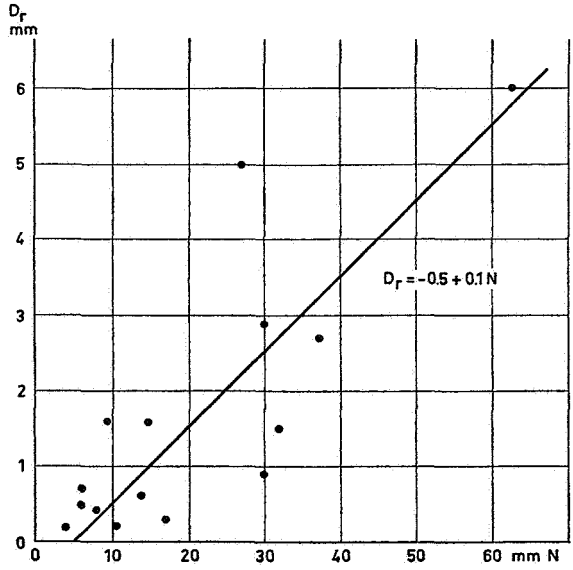


*Fig. 2.* Rapid percolation $(D_r)$ as a function of precipitation $(N)$.

b. *Root growth*

Root growth, root dying and decay is treated before evapotranspiration, expressed in mm's of maximally available water, according to the function

$$B = f_4(t) \tag{6}$$

where $t$ is time. This function is estimated from summer depletions in the soil concerned and information about the state of the cultivated crop(s) (fig. 3).

c. *Evapotranspiration*

The phenomenon of evapotranspiration is considered a process of drying of the soil by the plant and treated in two alternative ways. In a simple model it is assumed that if the available quantity of water within reach of the roots or atmosphere allows potential evapotranspiration, evapotranspiration is potential and therefore easy to calculate. If the quantity of available water within reach

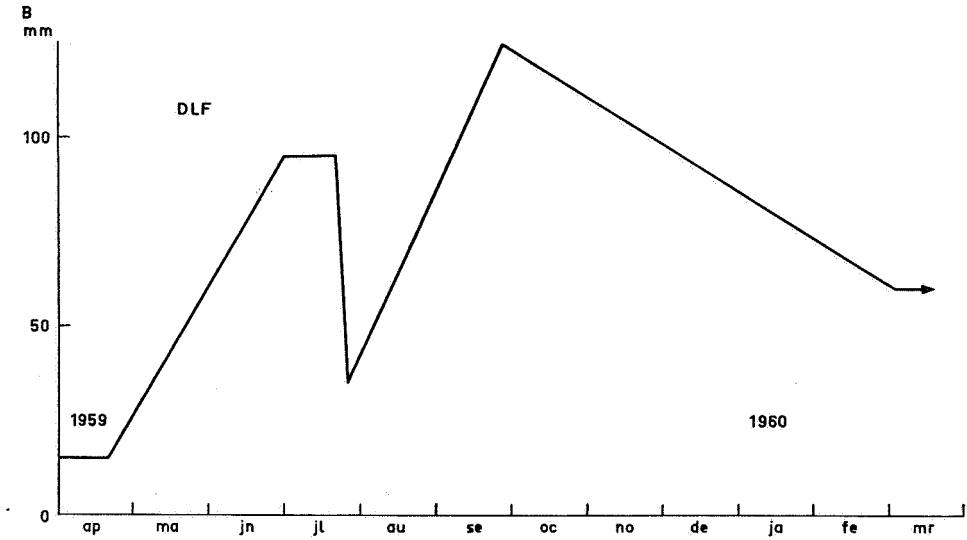


*Fig. 3.* Maximum of available water *(B)* within reach of roots or evapotranspiration resp. as a function of time (1959—1960)

of the roots, however, is not potential, all the available water is allowed to de depleted.

The alternative model is based on the assumption that the depletion of soil is a gradually decreasing process, occurring according to the technological formula for the drying of a thin layer of material. The soil water within reach of the roots is present in a layer which is considered thin because it covers the complicated system of active roots with a thin layer. The following equation is used

$$-dV/dt = k \cdot V/B \qquad (7)$$

where $V$ is the quantity of available water within reach of the roots, and $k$ a constant, supposed to represent the value of the potential evapotranspiration rate if $V/B = 1$. The spacial distribution of water is neglected in the range of the roots. Although no special attention was paid to the influence of the aperture of the stomata on transpiration, it is manifested in the rectilinear relation between evapotranspiration rate (drying rate) and the relative available water content in the soil. This relation reflects the regulation ability of stomata.

It is difficult to work with the stomatal aperture in a crop, because measurements have not been and could not be made in the field. It was impossible to find a mean value for stomatal aperture (or stomatal resistance) over a period of about 5 days, since it varies with the water content of the leaves and the light intensity. Therefore, it also varied with the angle of light incidence on the leaf, being a function of the inclination of the sun, and the

position of each individual leaf. Light intensity varies also according to the depth in the vegetation. Since all values vary continually during the day, it is difficult to find a reliable integrated average value.

Integration of eq. 7 and adaptation to the conditions of the model gives the following equation

$$E_i = (b_{V_{i-1}} + N_{r2,i} + \Delta W_i + b_{C_{i-1}}) \cdot (1 - \exp(-E_{p,i}/(B_{v,i-1} + \Delta B_{v,i})) \qquad (8)$$

where $E$ is the actual evapotranspiration, index $b$ denoting only water $V$ present within reach of the roots and atmosphere, $\triangle W$ representing the quantity of water, available by root growth in period $i$ (not always identical with $\triangle B$, because the roots can penetrate a layer depleted by a former crop and not completely refilled by percolation water), $b_{C_{i-1}}$ the quantity of capillary rise into the root zone from the preceding period, $E_p$ the potential evapotranspiration, $B_v$ the capacity of the soil within reach of the roots and atmosphere, corrected for dehydration and rehydration and $\triangle B_v$ the increase of this capacity in the period in question.

Whether the formula based on a simple rectilinear relationship between drying rate and concentration holds in all cases is not certain. There are, however, cases to which it does apply (MAKKINK and VAN HEEMST, 1964) (fig. 4).

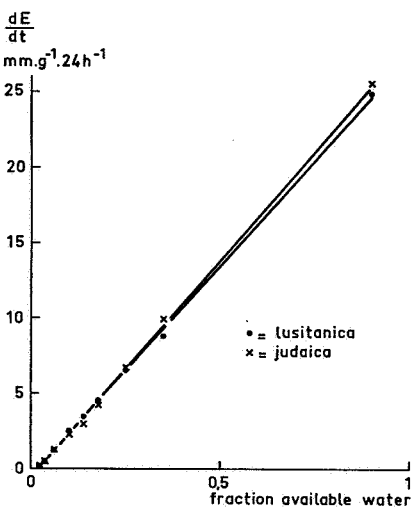


*Fig. 4.* Evapotranspiration as a function of the fraction of available water in the soil, for two varieties of Dactylis glomarata according to data of PERRIER, MCKELL and DAVIDSON

Potential evapotranspiration is calculated according to

$$E_{p,i} = g_i \cdot E_\pi \tag{9}$$

$g_i$ representing a crop factor and $E_\pi$ the potential evapotranspiration of a short grass crop covering the soil completely and optimally provided with water. This magnitude is deduced for the locality from the subsequent magnitude at Wageningen by a locality factor $(E_{0,loc.} / E_{0,Wag.})$
$E_{\pi,Wag.}$ is calculated according to a simplified formula

$$E_{\pi\ ,Wag.} = 0{,}83 \{\triangle / (\triangle + \gamma)\} R_m/L - 0{,}50 \tag{10}$$

$R_m$ is the total global radiation per month, $L$ the latent heat of water, $\triangle$ increase in maximum vapour pressure in mm Hg per °C, $\gamma$ psychrometric constant.

The crop factor is a function of time

$$g_i = f_5(t) \tag{11}$$

and estimated from the crop condition (height, density, wetness due to precipitation) (fig. 5).

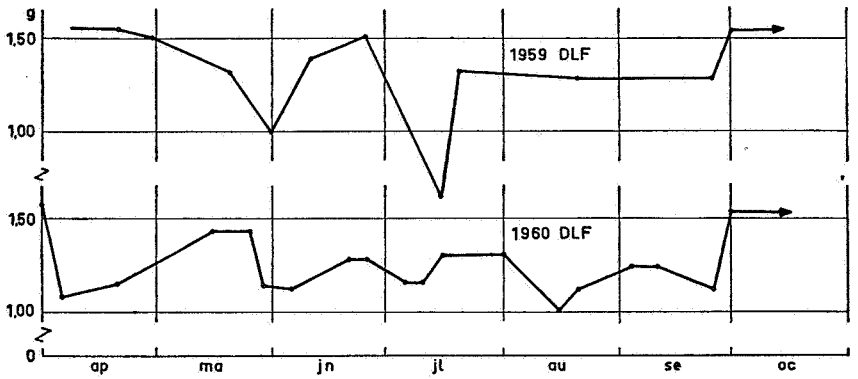


*Fig. 5.* Crop factor of flachs in 1959 and grassland afterwards on the drainage-lysimeterfield

d. *Infiltration from below*

Since depletion from above creates the required capillary gradient for infiltration from below, the calculation of the latter follows. The function $h = f(\log \Psi, C)$ for unsaturated flow was graphically transformed for 2 clays of WIND (1955) and WESSELING (1957) into a function $\log C = f(\log h, \log \Psi)$ (fig. 6). Here $h$ is the height above the water table and $\Psi$ the capillary potential at height $h$ (MAKKINK, 1962). The line for $\log \Psi = 4.2$ is approximated by

$$C = K \cdot h^{-n} \tag{12}$$

where $K$ is a constant and $n$ the constant of the authors mentioned with a

value of 1.5 – 2.0 for clays. Since $\Psi$ was not measured, $C$ was calculated with equation (12) only. This applying to $\log \Psi = 4.2$ only, a supposed depth was calculated in cm to which all water available to the plant roots is depleted. This was calculated by cumulative addition of the sum of the depletion from period to period. Because partly refilling by precipitation does not alter the potential at the bottom of the depleted zone, refilling due to precipitation, was neglected until it equated total depletion (fig. 7). The depth of the assumed wilting point was found by converting the depleted mm of water into cm of soil depth with a value derived from the pF-curve of the soil.

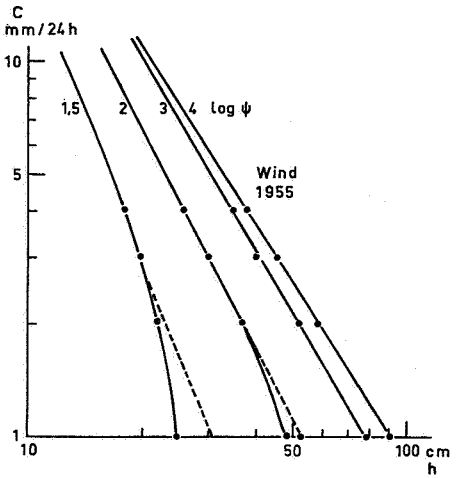


*Fig. 6.* Capillary rise *(C)* as a function of height above water table *(h)* and pF (= log $\psi$) on logarithmic scales for a heavy basin clay soil according to WIND (1955)

Value $h$ now equals the difference between depth $P$ of the water table and depth $d$ of depletion. The constant $n$ was provisionally put at 1.8. The value 2800 was graphically found for $K$. Calculation showed, however, that $I$ became much too high. Therefore the value 1.35 mentioned by RIJTEMA (1965) with a corresponding value of 150 estimated for $K$ was tried, which gave the results mentioned in paragraph 6.

The capillary rise is averaged for a period of 5 days. In figure 7 the slope of the line, indicating the water front from below, suggests a constant rate throughout the period the soil is not refilled. For each short period capillary rise is actually calculated according to the prevailing values of vacancy and depth of the water table.

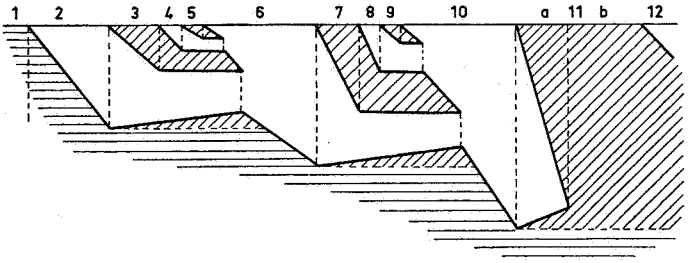


*Fig. 7.* Scheme of drying and refilling of a soil. In odd-numbered periods the soil is refilled by rain, supposedly moving from the top downwards (rapid percolation through cracks and holes excluded). In period 1 and 11b normal drainage occurs. In even-numbered periods depletion by evapotranspiration occurs, supposedly progressing from the top downwards. So long as the refilling water does not contact deeper soil water, capillary infiltration from below is supposed to occur. Horizontally shaded: water-formerly present ("old" water), obliquely shaded: refilling water ("new" water).

e. *Variation of the water table*

Variation in the depth of the water table in period $i$ is taken into account, as well, assuming that the water in the profile is in hydrostatic equilibrium at the beginning as well as the end of the period. The intake and release quantity of water ($Z$), corresponding to the change in the water table between the levels $P_{i-1}$ and $P_i$, is calculated according to MAKKINK (1962) with

$$Z = f_Z \cdot (P_i^z - P_{i-1}^z) \text{ *} \tag{13}$$

where $f_Z$ and $z$ are constants to be estimated. In this approximating model hysteresis is neglected in taking one $Z$ for intake from and release to the ground

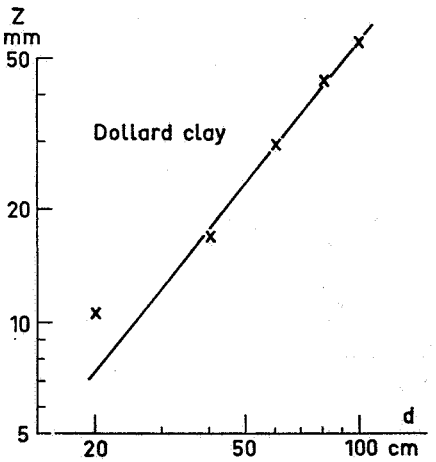


*Fig. 8.* Quantity of water released (Z) from clay soils as a function of the depth of the water table below the surface, according to data of VAN HOORN (1960)

*) In this equation $P_i$ and $P_{i-1}$ should be replaced by $P_i - D_{M,i}$ and $P_{i-1} - D_{M,i-1}$ resp. with $D_M$ for the depth of the vacancy in the evapotranspiration zone.

water. The values for $f_Z$ and $z$ were taken from a curve of Dollard clay according to data of VAN HOORN (1960) (fig. 8).

f. *Dehydration and rehydration*

The last phenomena taken into account are those of dehydration and rehydration. Dehydration concerns water that can only be reabsorbed at a very slow rate after being depleted. Only a part of the depletion can be immediately refilled. The maximum quantity of this immediately reabsorbable water is a constant fraction of the maximal depletion, after very much drying somewhat less (fig. 9) (MAKKINK en VAN HEEMST, 1965). For this fraction has been found for the Rottegat-clay a value of 0.61.

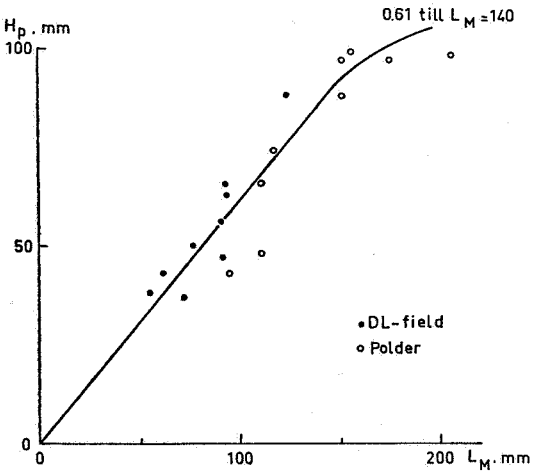


*Fig. 9.* Maximum quantity of water that can immediately be absorbed by a dried soil ($H_p$) as a function of the maximum water deficit in summer ($L_M$) in the clay of the polder Rottegat

The equation used is

$$S_{v,i} = f_{Sv} \cdot L_{M,i} \tag{14}$$

where $S_{v,i}$ is the dehydrated field capacity of the soil maximally depleted by $L_{M,i}$ mm water, and $f_{Sv}$ being the factor of immediate resorbability.

Rehydration occurs with a constant rate; it was found to be 0.0036 mm . day$^{-1}$ . mm$^{-1}$ for the clay concerned (fig. 10). Therefore the rehydration rate for the whole profile depends on the quantity of refilling water present after the moment of maximal depletion. This quantity can be calculated in every short

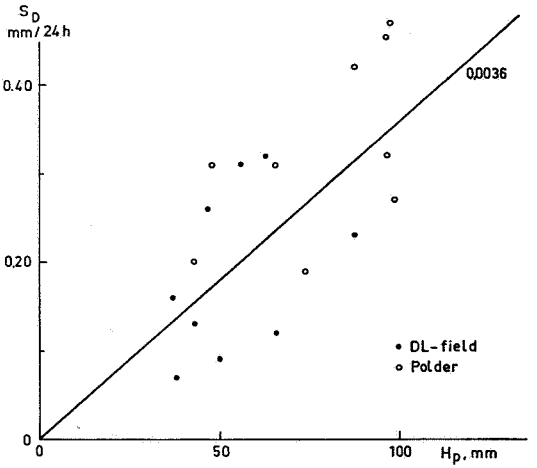


*Fig. 10.* Rate of rehydration of the whole profile in the drainage period ($S_D$) as a function of the maximum amount of water that can immediately be absorbed ($H_p$) in the clay of the polder Rottegat

period by multiplying the total water reabsorbed by the profile since the moment of greatest depletion, with the rate of rehydration per mm per short period

$$S_{v,i} = S_{v,i-1} + \triangle S_{v,i} \tag{15}$$

$$\triangle S_{v,i} = n \cdot S_{hy} \cdot (L_{M,i} - \overline{L}_i) \tag{16}$$

where $\triangle S_{v,i}$ is the quantity of water rehydrated in period $i$ and increasing the field capacity, $n$ the number of days in period $i$, $S_{hy}$ the rehydration rate in mm . day$^{-1}$ . mm$^{-1}$, $\overline{L}_i$ is actual average depletion in period $i$, $L_{M,i}$ the maximum depletion in summer, $L_{M,i} - \overline{L}_i$ is the quantity of water, reabsorbed after the moment of maximum depletion. Introduction of the magnitudes $L_M$ and $\overline{L}$ is motivated because they are rather easy to calculate.

## 4. The calculation procedure

The water bookkeeping of the storage deals with three separate zones: the zone in which evapotranspiration takes place (maximum capacity $B$), the transito-zone through which water passes percolating from above and rising by capillary forces, and the groundwater zone, lying between the phreatic level and the basic level at a constant depth from the surface (fig. 1). The boundary planes between the three zones change by root growth and decay and by releasing

or receiving water to or from the ground water respectively. Those changes are corrected in the model. It is impossible to treat the complete model in all details in the scope of this paper. It was necessary to introduce 80 symbols and to set up 40 decisions to warrant that the right equation was applied in the situation concerned.

A few simple examples are given here. The last two are simplified.

a. *Percolation*

Figure 11 shows the calculation procedure of the amount of water percolating from the evapotranspiration zone into the transito-zone ($b_D$).

First the rapid percolating ($D_r$) is calculated according to eq. 5. A decision is made to find out whether $D_r$ is positive or 0, or negative. In the latter case rapid percolation is put at 0, because negative values have no use. The first precipitation remainder ($N_{r1}$) is calculated. The result is examined to see whether the amount of water can be completely stored into the emptied soil,

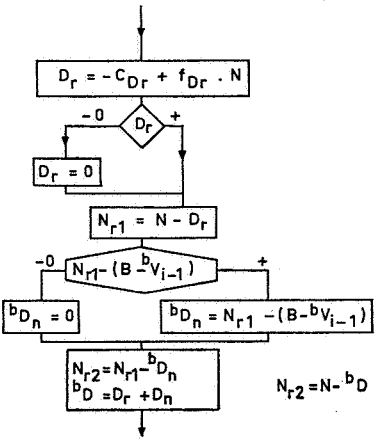


*Fig. 11.* Block diagram of drainage *(D)*. Calculations in rectangulars, decisions in rhombs or sidely truncated rhombs. $D_r$ rapid drainage, $C_{Dr}$ constant, $f_{Dr}$ factor, $N$ precipitation, $N_{r1}$ 1st remainder of precipitation, $B$ capacity of evapotranspiration zone. $b_{Vi-1}$ water content in $B$ at the beginning of short period, $b_{Dn}$ normal drainage from $B$-zone, $N_{r2}$ 2nd remainder of precipitation, $b_D$ total drainage from $B$-zone.

or not. The depletion or vacancy is indicated by the difference $B - b_{Vi-1}$. If the precipitation remainder is too large to be stored, there is normal percolation out of the evapotranspiration zone $b_{Dn}$, its amounts is equal to the surplus $N_{r1} - (B - b_{Vi-1})$. If all water can be stored, there is no percolation into the transito-zone.

Now, the second precipitation remainder ($N_{r2}$) is calculated (first remainder deminished by the normal percolation into the transito-zone). Also the total percolation into the transito-zone ($b_D$) is known. It will be clear that $N_{r2}$ is identical with $N - b_D$.

b. *Infiltration from below*

The second example concerns the calculation of infiltration ($I$) from and run-off ($A$) to the ground water and the changing water content in the layer of the bookkeeping ($\triangle V_i$) (fig. 12). First the quantity of water involved in the changing water table is calculated ($Z_i$). Then the change in water in the transito-zone is calculated which is the capillary rise ($^tC_i$) into this zone, diminished by the capillary transport out of it into the evapotranspiration zone ($^bC_i$), and increased by the percolation water into the transito-zone ($^bD_i$). After

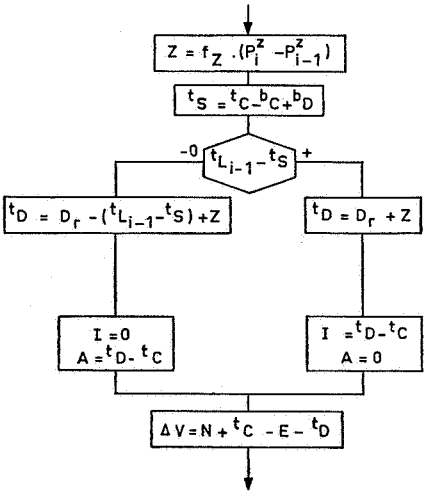


*Fig. 12.* Block diagram of run-off ($A$) to ground water, infiltration ($I$) from ground water and change in total moisture content ($\triangle V$). $Z$ water released by lowering the water table, $f_Z$ factor, $P$ depth of water table below surface, $z$ exponent, $t_S$ change in vacancy in the transito-zone, $t_O$ capillary rise into transito-zone, $b_C$ capillary rise into evapotranspiration zone ($B$-zone), $b_D$ drainage from $B$-zone, $t_{Li-1}$ vacancy in transito-zone at the beginning of period $i$, $t_D$ drainage from transito-zone, $D_r$ rapid drainage, $E$ actual evapotranspiration. For clarity sake the indices $i$ have been dropped.*

this the question has to be decided if the vacancy in the transito-zone at the beginning of the period ($t_{Li-1}$) can store the change in water. If so there is no drainage from the transito-zone to the ground water besides the amount of rapid percolation and the amount of water released by a decrease of the water table

* See note on page 98.

$(Z_i)$ (this value may be negative in case of a rising water table). If the transito-zone is completely filled $(t_{Li-1} - t_S) \leqq 0$, the drainage to the ground water is increased with the surplus. In that case there is no capillary rise from the ground water. Infiltration $(L_i)$ and run-off $(A_i)$ are easily found.

Finally the change in the water content of the whole layer of bookkeeping can be found by the concerning water balance equation.

*c. Dehydration and rehydration*

The third example concerns dehydration and rehydration (fig. 13).

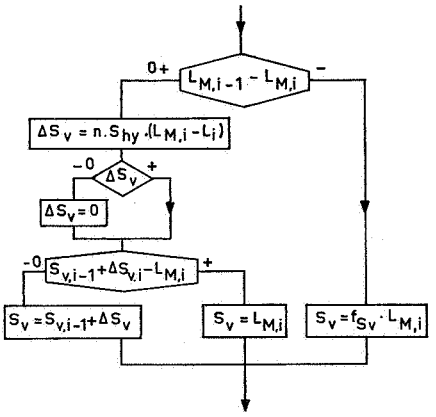


*Fig. 13.* Block diagram of dehydration and rehydration. $L_{M,i-1}$ maximum vacancy at the beginning of period $i$, $L_{M,i}$ the same at the end, $\Delta S_V$ increase in rehydration in period $i$, $n$ number of days in short period, $S_{hy}$ rehydration rate per mm rehydration water, $S_v$ actual field capacity, $f_{Sv}$ factor of refillability.

The first to find out is whether the soil is becoming dryer or wetter in period $i$. This is done by comparing the maximum depletion at the end of the period $(L_{M,i})$ with the maximum depletion at the beginning $(L_{M,i-1})$. Both values are rather easily found. If the later value exceeds the earlier one the soil is drying, in case of the reverse the soil is becoming wetter.

In the first case eq. 14 is used, in the second eq. 15 and 16 (fig. 13). To prevent negative values in the exceptional cases of $L_{M,i} - L_i$ being somewhat below 0, $\triangle\ C_{v,i}$ is put at 0. Rehydration should not reach a value so that the maximum field capacity of the soil is exceeded. Therefore the capacity of that part of the soil, involved in depletion is compared to the maximum vacancy $L_{M,i}$ after rehydration at the end of period $i$. If the soil is not at maximum field capacity, the value of actual field capacity is equal to the value $C_{v,i-1}$ increased by the rehydration during period $i$, $\triangle\ C_{v,i-1}$.

## 5. Weather and observations in 1959 and 1960

The following figures show the weather in the years 1959 and 1960. In figure 14 the average precipitation per day is plotted against time for each balance period. The average $E_0$ per day is plotted in the same way. The figure shows that 1959 had a precipitation deficit throughout the summer, whereas 1960 had a precipitation deficit in early summer, but a surplus in late summer. Run-off to the ground water at the drainage-lysimeter field is shown in figure 15 together with precipitation. There scarcely is any run-off in the summer and a part of the autumn in 1959, whereas in 1960 run-off starts again in the middle of the summer.

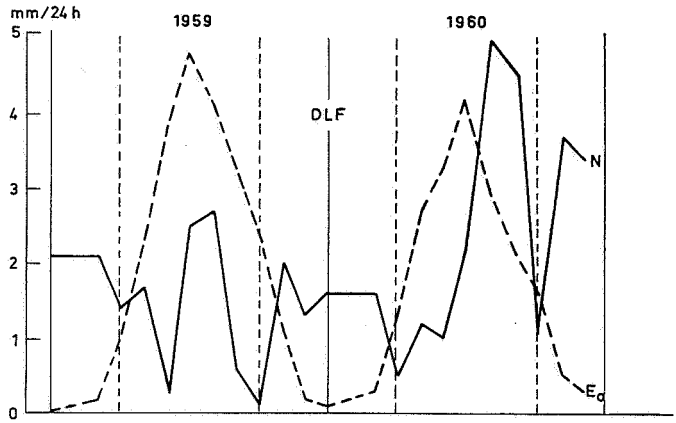


*Fig. 14.* Time curves of observed precipitation *(N)* and calculated $E_0$ (according to Penman) in mm per day for the balance sheet periods of the drainage lysimeter field in the polder Rottegat in 1959 and 1960. Full drawn vertical lines at 1st of Jan., interrupted vertical lines at 1st of April and 1st of Oct.

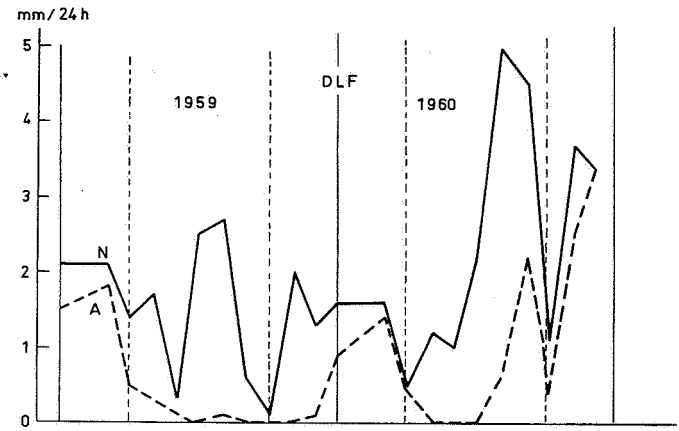


*Fig. 15.* Time curves of observed run-off *(A)* and precipitation *(N)* in mm per day

Actual evapotranspiration $E$, calculated from the observed data on water balance is compared with $E_0$ in both years (fig. 16). Throughout the summer of 1959 evapotranspiration was reduced continuing during a good deal of the autumn. In 1960, however, evapotranspiration is reduced through part of the summer, but afterwards evapotranspiration is approximately $E_0$. In the winters 1958/'59 and 1959/'60 actual evapotranspiration exceeds $E_0$, which is not likely. It is a reason to suspect these "observed" values of the evapotranspiration:

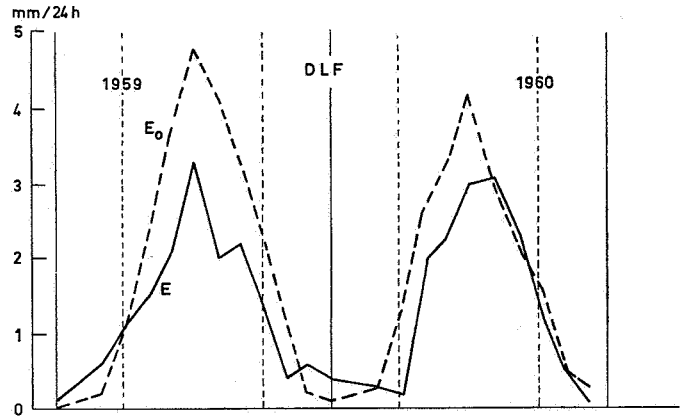


*Fig. 16.* Time curves of $E_0$ (Penman) and of actual evapotranspiration *(E)* based on the observed water balance in mm per day

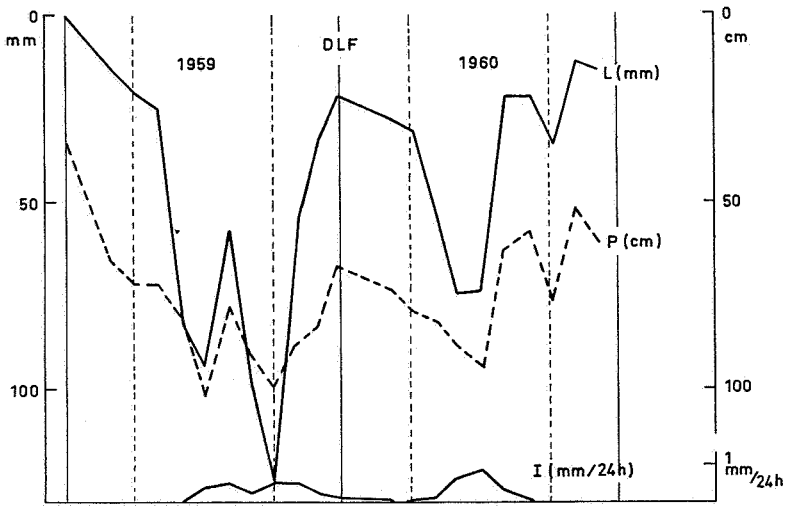


*Fig. 17.* Time curves of moisture deficit (vacancy, $L$) in mm, of the depth of the water table (cm below surface), and of observed infiltration rate from ground water (mm per day). The former two concern the end of the balance sheet period, the later the balance sheet period.

measurement of snow is not reliable in winter and it was likely that no snow was blown from the field into ditches.

In figure 17 there is a line for the infiltration rate ($I$) from the ground water observed at the drainage-lysimeter field, showing that the tops coincide with the greatest depths of the groundwater table ($P$). It is remarkable that the highest rate (1960) does not agree with the deepest water table (1959) nor with the greatest soil moisture deficit ($L$), occurring at the end of the summer of 1959. This may indicate that the properties of the soil have changed after the dry summer of 1959.

## 6. Calculations and observations in 1959 and 1960

Due to rounding the balance sheet of the calculated figures fits with a difference of $-7$ and 3 mm in both years (table 1). The other discrepancies will be discussed on the base of the graphs (fig. 18–21).

Table 1. Annual data on the drainage-lysimeter field in the polder Rottegat for 1959 and 1960 (in mm)

| | 1959 | | 1960 | |
|---|---|---|---|---|
| | calc. | obs. | calc. | obs. |
| Precipitation | (578) | 578 | (853) | 853 |
| Amount | 73 | 73 | 26 | 56 |
| Run-off | 223 | 186 | 382 | 400 |
| Evapotranspiration | 472 | 484 | 446 | 491 |
| Change moisture content | −37 | −19 | 54 | 18 |
| Sheet deficit | − 7 | 0 | 3 | 0 |

Capillary rise starts too early in both years, because the formula for a stationary condition has been used and no delay has been applied in the model (fig. 18)*. On an average the capillary rise was too low in 1960. Since capillary rise feeds evapotranspiration many of the deviations in evapotranspiration can be explained by the deviations in capillary rise (fig. 18), except for the second period of 1959 and 3 values in the winter 1959/'60. In the second period of 1959 snow had fallen which had partly been blown away by the wind. This suggested an observed** evapotranspiration which was too high and led to an observation of too little run-off. The observed** values in the winter 1959/'60 are much above the values of $E_0$ (fig. 14), which is unlikely and may be due to errors in any of the observed terms of the balance sheet equation. The

* Capillary rise is indirectly affected by the value of $Z$ (see note on page 98).
** i.e. calculated from observed values in the balance sheet equation.

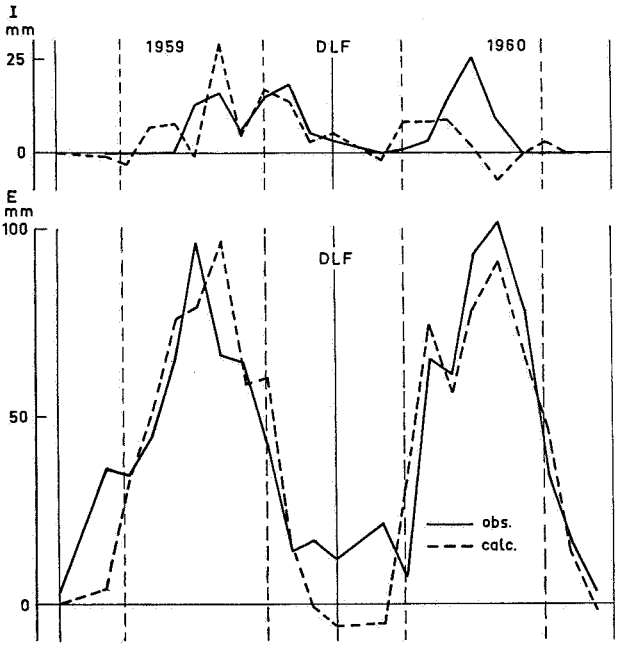


*Fig. 18.* Observed and calculated infiltration from ground water in mm per balance sheet period (above) and observed and calculated actual evapotranspiration in mm per balance sheet period (below)

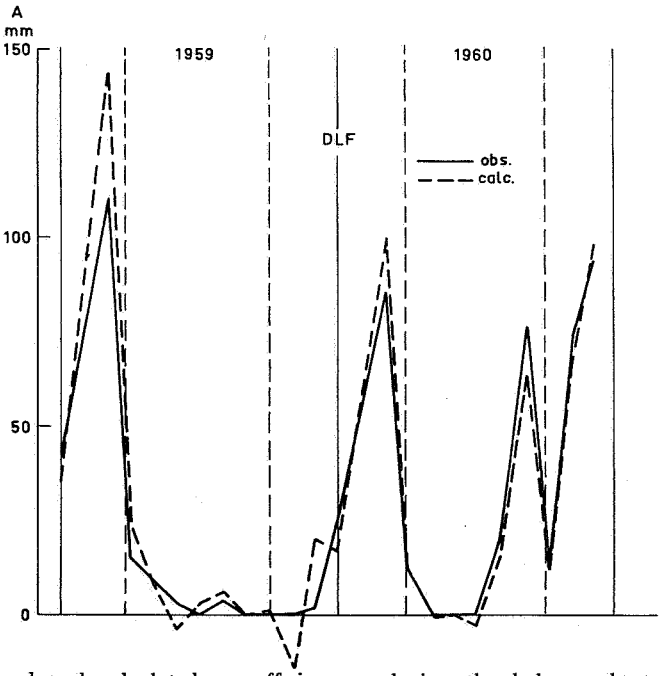


*Fig. 19.* Observed and calculated run-off in mm during the balance sheet period of the drainage lysimeter field in 1959 and 1960

calculations of evapotranspiration are made with the simple evapotranspiration model (3.3). Calculated run-off is in good agreement with the observed one; the second calculated value may be correct (fig. 19).

Change in moisture content of the profile shows much agreement between calculation and observation, except for a few winter data (fig. 20). As far as calculation is concerned, the values are obtained as the closing item of the balance sheet.

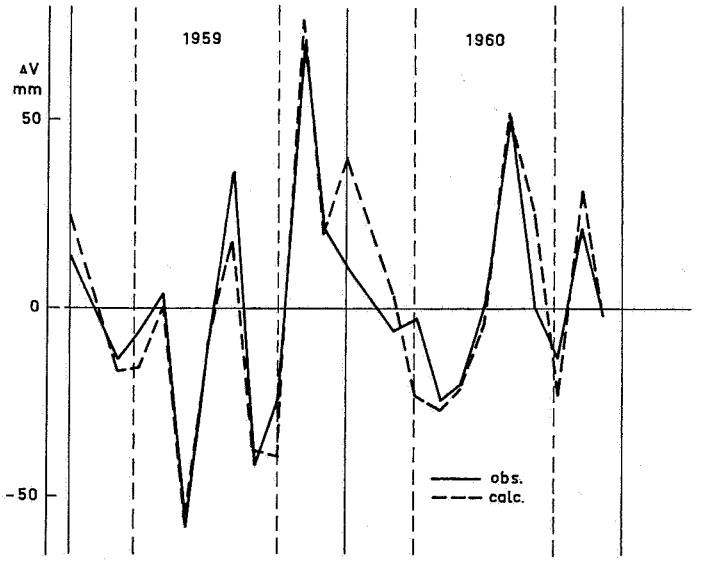


*Fig. 20.* Observed and calculated change in moisture content of the water balance layer in mm per balance sheet period

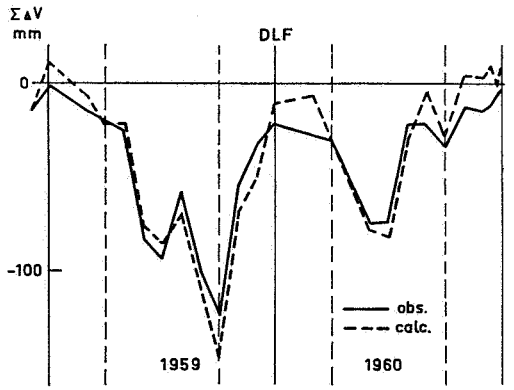


*Fig. 21.* Observed and calculated accumulated change in moisture content of the water balance layer at the moments between the balance sheet periods

Represented cumulatively, the calculated moisture deficit of the profile runs almost parallel to the observed one (fig. 21), but in winter the calculated values are closer to maximum field-capacity than the observed ones. This may be due to the fact that water percolating through capillaries to the ground water is lost from the soil with sampling, whereas it is calculated in the model. The results presented in the figures 18–21 are better than those published before (MAKKINK, 1958, 15th report), because a number of errors in the program were corrected since.

## 7. FURTHER COMMENT

The calculation is not yet in its final stage. The discrepancies in the observations suggest that improvement is possible.

Several simplifications were applied, which perhaps were not admissible. Several factors were neglected without knowing how the results were affected. Advective energy, for instance, was not taken into account (which was perhaps necessary in 1959). Other phenomena neglected in the model are rain intensity (which could have affected rapid drainage), the seasonal temperature wave in the soil (which could have caused upward destillation of water in winter and downward in summer), air remaining for some time below the phreatic level after a rise in the water table, capillary hysteresis (in contrast to the dehydration/rehydration hysteresis, giving rise to different values of $Z$ with a rising and falling water table), interception (increasing evapotranspiration in periods with limited water supply), fluent gradients in soil moisture content, the pF/water content relationship (affecting the evapotranspiration calculated with the drying equation 8), the daily rhythm of evaporation, botanical characteristics of stomatal reaction (reaction to water content in the leaves and to light intensity) a.s.o. Furthermore, a simplified evaporation formula was used in which wind velocity, long wave back-radiation and vapour-pressure deficit of the air were neglected. The roughness of the crop accounted for in the crop factor was applied to the total evaporation instead of to the vapour-transmission term only.

Considering this list of imperfections would be enough to give up all hope of finding a reasonable model to calculate the items of the water-balance sheet on the basis of simulating the processes involved. However, the tentative results are encouraging.

A number of improvements may still be made. The merit of the electronic computer is that it multiplies the rapidity of our mental arithmatic by 1000 or 1500 and avoids the errors made in tedious and prolonged calculations. Consequently, disgust will not prevent repeating the whole procedure several

times changing one or another constant, coefficient, or exponent or leaving out some elementary process. Thus achieving a numerical check on the relative importance or the accuracy of any magnitude used in the model and the relative importance of each participating phenomenon.

This work has not yet been done at this moment.

## 8. Summary

A simulation model was designed to calculate by a computer the run-off ($A$) to the ground water, infiltration ($I$) from the ground water, actual evapotranspiration ($E$) and the change of moisture content in a constant soil volume ($\triangle V$) as functions of time. Four independent functions (eq. 1 to 4) were applied ($B$ maximum of available water within reach of the evaporation capacity of the atmosphere with or without mediation of the vegetation and without completion by capillary rising water, $b$ index indicating the storage capacity $B$, $E_p$ potential evapotranspiration, $N$ precipitation, $\triangle P$ change in depth of water table, $\overline{P}$ average depth of water table, $i$ elementary period of calculation (about 5 days), $i-1$ moment of the beginning of period $i$).

Percolation, evapotranspiration, capillary rise, change in soil water content by changes in the depth of water table, dehydration and rehydration of the heavy clay soil were separately studied and numerically introduced into the program. About 80 symbols and 40 decisions were necessary. Water balance equations were applied in the three separate storages of the total water balance-sheet volume of the soil, viz. the evapotranspiration zone (capacity $B$), the transito zone (between the lower level of $B$ and the phreatic level) and the groundwater zone (between phreatic level and the constant basis plane). Observed time series of a few weather factors and estimated time series concerning the over- and underground parts of crop were applied. Constants of the soil were estimated or determined. The results were checked by means of the observed terms of the water balance sheet of the experimental field in the polder Rottegat (province of Groningen) for periods of about a month.

Observations and weather factors are presented in the figures 14—17; observations and results of the calculations are presented in the figures 18—21.

RESUMO: AKVOBILANCO KAJ AKVOLIBROTENADO DE REGIONOJ

Simula modelo estis kunmetata por kalkuli per automata kalkulilo defluon ($A$) al la terakvo, enfluon ($I$) el la terakvo, efektivon evapotranspiron ($E$) kaj shanghighon de la akvoenhavo ($\triangle V$) de konstanta tervolumeno kiel funkcion de la tempo. Kvar sendependaj funkcioj (ekw. 1 ghis 4) estis aplikataj ($B$ maksimuma sorbebla akvo en atingopovo de la vaporiga povo de la atmosfero kun au sen perado de la vegetacio kaj sen kontribuo de kapilare levighanta akvo, $b$ indekso indikanta la magazenkapaciton $B$, $E_p$ potenciala evapotranspiro, $N$ falajho, $\triangle P$ shanghigho de la profundeco de la terakvonivelo, $\overline{P}$ mezuma profundeco de la terakvonivelo, $i$ elementa kalkulperiodo (pr. 5 tagojn), $i-1$ momento de la komenco de periodo $i$.

Perkolado, evapotranspirado, kapilara levigho, shanghigho de akvoenhavo sekve de shanghigho de la frea nivelo, dehidracio kaj rehidracio de la peza argiltero estis studataj aparte kaj numerike enigataj en la programon. Chirkau 80 simboloj kaj 40 decidoj estis necesaj. Akvobilancaj ekvacioj estis uzataj por la tri magazenoj en la tuta bilancvolumeno de la tero, nome la evapotranspira zono (kapacito $B$), la transitzono (inter la malsupra nivelo de $B$ kaj la frea nivelo) kaj la terakvozono (inter la frea nivelo kaj la konstanta baza ebeno).

Observitaj temposerioj de kelkaj veterfaktoroj kaj taksitaj temposerioj de grandoj koncerne superterajn kaj subterajn partojn de la vegetacio estis uzataj. La rezultoj estis komparataj kun la observitaj eroj de la akvobilanco pri la eksperimenta kampo en la poldero Rottegat (provinco Groningen) por periodoj de proksimume unu monato (fig. 18–21).

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