COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO

COMMITTEE FOR HYDROLOGICAL RESEARCH TNO

VERSLAGEN EN MEDEDELINGEN No. 18

PROCEEDINGS AND INFORMATIONS No. 18

HYDRAULIC RESEARCH FOR WATER MANAGEMENT

TNO

HYDRAULIC RESEARCH FOR WATER MANAGEMENT

COMMISSIE VOOR HYDROLOGISCH ONDERZOEK TNO
COMMITTEE FOR HYDROLOGICAL RESEARCH TNO

VERSLAGEN EN MEDEDELINGEN NO. 18
PROCEEDINGS AND INFORMATIONS NO. 18

HYDRAULIC RESEARCH FOR WATER MANAGEMENT

PROCEEDINGS OF

TECHNICAL MEETING 26

VERSL. MEDED. COMM. HYDROL. ONDERZ. TNO 18 – THE HAGUE – 1973

This issue is also Publication No. 100 of the Delft Hydraulics Laboratory, P.O. Box 177, Delft, The Netherlands.

CONTENTS

AUTHORS

H. de Groot	Director of Rijnland Water Board, Leiden
E. Allersma	Head Hydrodynamics and Morphology Branch, Delft Hydraulics Laboratory
C. B. Vreugdenhil	Head Mathematics Branch, Laboratory De Voorst, Delft Hydraulics Laboratory
J. C. Berkhoff	Mathematical Engineer, Delft Hydraulics Laboratory
C. Veeningen	Project Engineer, Delft Hydraulics Laboratory

FOREWORD

Water management is more and more dealing with aspects of water quantity control as well as water quality control. The human interference in the aquatic environment requires careful consideration in order to give the water management a sound scientific base. The series of contributions given on October, 28, 1971 at the 26th Technical Meeting regard hydraulics research, necessary in this respect expecially with regard to the use of mathematical models.

Chapter I deals with the statement of the problems encountered in practice. In chapter II an over-all view is given on the hydraulic problems met in this respect. This is deepened into a mathematical description for quantity control (chapter III) and quality control (chapter IV). Both chapters treat the questions into detail in order to make them ready for numerical computations.

Finally chapter IV gives a synthesis into the use of these mathematical models for a number of practical problems.

M. de Vries

I. PROBLEMS FACING THE POLDER RESERVOIR CONTROL

H. DE GROOT

1. What is reservoir control?

The term "reservoir control" in connection with the polders of the Netherlands comprises the supply of water and the discharge of water in such a way that both quantitatively (level control) and qualitatively (quality control) the best possible situation in the reservoir is created and maintained.

In a wider sense reservoir control actually comprises all activities aimed at meeting the purpose of the reservoir to the greatest possible extent. This is essential because effective reservoir control serves many different interests such as, for example, agriculture and horticulture, industry, shipping, groundwater regulation, provision of drinking water, recreation and fishing (both professional and in the sporting field). In addition, by efficient control of the level, the flooding of polders is prevented whenever possible, thus greatly increasing the safety of the various interest established in those areas. In the past there were only agrarian interests to be mainly considered, but nowadays there are many aspects of society connected with housing, working, and traffic which are now very closely concerned with protection of the polders against inundation, not only in a local context but also regionally.

In considering reservoir control as a whole, it is not only the manipulation of supply and discharge waters which must be thought of, because the network of the reservoir water formed by canals, lakes and small waterways must also be maintained in such a state that there can be effectively play with the means of both supply and discharge. For this, on the one side the water must conform to the measurements laid down and be free of plant vegetation, while on the other side precise consideration must always be given to what profile interferences can be permitted in the form of bridge openings, narrowings, fillings, culvert construction and similar works.

2. The historic development

In the past, when there were no means of mechanical discharge, the reservoir functioned primarily as a place for storing water. When there was too much water, the superfluous amount was temporarily stored in the reservoir and then, when sea water levels were sufficiently low, it was discharged in a natural way outside the area.

In periods when water was needed, because of the lack of sufficient inlet capacity, the reservoir was used as a water storage basin. It is obvious that when used in this way, namely, in the seasons, the level of the reservoir showed large fluctuations.

The buffering effect of the reservoir was essentially important, and this is why the Rijnland Water Board so strongly opposed the reclamation of the Haarlemmermeer in the 18th and 19th centuries, because the storage capacity of the reservoir of about 22,000 hectare acres would have been reduced by more than 80% to about 4,000 hectare. It was only when compensation was offered in the form of steam pumping-stations that the Rijnland Water Board could agree to this interference in the control of its reservoir level.

After the installation of the pumps and their greater reliability, the value of the storage function became more relative; the essence of the level-controlling capacity no longer lay passively with the reservoir itself but was transferred to the hands of the reservoir controller through the mechanical means thus provided. Moreover, with such an active power in his hands, the reservoir controller was able to have a better grip on the qualitative control of the reservoir water.

3. The functioning of the network of reservoir waters

Before dealing with the direct practice of reservoir control it is advisable to explain clearly the actual functioning of the network of reservoir waters. In the polder areas of the Netherlands — with their intrusion of salt via the salt water adjacent to the hydraulic works and their (salt) seepage — the system of reservoir waters not only has the original function of the discharge and supply of superfluous or necessary water respectively, but has also obtained the function of discharging the salt which has penetrated into the area. To accomplish the salt discharge, there has been developed the system of flushing the network by admitting a larger quantity of water than would be necessary for the level control and then simultaneously discharging the same quantity of water through the network's discharge points located opposite the inlet place(s). With the increase of the organic charge on the reservoir water in the form of dispersed and concentrated discharges of domestic and industrial waste water, the flushing system also started to function as part of a waste water system. The bottleneck of the re-freshening system naturally lies in the circumstance that there is no functional splitting in the waters for the supply of good water and for the discharge of used or polluted water. Because of this, reservoir control has become a precise system which is still further complicated by the requirement that the most effective use is made as possible of the supply water, which in certain periods is scarce.

All this means that the manipulation of the means of supply and discharge must be carried out with great care and with a strategy adapted to every situation and selected in advance.

It is not strange, therefore, that the reservoir controller who is fully aware of his responsibilities wishes to form a picture of the full functioning of his reservoir water network, and in this connection desires to become acquainted with the nature of the currents and stream conditions of his reservoir waters under all circumstances. The first steps along the road of research in this direction were taken in the 1930's and 1940's when discharge measurements were made at pumping-stations and locks, along with current measurements in the main waters of each network and a series of water-balance calculations. With these and a number of other observations and calculations the reservoir controller built up a framework within which he could control the purpose and the direction of his reservoir system.

4. THE PRACTICE OF RESERVOIR CONTROL

For the "daily" and the "long-term" reservoir control policy the controller must have available a network of observation points in the reservoir and along its borders. This is essential because rapid information is needed on such things as:

— Meteorological conditions.
— The quantitative situation in the reservoir (levels).
— The qualitative situation in the reservoir (chloride and oxygen contents).
— Quantitative and qualitative facts about the water to be let in.
— Qualitative details about the water to be discharged.

It may at first sight seem strange that the reservoir controller must consider the quality of the water to be discharged. However, if for some cause or other it is necessary in the interests of the reservoir to discharge dirty water outside, it is advisable to take into consideration the interests of the external region which could be harmed by the discharge.

Here is an example: To ensure causing as little nuisance as possible during the bathing season at Katwijk beach by the discharge of organically and/or bacteriologically polluted water, a pumping regime has been developed with discharge times in the evening hours when the beach is not being used for recreation. This means that the water in the coastal strip will be able to be in a reasonable condition, as a result of diffusion and convection, before the daytime period when the beach is used again.

The continuous study of the incoming information and of the results of the reservoir policy lead to the determination of actions directed at achieving the maximum possible results in every situation.

To enable further hydrological insight to be had into the reservoir use over a longer period, it is important to make water and chloride balance sheets not only every year but also for every winter half-year and every summer half-year. Then, too, an analysis of periods of extended drought or of excessive water helps to contribute to a widening of knowledge about the control as well as to the evaluating of the records which appear on the positive and negative sides of the water balance-sheets.

The execution of reservoir control — that is, the daily use of the means of letting and letting out the water — as at present practised is still considered as a skilled and experienced activity, especially in those cases where it concerns complex reservoir networks. The control is carried out centrally, and by one man, in such a way that in respect of the qualitative aspects he is also cooperating with someone controlling the chemical-technological side of the work.

This daily work is — not yet (?) — automatised. To determine the actual control required, interpretation is necessary of the available data about such things as wind strength and direction, the actual and expected rainfall, the nature of the water (snow or rain), the course of the temperature, the situation in the polders, the agrarian requirements of water, the quality in the various sectors of water, and, very frequently, the complaints received about pollution and the dying of fish. And many other factors as well. Moreover, all these things have to be seen against the background of the climatic factors of the previous days and of the metereological forecasts or expectations for the coming few days. These are all helpful when the reservoir controller wishes to try and anticipate drought or excess rain, or perhaps wind direction and force.

All this shows that reservoir control is not determined only by exact and precise information but also that less accurate factors often play a role.

This is why, therefore, the work cannot be fully automised, and that much of it has to be done "live", supplemented by a great deal of background experience and a certain amount of intuition. In extreme circumstances the control even requires a non-stop personal accompaniment.

5. Quantitative aspects

There is a tendency to think that quantitative control now-adays creates few problems. In general, of course, there are available a large discharge pumping capacity, reliable pumping-stations, and experienced reservoir controller, extensive and continuous observations, and a good insight into the functioning of the whole reservoir system. To a certain extent these facilities are in regular use, but the trend to a refinement of quantitative reservoir control (that is, within narrow limits variations in reservoir level when there is a small storage capacity) is making the system more vulnerable and sensitive. Because the level control is kept well in hand, more variations (more discharges and more fillings) are accepted, but the result is that the reserve in the quantitative control system becomes less while the situation can at times become more risky, such as when a pumping station is out of action.

There is also a clear alternating effect to be observed between the increasingly refined level control with small level fluctuations and the interests which are often connected with the reservoir, such as polder intakes, industrial outlet and intake levels, and the freeboard of the dikes in connection with the tendency to have lower protection height.

For these reasons it is necessary to be continually aware of margins that are becoming smaller and smaller, and therefore not to neglect the possibility of disturbances or even calamities. This means that too great a dependence should not be put on the available storage capacity, because, for example, high levels can occur locally as a result of a sudden wind storm. From this point of view a "pumping level" — that is a reservoir level at which the polders are compelled to stop their pumping — is no longer an antiquated conception but is a desirable requirement within the built-in safety system to limit as far as possible emergency situations during periods of great excess of water.

The situation which results from the increasingly refined level control makes it advisable for there to be critical judgment of applications for permits for works which would decrease the water-transporting capacity of the component parts of the reservoir network. Examples of works included in this category are narrowings, filling-in, and sudden changes of profile. More than ever before there must be advance investigation of the hydraulic and hydrological results of such works.

6. The qualitative aspects

Although a number of the problems concerning the quantitative aspects of reservoir control have been summed up in the foregoing paragraphs, in practice these problems of level control are usually interwoven with those of quality control. In fact, each of these types of control interferes with the others. Every pumping or discharge, irrespective of whether there is a counterbalancing inlet supply, influences the quality of the reservoir water. Here are some examples:

- Discharging in a period of excess water also means the discharge of salt through either salt seepage or via locks water; admission of water for level control during a drought period can result either in a re-freshening or (with the present quality of Rhine water) the salination of the reservoir water.
- Admission intended for re-freshening can be disturbed by rainfall.
- The need for re-freshening is, among other things, dependent on the amount of the deposition of rain, or other forms of moisture, as well as on the actual rain and water pattern (torrents or equally distributed rainfall).
- Calamities such as poisonous discharges or dead fish can necessitate either the pumping out or the inletting of water, which of course affects the maintenance of the level.

These examples show that in principle the idea of having "quantitative and qualitative control in one hand" is right. From this it follows that in addition to giving considerations to results of quantitative control, every change in the components of the reservoir network must also be looked at in the light of the results of quality control.

7. The problems

After these explanations about the various aspects of reservoir control, it will probably be useful within the framework of this survey to explain a number of problems connected with the actual tasks of the controller, especially those regarding the safeguarding of the interests of his reservoir against a continuous series of demands. Outstanding in this respect is the fact that with the large reservoirs under consideration here, each demand in itself need not be generally regarded as of great importance: the real problem is to be found in the cumulation of the results of many demands. Here the so-called "salami procedure" is very frequent: a small slice is cut off each time until the sausage is appreciably smaller in size. This is the case in a hydrological respect in connection with the increasing resistance which causes profile changes of the waters by the intrusion of the banks, the construction of bridges and of dams with culverts, and the filling in or narrowing of the waterways and the canals.

It will not be a matter of surprise that — especially where it concerned marked interference such as the filling in or narrowing of city canals in places like Leyden and Haarlem — the Rijnland water control authorities did not in former times give their general approval to such activities. Their watchword was, Keep what you have and do not risk retrogression.

However, as the calculation techniques became more effective, it was possible to judge the applications better, and today it is possible by means of a mathematical model to check the results of such interferences and to introduce such compensatory measures elsewhere to ensure that there is ptracically no change in their total influence on the water levels and conditions in the surrounding area. Also it can be calculated in an analogical way how certain large discharges and withdrawals from the reservoir network (such as the cooling water area of an electric power plant) can change the existing pattern of re-freshening and flushing.

Also various questions about the construction of a new pumping-station can be solved in this way so that the best possible choice of location and capacity can be determined. Here is an example:

In the preparation of the design for a new pumping-station at Halfweg (between Amsterdam and Haarlem), the following questions arose:

- Was the establishment of one single pumping-station in Spaarndam to replace the units at Spaarndam and Halfweg both possible and desirable from the point of view of water levels and stream conditions?
- What would be the results for the water levels and stream conditions of installing

a new pumping station at Halfweg with a capacity one-and-a-half times as large as the existing pumping-station?

The decisions about whether these or similar steps and changes were or were not permissible or desirable could be made much more easily and more justifiedly by the advanced new calculating techniques which were actually available for more complex systems. To be able to translate complex systems into a system of reservoirs in a network, however, close cooperation is required between the mathematical translator and the expert of the reservoir water system. From the controller's side an indication will have to be given of what grades of precision are desired in the results of the investigation, and this, of course, depends on the nature of the problem. Then, setting out from the desired precision and from the geometry of the system, the translator will draw up a schematic network with the desired degree of refinement. From his own side, the reservoir controller will supply the information about the channel network, such as length, cross sections, resistance, etc.

To be able to investigate in the model results the of new situations, changes, etc., it is necessary to verify in advance the original situation according to the behaviour of the operating reservoir system. For this purpose, recorded data from certain periods of the past can be used, or alternatively certain actual stream situations in reality and in the model can be produced and measured.

Once a network model of a reservoir system has been prepared, it is obvious that problems and questions arising from time to time can be investigated and dealt with by means of the model. According as the use being made of the reservoir and the reservoir water becomes more intensive, and as the density of the inhabitants and traffic increases, the number of demands on the reservoir, both quantitatively and qualitatively, become more frequent. To be able to continue to evaluate the results of each demand and of the cumulation of all the new demands, the regular consultation with the network model has become a matter of great importance.

Although a number of regular problems have already been discussed here, there are other possible problems which will have to be tackled. These include:

- The adaptation of the capacity of the reservoir pumping-stations and its results for the reservoir.
- A change in the profile of the waters, canals, etc., (locally by hydraulic works) and over a considerable length.
- The filling in of waterways.
- The maximum possible investigation of compensatory measures for narrowing or filling in the waterways.
- The dispersal of salt by a new pumping-station for a polder with much salt seepage.
- The dispersal of waste materials form the discharges of municipalities and industries, and its influence on the oxygen, nitrate and phosphate content of the surface water.

- The results of the thermal effect of the discharge of large quantities of cooling water, both on the stream picture and the temperature curve in the cooling circuit.
- The bacteriological aspects of the discharge of faecal waste through sewerage systems, effluent pipelines of purification systems, and emergency outlets. In this connection, it can be mentioned that in view of the recreational use being made of certain waters and lakes, it is desirable to prevent the bacteriological pollution of such waters. To do this it is of interest te bocome acquainted with the travelling times of these bacteria from the discharge points to the recreational water, and also the dilution the discharge water undergoes during the journey.
- The measures to be taken in case of disaster: for example, a large discharge of poison (such as the cyanide poisoning near Schiphol Airport) can be investigated by means of a network calculation.

Then, too, the results of a possible breach in a dyke of a deep polder in a reservoir area can be successfully fore-shadowed by means of a network model, especially the size and the way the levels will fall at various points in the reservoir. Such a study will enable judgment to be decided about the possible occurrence of the loss of stability of reservoir dykes.

It can be mentioned here that the problems just summed up are taken from actual practice and that they were evaluated because a network model was available, thereby enabling responsible decisions to be taken. This shows the great value of such mathematical models.

8. Future developments

Water is an environmental factor of the first order. As water, judged by both quantity and quality aspects, is becoming more and more scarce, a great responsibility rests on the controller to exercise his control in such a way as to reduce the chances of wastage and pollution to the minimum. Such an efficient control demands a good internal organization which can act promptly. Observations of rainfall, level, chloride content, oxygen content and other quality parameters must be automatized, making use of telecommunication systems for the transport of information. These observation results, which are actually reproducing the effect of the control, must be used in the fullest possible way, preferably statistically, although this could produce certain problems because of the many variables.

It is here that the automation of control through the mathematical model comes in. Without a doubt the water control bodies will have to introduce this method of automation, because experience has already shown that the mathematical model of the reservoir network can increasingly facilitate the task of the reservoir controller.

The interaction between experts of various disciplines (hydrologists, water technologists, biologists and hydraulic engineers, for example) will certainly lead to a system of directives for the hydrological policy in which a number of factors will have to be accepted. Taking the dynamic character of quality control into consideration, these directives, however, will always have to be adapted to the changing circumstances and surrounding conditions. The continuous optimalization of the water control policy is a task which perhaps in the long run can be better tackled by a computer than by a human controller. But the solving of this problem — seen also from a financial-economic viewpoint — is a task for specialists nowadays.

The biggest problem will be the techniques with mathematical models which will be developed for both quantity and quality investigation from the experience of practical application. Good research can often be hampered by difficulties in its practical application, because the man who will have to work with it daily often finds it very difficult to understand and use the new method, often thinks it expensive, and frequently regards it as not suitable for his particular requirement.

It is, therefore, the responsibility of the research worker to take these (and other) factors into account, and to help the water controller step over these threshold.

II. HYDRAULICS OF OPEN-WATER MANAGEMENT

E. ALLERSMA

1. Introduction

The management of open waters is an important aspect of life in flat densely-populated plains. Drainage, irrigation and flood control require control of the movements of water, while the quality of the water is becoming more and more an additional problem.

This survey introduces some basic hydraulic aspects of the behaviour of water in open water courses, sewers, lakes, control structures, etc., as well as the phenomena involved in the transportation of matter (pollutants, sediments, salt) and heat by the water. These basic concepts form a starting-point for the more mathematically-oriented discourses of Vreugdenhil and Berkhoff.

2. Hydrodynamic aspects

2.1 Basic equations

The basic principles of the hydraulics of flows in open conduits are the continuity of an incompressible medium and Newton's second law of motion. These concepts lead to differential equations for a unit volume of water which can be used for the derivation of differential equations for digital computations and analytic solutions for everyday use in simple cases.

Experimental work provides information about empirical parameters, which inevitably appear in the integration of the basic equations.

Continuity

The condition of incompressibility leads to the following equation of flows for a unit of volume:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{1}$$

with

x, y, z = co-ordinates (horizontal, horizontal, vertical)
v_x, v_y, v_z = velocities (horizontal, horizontal, vertical)

Integration of this equation between the bottom (0) and the water surface (h) leads to:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial h}{\partial t} = 0 \tag{2}$$

with

q_x, q_y = rates of flow per unit of width
t = time
h = water depth

This equation describes the continuity of water flowing horizontally between the bottom and the surface of a large body of water.

Further integration over a cross-section along the y-axis leads to the expression for the continuity in a prismatic canal along the x-ordinate

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{3}$$

with

Q = discharge or rate of flow in the canal
A = area of the cross-section

Integration of equation (2) in both horizontal directions leads to the equation of continuity for a lake with area F

$$\sum Q + F\frac{\partial h}{\partial t} = 0 \tag{4}$$

which expresses that the storage equals the balance of flows into and from the lake.

Motion

The relationship between the acceleration (a) of a body (mass: m) acted upon by a force (F) was expressed by Newton in his second law as

$$F = ma = \frac{d}{dt}(\overrightarrow{mv}) \tag{5}$$

Application of this equation to a unit volume of fluid with a constant density (ρ) leads to the general equations of Euler:

$$f_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right)$$

$$f_y - \frac{\partial p}{dy} = \rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) \tag{6}$$

$$f_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right)$$

in which the external forces (f) and the pressure gradients act upon the unit mass (ρ). Viscous friction has been neglected because only turbulent motions are considered.

In the present discussion, in which only horizontal or practically horizontal motions play a role, and the x-ordinate can be chosen along the stream line, these equations simplify to:

$$f_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x}\right)$$

$$f_y - \frac{\partial p}{\partial y} = \rho v_x\frac{\partial v_y}{\partial x} \tag{7}$$

$$f_z - \frac{\partial p}{\partial z} = 0$$

The second of these equations includes the centrifugal force

$$\rho v_x\frac{\partial v_y}{\partial x} = \rho\frac{v_x^2}{r} \tag{8}$$

with r being the local radius of curvature of the stream line.

The forces (f) acting on the unit volume of fluid are, in this case, resulting from the fact that the motion takes place on the surface of the rotating earth.

The first component acting vertically is the weight g caused by gravity.

The second one called the Coriolis force is caused by the geostrophic acceleration. Its vertical component is negligible compared with gravity but its horizontal component must be taken into account. Its magnitude is:

$$2\rho v_x\omega\sin\varphi \tag{9}$$

with

ω = angular velocity of the earth's rotation
φ = geographic latitude.

The Coriolis force acts towards the right-hand side of v_x on the northern hemisphere and to the left-hand side on the southern hemisphere.

The effect of the centrifugal force is included in the local value of g.

Substitution of these forces into equations (7) leads to

$$
\begin{aligned}
& f_x' - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x}\right) \\
& f_y' - \frac{\partial p}{\partial y} \pm 2\rho\omega v_x \sin\varphi = \rho \frac{v_x^2}{r} \\
& f_z' - \frac{\partial p}{\partial z} + \rho g = 0
\end{aligned}
\tag{10}
$$

in which f_x', f_y' and f_z' are forces per unit of volume exerted on the boundaries of a body of fluid, as will be shown below.

Integration of equations (10) from the bottom ($z = z_0$) to the water surface ($z = z_0 + h$) leads to the equations of motion for a vertical column of water with unit basic area:

$$
\begin{aligned}
& \tau_{sx} - \tau_b - \rho g h I_x = \rho\left\{\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\alpha \frac{q_x^2}{h}\right)\right\} \\
& \tau_{sy} - \rho g h I_y \pm 2\rho q_x \omega \sin\varphi = \rho \frac{q_x^2}{hr} \\
& p = \rho g(z_0 + h - z)
\end{aligned}
\tag{11}
$$

in which the shear stresses at the surface (τ_{sx}, and τ_{sy}) and the bottom (τ_b), together with the pressure at the bottom (p_0), appear in equations (10) as f_x', f_y' and f_z'.

The external stresses are in practice:

- Surface friction caused by the wind

$$
\begin{aligned}
& \tau_{sx} = C_w W_x W \\
& \tau_{sy} = C_w W_y W
\end{aligned}
\tag{12}
$$

with W being the wind speed with components W_x and W_y.

- The hydraulic resistance of the flow along the bottom

$$\tau_b = C_b v_x^2. \tag{13}$$

- The hydrostatic pressure at the bottom

$$p_0 = \rho g h \tag{14}$$

The horizontal component of the gravity via the hydrostatic pressure appears in the first two equations in terms with the slopes of the water surface I_x and I_y. The coefficient α accounts for the uneven distribution of v_x over depth.

Integration of equations (11) over the cross-section of a canal which itself runs along the x-axis yields:

$$B_s\tau_{sx} - P_s\tau_b - \rho g A_s I_x = \rho\left\{\frac{\partial Q_x}{\partial t} + \frac{\partial}{\partial x}\left(\alpha'\frac{Q_x^2}{A_s}\right)\right\} \tag{15}$$

for the motion along the canal and

$$B_s\tau_{sy} - \rho g A_s\frac{\Delta h}{B_s} \pm 2\rho Q_x\omega\sin\varphi = \rho\frac{Q_x^2}{A_s r} \tag{16}$$

for the equilibrium across the canal, with

B_s = width of the canal
P_s = wetted perimeter of the canal
A_s = cross-sectional area of the canal
Q_x = discharge in the canal and
Δh = difference in water-level across the canal.

These equations pertain to the part of the cross-section of a water course which actually conveys water, excluding parts of flood plains in which water is stored without a through flow.

Equations (15) governs, with equation (3), the motion of water along a canal. It can be re-worked to:

$$\frac{\tau_s}{\bar{h}} - \frac{\tau_b}{R} - \rho g I = \rho\left\{\frac{\partial\bar{v}}{\partial t} + \frac{\partial}{\partial x}(\alpha'\bar{v}^2)\right\} \tag{17}$$

with

$\bar{h} = \dfrac{A_s}{B_s}$ = average depth

$R = \dfrac{A_s}{P_s}$ = hydraulic radius

$\bar{v} = \dfrac{Q}{A_s}$ = average velocity

Equation (16) is important only in wide canals where it provides an expression for the slope of the water surface across a canal due to geostrophic effects, centrifugal force and wind.

$$\frac{\Delta h}{B_s} = I_y = 2\omega\bar{v}\sin\varphi/g - \frac{\bar{v}^2}{gr} + \frac{C_w W_y W}{\rho g\bar{h}} \tag{18}$$

2.2. *Modes of flow*

The formulas used in practice for the computation of flows in water courses can be derived from the equations of continuity (3) and motion (17). The relative magnitudes of the terms in these equations are different for the various modes of flow. The following classification can be made according to the importance of $\frac{\partial\bar{v}}{\partial t}$ and $\frac{\partial\bar{v}}{\partial s}$:

$\frac{\partial\bar{v}}{\partial t} = 0 \quad \frac{\partial\bar{v}}{\partial x} = 0$ steady uniform flow

$\frac{\partial\bar{v}}{\partial t} = 0 \quad \frac{\partial\bar{v}}{\partial x} \approx 0$ steady gradually changing flow

$\frac{\partial\bar{v}}{\partial t} = 0 \quad \frac{\partial\bar{v}}{\partial x} \neq 0$ steady rapidly changing flow

$\frac{\partial\bar{v}}{\partial t} = 0 \quad \frac{\partial\bar{v}}{\partial x} = 0$ but $\frac{\partial Q}{\partial x} \approx 0$ and $\frac{\partial Q}{\partial t} \approx 0$

quasi-steady and uniform flow

$\frac{\partial\bar{v}}{\partial t} \approx 0 \quad \frac{\partial\bar{v}}{\partial x} \approx 0$ non-steady flow, slowly changing

$\frac{\partial\bar{v}}{\partial t} \neq 0 \quad \frac{\partial\bar{v}}{\partial x} \neq 0$ non-steady flow

(Remark: $\approx$ means "small" and $\neq$ means "large")

In the cases where $\frac{\partial\bar{v}}{\partial t}$ and $\frac{\partial\bar{v}}{\partial x}$ are small or negligible gravity, bottom friction and wind forces mainly govern the flow. The action of the wind can be illustrated by two extreme cases:

$\frac{\partial\bar{v}}{\partial t} = \frac{\partial\bar{v}}{\partial x} = 0 \quad I = 0$ steady uniform drift flow

$v = 0 \quad I \neq 0$ pure wind set-up.

2.3. Steady uniform flow

In this case the equation of motion obtains the form

$$\frac{C_w W_x W}{\bar{h}} - \frac{C_b Q^2}{R A_s^2} - \rho g I = 0 \tag{19}$$

Without wind this leads to the well-known formulas of:

Chézy $$Q = A_s C \sqrt{RI}$$

Kutter $$Q = A_s \frac{100\sqrt{R}}{6 + \sqrt{R}} \sqrt{RI} \tag{20}$$

Gauckler $$Q = A_s \frac{R^{1/6}}{n} \sqrt{RI}$$

in which g/C_b is determined experimentally and laid down in different expressions. The value of C varies between 30 and 70 $\mathrm{m}^{1/2}/\mathrm{s}$, depending on conditions.

The last of the three formulas was also devised but later discarded by Manning whose name, however, is still associated with the relationship.

In the case of a pure drift flow without gravity effects equation (19) leads to:

$$Q = A \sqrt{\frac{RC_w}{\bar{h}C_b} W_x W} \tag{21}$$

which shows that the rate of flow is linearly proportional to the wind speed.

2.4. Steady gradually changing flow

In this case the rate of flow changes gradually from place to place but the pattern remains the same. The equations of continuity and motion take the form:

$$Q = \bar{v} A_s \tag{22}$$

$$\bar{v} \frac{\partial \bar{v}}{\partial x} = -g \frac{\partial h}{\partial x} + g I_b - \frac{g}{C^2 R} \bar{v}^2 \tag{23}$$

in which I_b is the slope of the bottom.

The combining and re-working of these equations leads to expressions for the flow in gradual transitions in canals. The profiles of the water surface are expressed in:

$$\frac{\partial h}{\partial x} = \frac{h^3 - h_e^3}{h^3 - h_g^3} \tag{24}$$

which was first integrated by Bresse. Cases of special interest are:

- $h = h_e = \sqrt[3]{\dfrac{q^2}{C^2 I}}$ which leads to $\dfrac{\partial h}{\partial x} = 0$ and a steady uniform flow with the surface of the water parallel to the bottom.
- $h = h_g = \sqrt[3]{\dfrac{q^2}{g}}$ with $\dfrac{\partial h}{\partial x}$ being very great and $v_g = \sqrt{g h_g}$.

For $\bar{v} > v_g$ the flow is super-critical and for $\bar{v} < v_g$ it is sub-critical. In the case that $\bar{v} = v_g$ the assumption that $\dfrac{\partial \bar{v}}{\partial x}$ is small is not valid; it is the case of a rapidly changing flow.

2.5. Steady rapidly changing flow

This type of flow exists in the cases of rapid transitions in channels such as weirs, gates, local contractions, sills, culverts, sluices and even pumping stations.

With sub-critical flow the general expression for the flow-head relationship is:

$$Q = mA\sqrt{2g\Delta h}$$

in which

Δh = difference in head over the transition
m = empirical coefficient depending on the kind of transition and the conditions of flow.

Often, the cross-section A also is a function of the difference in head.

An example is a broad crested weir with length B for which the formula reads:

$$Q = \mu B(h_1 - \Delta h)\sqrt{2g\Delta h}$$

in which h_1 and h_2 are the elevation of the upstream and downstream water-level above the crest and $\Delta h = h_1 - h_2$.

If the flow becomes super-critical, the downstream water-level does not influence the rate of flow. The water depth in the transition attains a value of about 2/3 h, and Δh does not exceed about 1/3 h_1. Then the formula reads:

$$Q = \mu B \tfrac{2}{3} h_1 \sqrt{2g\tfrac{1}{3}h_1}$$

in which μ depends on the shape of the transition and the flow conditions.

Often the rate of flow and the water-levels vary slowly with time. Then the flow conditions may change from sub-critical (equation 26) to super-critical (equation 27) and back. This involves a choice between the two equations. Super-critical conditions, in fact, mean a discontinuity in the water surface.

2.6. Quasi-steady uniform flow

This is the case of a long flood wave in which the flow is virtually steady and uniform because the variations take place very slowly. Although the velocity remains practically the same, a variation of the discharge occurs in connection with a variation in the cross-sectional area.

The equation of continuity:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{3}$$

leads to an expression for the celerity of the flood wave:

$$c = \frac{\partial Q}{\partial A} = \frac{1}{B}\,\frac{\partial Q}{\partial \bar{h}}$$

The value of $\frac{\partial Q}{\partial \bar{h}}$ may be derived from equations (20) but also an observed discharge rating curve may be used for the purpose.

2.7. Non-steady flow

In this case the variations in the velocity are able to cause marked effects of inertia. The phenomenon is a translation wave as may be caused by the filling of a lock chamber or the start of a pumping-station. Also tidal waves are of this nature.

The equations of continuity:

$\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = 0$ and motion $\frac{\partial \bar{v}}{\partial t} + g\frac{\partial h}{\partial x} = 0$ combine to the differential equation

$$\frac{\partial^2 h}{\partial t^2} = gh\frac{\partial^2 h}{\partial x^2} \tag{29}$$

This equation has solutions in the form of:

$$h = f(x \pm ct) \tag{30}$$

in which

$$c = \sqrt{gh}$$

is the celerity of the wave in the case that the wave height (z) is small compared with the water depth (h).

The velocities within the wave are proportional to the local value of z according to:

$$\bar{v} = \frac{cz}{h}. \tag{32}$$

The effects of acceleration and the gradient of the flow are much more important in short surface waves.

Extreme cases of non-steady flow are breaking waves and bores in which very abrupt changes of velocity and water-level take place.

2.8. *Effects of wind*

The drift currents of a steady uniform nature have already been mentioned in Section 2.3. Generally the velocity of flow is a few percent of the wind speed.

Wind blowing over an enclosed body of water, such as a lake, cannot generate a steady current on its own and therefore $v = 0$. The equation of motion reduces to:

$$\frac{\tau_s}{\bar{h}} - \rho g I = 0 \text{ with } \tau_s = C_w W_x W = C_w W^2 \cos\alpha$$

and, therefore,

$$I = \frac{C_w W^2 \cos\alpha}{\rho g \bar{h}} \tag{33}$$

is the slope of the water-level.

Internal flows in the enclosed body of water, which may also be a system of canals, will hardly be influenced by a steady wind, although temporary and local currents can occur as a result of variations in the wind and differences in water depth.

The value of $C_w/\rho g$ is of the order of magnitude of 0.2×10^{-6} sec^2/m in sheltered canals and 0.3 to 0.4×10^{-6} sec^2/m in open water.

3. Transportation phenomena

3.1. *Basic equations*

In modern society a great variety of pollutants can get into the water. These can roughly be divided into:

- Dissolved matter, such as chemicals and salt;
- solid matter; e.g., sediments; and
- heat, mainly from power plants.

As to their behaviour in water they can be sub-divided as follows:

- Dissolved matter, heat and very fine particles which move with the water in solution or suspension; and

- coarser materials as sediments which have their own specific modes of transportation.

In all cases high concentrations or temperatures give rise to differences in density and, therefore, to density currents under the action of gravity. The stratification of lakes and the salt wedge in estuaries are examples of these phenomena.

Organic matter and other chemicals can be broken down in the course of time, causing a gradual decrease of the concentration. Heat can be transferred to the atmosphere through the water surface. Sediments can be eroded and deposited.

Low concentrated solutions of matter and heat move with the motions of the water. In this case there are several modes of transport:

- Transportation with the average velocity ($\bar{u}$) of the water;
- molecular diffusion;
- turbulent diffusion;
- dispersion caused by the uneven distribution of the flow over a cross-section;
- dispersion caused by actions of wind and waves;
- dispersion caused by temporary storage besides the main stream; and
- dispersion caused by the intermixing in oscillating tidal flows.

The last six processes only cause a resultant transport when there is a spatial gradient in the concentration.

The transport through a cross-section of unit area caused by the velocity ($\bar{v}$) is:

$$\theta_1 = \bar{v}c \tag{34}$$

The transport of matter by the mixing processes can be regarded as a continuous exchange of unit volumes of water with concentration c_1 and c_2 ($c_2 > c_1$) over a distance (mixing length) l with a velocity v'. Then the transport is:

$$\begin{aligned} \theta_2 &= -v'(c_2 - c_1) \\ &= -v'l\frac{\partial c}{\partial x} \\ &= -D\frac{\partial c}{\partial x} \end{aligned} \tag{35}$$

The negative sign occurs because the direction of transport is towards the area of lower concentration.

The total transport caused by all the processes can, in first approximation, be described by:

$$\theta = \bar{v}c - D\frac{\partial c}{\partial x} \tag{36}$$

where the diffusion coefficient (D) depends on the process involved in relationship with the scale of the whole system. In prismatic canals horizontal dispersion by an uneven velocity distribution and vertical turbulent diffusion play the main roles. Larger scale factors such as wind and tides become important in lakes and seas.

Apart from this equation of motion, a condition of continuity is needed to describe the behaviour of dissolved matter and heat. Three factors appear in the equation for a unit of volume:

- The gradient of the transport $\dfrac{\partial \theta}{\partial x}$;
- the change of concentration $\dfrac{\partial c}{\partial t}$; and
- the breaking down of the matter which is taken proportional to the concentration $\dfrac{c}{\gamma}$ leading to:

$$\frac{\partial \theta}{\partial x} + \frac{\partial c}{\partial t} + \frac{c}{\gamma} = 0 \tag{37}$$

Integration of equations (36) and (37) over a cross-section A leads to

$$\theta = A\bar{v}\bar{c} - AD\frac{\partial \bar{c}}{\partial x} \quad \text{and} \quad \frac{\partial \theta}{\partial x} + \frac{\partial A\bar{c}}{\partial t} + A\frac{\bar{c}}{\gamma} = 0.$$

Elimination of θ gives:

$$\frac{\partial c}{\partial t} + \bar{v}\frac{\partial \bar{c}}{\partial x} - \frac{1}{A}\frac{\partial}{\partial x}\left(AD\frac{\partial \bar{c}}{\partial x}\right) + \frac{\bar{c}}{\gamma} = 0 \tag{38}$$

which is the basic equation for the computation of the concentration (temperature) in a system as a function of time.

Density differences exceeding about 1 kg/m^3 give rise to density currents which can be described by a set of equations such as (3) and (17) for each layer, taking into account the difference in density and the interfacial friction. A simple analytical solution of these simultaneous equations is not possible. In many cases where pollutant drains into a body of water, the zone within which density effects prevail is restricted. Outside that area of initial mixing, the phenomena of convection and diffusion predominate in the transportation process.

The processes of sediment transportation are complicated and, depending on conditions and the nature of sediment, a great number of mathematical models are in use. For computations within a limited range it is often useful to describe the (experimentally obtained) relationship between sediment transport (θ) and velocity (v)

with a simple power function:

$$\theta = mv^n \tag{39}$$

Sediment transports generally occur slowly and subsequent changes in the bed take a long time to develop. For this reason the influence of the sediment transport upon the flow is weak.

3.2. *Transportation in solution*

It is assumed that none of the dissolved matter is broken down or transferred during the process of transportation.

Then, for example, the dumping of a mass M of the matter at a position $x = 0$ and time $t = 0$ in a steady uniform stream with an average velocity v leads to the following distribution of the concentration in time and space:

$$C = \frac{M}{A(4\pi Dt)^{1/2}} \, e^{-\frac{(x-\bar{v}t)^2}{4Dt}} \tag{40}$$

which shows a gradually spreading cloud with a decreasing maximum concentration floating down with the average velocity. This velocity and the diffusion coefficient are kept constant in this case.

3.3. *Transportation and transmission of heat*

A process with a gradual breaking down of the transported matter occurs with the cooling water of power stations where the heat is transported by the water and transferred to the atmosphere in the cooling circuit.

The source of heat is assumed to be constant (R) causing a temperature

$$T_0 = \frac{R}{\rho s_w Q} \tag{41}$$

at a location $x = 0$ along a canal with a width B carrying a discharge Q. Then

$$\gamma = h \rho s_w / \varphi$$

with

h = average depth of the canal
s_w = specific heat of water
φ = transmission coefficient from water to air.

The solution of equation (38) becomes

$$\begin{aligned} T &= T_0\, e^{-\frac{\varphi t}{\rho s_w h}} \\ &= T_0\, e^{-\frac{\varphi B x}{\rho s_w Q}} \\ &= T_0\, e^{-\frac{\varphi F}{\rho s_w Q}} \end{aligned} \tag{42}$$

which shows how the temperature of the water decreases with time (t), with the distance (x) from the source and after traversing a certain area (F).

Under average conditions φ is about 10 cal/m^2 °Cs.

4. Networks

4.1. Components and structures

The system of canals, channels, rivers, flood plains, lakes, reservoirs, culverts, bridges, gates, weirs, pumping-stations, etc. which serves the irrigation or drainage of an area can be reproduced as a system of interdependent equations.

In principle, the system of canals can have two different structures or a combination of the two. These are:

- The structure of a tree or a river basin in which the water can follow only one path from its point of introduction in the system to the outfall, and
- the structure of a net in which meshes occur and in which the water can follow various paths towards the outfall.

The latter structure leads to a much more complicated set of equations.

As is the case in electronic networks, the system is subject to the Kirchhoff laws in which the water-levels and the rates of flow figure instead of the electric voltage and current.

The solution of the system of equations is expected to produce the water-levels and the rates of flow in relevant locations in the network.

4.2. Boundary conditions and data

The solution of the equations depends greatly upon the boundary conditions and data. These can include a great number of aspects, the most important of which are mentioned here.

The geometry of the system is not fixed. Storage capacity and hydraulic resistance may depend on the water-level. The change from sub-critical to a super-critical flow in a transition may virtually cut a branch in the network with respect to the continuity of the water-level. Management may influence the geometry (even automatically) by closing gates, starting pumps, adjusting weirs, etc. at any instant.

The input of water may originate from precipitation but also discharges from upriver, sewage system pumping-stations, etc., may supply water to the system at any location and with an arbitrary variation in time. Similar considerations apply for the discharges of heat and transportable matter.

Water can leave the system under consideration through one or more canals, drainage sluices, pumping-stations, weirs, etc., but also by infiltration into the soil.

Open-water connections such as canals and sluices provide a condition in the form of a water-level which may be constant or variable with time. A pumping-station will draw an almost constant discharge of water.

Other data are the wind, temperature, moisture content of the air, properties of dissolved matter, etc.

4.3. *Steady and non-steady phenomena*

If the system with a dimension L is traversed by phenomena travelling with a velocity c, the time $t_s = L/c$ can be regarded as a characteristic reaction time of the system. It is the time after which a translation wave, a flood wave or a cloud of pollutants leaves the system.

A similar characteristic time (t_i) can be attributed to the inputs into the system. This may be the duration of a period of precipitation or the period of variations of the tide at an outfall or the duration of a discharge of pollutants.

Depending on the relative magnitude of these characteristic times, the phenomena in the system can be regarded as steady or non-steady. The following classification can be made:

- Steady flows with ($t_i \gg t_s$).
- Quasi-steady flow as in a flood-wave ($t_i > t_s$).
- Non-steady flow as in a translation wave ($t_i \leqq t_s$).
- Impulsive phenomenon, as in a bore ($t_i \ll t_s$).

In each of these cases the appropriate equations must be used.

4.4. *Solution methods*

In the course of time a number of methods has been developed to investigate the phenomena in drainage and irrigation systems and in similar networks of canals or conduits. As the increased sophistication of the management and the execution of even greater and more complex works require more reliable and accurate results, a brief review of the main methods is now given:

a. Observations *in situ* with simple deductions can show the solution of many problems encountered in day-to-day management. The method also allows for estimates about the effects of relatively simple interventions. Use can be made of relationships as described in this Chapter.

b. Hydraulic models have proved to be very useful tools in the investigation of complex situations with important interventions to be studied. Steady as well as non-steady flows can be reproduced. Hydraulic models are indispensable in the detailed design of complex transitional structures such as sluices, weirs, syphons, pumping-stations, etc.

c. Hydraulic analogs, consisting of reservoirs interconnected by orifices and tubes, have been used to reproduce the flows and storages in a system of canals. Their uses are restricted, however.

d. Electric analogs are in operation. Two types can be distinguished.
   - The purely electrical analog consisting of electrical components. Difficult to reproduce are the quadratic resistance law and the fact that many properties, such as resistance and storage, vary with the water-level.
   - A combined electrical and mechanical analog has been developed to facilitate the introduction of all these non-linear elements, but this is at the cost of a much slower operation.

e. Digital mathematical models came into use with the construction of large computers and the development of digital mathematics. This type of model can be made to simulate hydraulic phenomena in very complex networks, steady as well as non-steady.

The methods described under (a), (b) and (e) can also deal with transport phenomena. A mathematical model can be combined with other programmes, e.g., to optimise a network of canals or conduits.

These various methods serve to study certain types of problems but large overlaps occur.

Generally the same amount of information is required for the application of different methods to the same problem.

In the order in which the methods have been discussed, an increase in abstraction of the model can be observed. This appears to be attended by an increasing flexibility of the model and, in view of costs, an increasing capacity for large and complex systems.

SAMENVATTING

WATERLOOPKUNDIGE PROBLEEMSTELLING

E. ALLERSMA

In vlakke dicht bevolkte gebieden is het beheer van het water een belangrijk aspect van het leven. Deze bijdrage behandelt enige fundamentele aspecten van de hydraulica van open waterlopen. Zij vormt een inleiding op de meer wiskundig gerichte verhandelingen van dr. Vreugdenhil en ir. Berkhoff, in het kader van een overzicht van de traditionele wiskundige en fysische modellen uit de vloeistofmechanica.

Uitgangspunten vormen de vergelijkingen van continuïteit en beweging (Euler) voor een eenheidsvolume van de vloeistof. Twee opeenvolgende integraties leiden dan tot de basisvergelijkingen voor de bewegingen van water in een open waterloop onder invloed van de versnelling van de zwaartekracht, de rotatie van de aarde, de wrijving langs de bodem en de sleepkracht van de wind over het oppervlak.

Afhankelijk van de relatieve grootte van de versnelling, $\partial\bar{v}/\partial t$, (massatraagheid) en de gradiënt van de stroomsnelheid, $\partial\bar{v}/\partial x$, worden dan de volgende verschijnselen onderscheiden en successievelijk kort beschreven:

$\dfrac{\partial\bar{v}}{\partial t} = 0 \quad \dfrac{\partial\bar{v}}{\partial x} = 0$ permanente eenparige stroom in een prismatisch kanaal met de formules van Chézy, Kutter en Gauckler (Manning).

$\dfrac{\partial\bar{v}}{\partial t} = 0 \quad \dfrac{\partial\bar{v}}{\partial x} \approx 0$ permanente geleidelijk veranderende stroom in niet prismatische waterlopen met verschillende verhanglijnen.

$\dfrac{\partial\bar{v}}{\partial t} = 0 \quad \dfrac{\partial\bar{v}}{\partial x} \neq 0$ permanente op korte afstand veranderende stroom zoals bij overlaten, stuwen, etc.

$\dfrac{\partial\bar{v}}{\partial t} = 0 \quad \dfrac{\partial\bar{v}}{\partial x} = 0$ maar met $\dfrac{\partial Q}{\partial x} \approx 0$ en $\dfrac{\partial Q}{\partial t} \approx 0$; een kwasi permanente en eenparige stroom zoals in een hoogwatergolf.

$\dfrac{\partial\bar{v}}{\partial t} \approx 0 \quad \dfrac{\partial\bar{v}}{\partial x} \approx 0$ niet permanente geleidelijk veranderende beweging zoals in translatiegolven en getijden.

$\dfrac{\partial\bar{v}}{\partial t} \neq 0 \quad \dfrac{\partial\bar{v}}{\partial x} \neq 0$ snel wisselende bewegingen zoals in korte golven met als extreme gevallen brekende golven en de bore.

In de gevallen waarbij de bovengenoemde factoren klein zijn wordt de waterbeweging beheerst door de wrijving aan de bodem, de zwaartekracht, de aardrotatie en eventueel de wind.

Naast de bewegingen van het water bestaat een toenemende interesse in de bewegingen van stoffen die door het water worden getransporteerd. De mechanismen die daarbij een rol spelen zijn:
- beweging met de gemiddelde stroom
- moleculaire en turbulente diffusie
- dispersie door menging op grotere schaal (getij, wind, tijdelijke berging, ongelijke snelheidsverdeling in een dwarsprofiel)
- dichtheidsstromen
- slepen langs de bodem (vooral sedimenten)
- afbraak van de stof, opname, afgifte aan de atmosfeer

Een tweetal eenvoudige gevallen wordt nader toegelicht.

In een polder, een boezem, een rivier of een irrigatiestelsel vinden de beschreven processen plaats in een systeem van kanalen die samen een min of meer ingewikkeld netwerk vormen. Men onderscheidt daarin de boomstructuur met vertakkingen en de netstructuur met gesloten mazen.

Deze netwerken geven bij de berekening aanleiding tot systemen van vergelijkingen die middels de wetten van Kirchhoff zijn gekoppeld. In hoeverre de verschijnselen kunnen worden beschouwd als permanent of niet permanent hangt af van de relatieve grootte van de reactietijd van het systeem en een karakteristieke tijd van de aan het systeem toegevoerde randvoorwaarden (getij, neerslagperiode, lozing van afval, etc.).

Hulpmiddelen bij het oplossen van deze stelsels van vergelijkingen zijn:

a. Waarnemingen in de natuur met eenvoudige berekeningen,
b. Een waterloopkundig model,
c. Een hydraulisch analogon,
d. Een electrisch analogon en
e. Een wiskundig model.

De onder (a), (b) en (e) beschreven methoden zijn ook geschikt om de bewegingen van meegevoerde stoffen te onderzoeken. In de gegeven volgorde nemen de mate van abstractie en de flexibiliteit van het model toe. Ook de omvang en de ingewikkeldheid van het systeem kunnen groter worden.

III. COMPUTATIONAL METHODS FOR CHANNEL FLOW

C. B. VREUGDENHIL

1. Introduction

In a great number of hydrological problems, such as are met in watersheds, drainage areas, etc., the flow of water in water-courses is of fundamental importance. The same is true in investigations of water quality. It is, therefore, desirable to have a reliable means of analyzing and predicting the water flow. In this chapter the term "channel flow" is used to comprise all kinds of watercourses like rivers, channels, creeks, tidal areas or even closed conduits. Of course there are situations where water flow cannot be described by channels: lakes, seas, tidal flats. For such areas computational methods have been developed (e.g., Leendertse, 1967); they are, however, not discussed in this contribution. In some respects the techniques are similar to those used for channel flow. Also in certain situations a stratification can occur, resulting in two or more layers of water with different densities flowing one over the other more or less independently. Examples are a salt water wedge in an estuary and a warm water layer near a cooling water outlet. Computational methods for such problems exist (Vreugdenhil, 1970) but they are not discussed in this contribution.

From the above it is evident that a computational method to be used in hydrological applications should be capable of covering different situations. An investigation can be concerned with very long flood waves in rivers, or with problems of tidal flow or with the effect of short but severe rainstorms. The method described in the following paragraphs has been designed and used at the Delft Hydraulics Laboratory to operate in all such circumstances. In addition, the method should be able to include complications like weirs, pumping stations, etc. No attempt has been made to give a complete treatment of all possible methods, but a discussion is given of the choice of this specific method. The principles of the methods are given in Pars. 2, 3, 6 and 7. Details contained in Pars. 4, 5 and 8, although essential for justified applications, can be skipped in first reading.

A rather fundamental difference exists between computational methods for steady and for unsteady flow. These are, therefore, discussed separately. The basis for all methods is found in the equations of motion, which in the relevant form for channels read

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\alpha' \frac{Q^2}{A_s}\right) + gA_s \frac{\partial h_s}{\partial x} + g\frac{Q|Q|}{C^2 R A_s} = 0 \tag{2}$$

as derived by Allersma in Chapter II. The notation is explained as follows:

A cross sectional area
Q discharge
t time
x location
A_s conveying cross-section (i.e. the part of A carrying the major portion of the discharge)
h_s water level
C Chézy coefficient for bottom roughness
R hydraulic radius
α' coefficient due to the non-uniform distribution of velocity
g acceleration due to gravity

2. Computational methods for unsteady flow

2.1. *General*

The differential equations (1) and (2), together with suitable initial and boundary conditions, give a complete description of the water flow at every location and at any time. Unfortunately it is not generally possible to solve these equations analytically. An approximate solution has to be found, which usually can be determined only at a number of specified locations and at certain intervals of time. Such a method is called a *finite-difference method.* Generally the solution is found at successive time levels: starting from the previously computed solution at a certain time t a step is made to $t + \Delta t$. Because of the repetitive character of this process it is ideally suited to a computer. For channel systems of some extent it is even practically impossible to work without a computer. Methods of this kind have been published, a.o. by Isaacson, Stoker and Troesch (1958), Meijer, Vreugdenhil and De Vries (1965), Baltzer and Lai (1968).

An extensive literature about the properties of finite-difference methods is available. It follows that a number of basic features should be present to have a reasonably working method. A closer analysis of these properties is given in Paragraph 5. The following properties are required.

(i) *Accuracy:* the representation of waves within the range of interest should be sufficiently accurate with respect to form, propagation and damping.

(ii) *Stability:* errors, introduced at any time level due to inevitable round-off and approximation errors should not be amplified during the process.

(iii) *Flexibility:* the method should be applicable to different situations. As a special point it should be possible to have unequal distances between the computational points because these often are dictated by geometrical or other considerations.

Concerning the question of accuracy it must be realized that the mesh size Δx (distance between computational points) and the time step Δt cannot be chosen at random; they must have a reasonable relation to the dimensions (wave length) and time scales (wave period) characteristic for the problem under investigation. This point is discussed further later on.

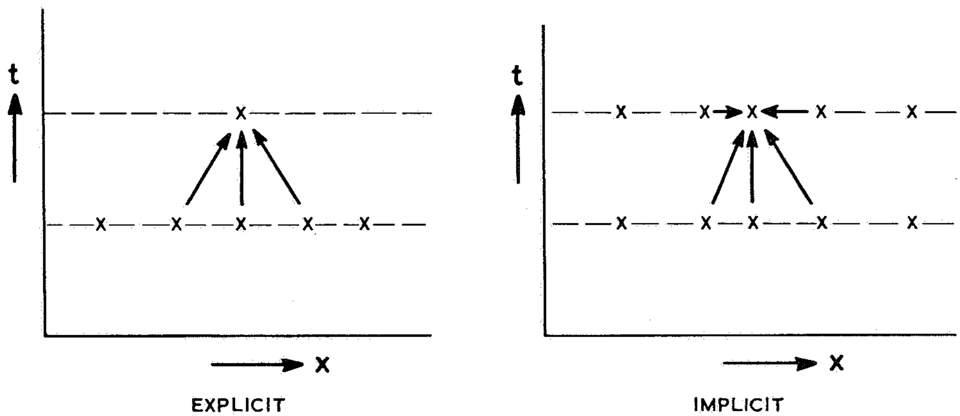


Fig. 1. Explicit and implicit methods.

With respect to stability a distinction is made between *explicit* and *implicit* methods (Fig. 1). In an explicit method the water level and discharge at a computational point are determined from the same point and its upstream and downstream neighbours, all *one time level earlier*. The values then generally can be computed explicitly. In an implicit method in addition the neighbouring points at the *same time level* as the unknown point are used. These, however, generally are still unknown themselves. There follows a system of relations between the unknown values at a new time level. This system has to be solved simultaneously, which would seem to be a considerable drawback when compared to the explicit method. However, explicit methods generally turn out to be stable only if a condition like

$$\sqrt{gh}\,\frac{\Delta t}{\Delta x} < 1 \tag{3}$$

is satisfied, meaning that the speed with which information from some point spreads in the computational scheme should not be smaller than the physical speed of propagation of a disturbance. For an implicit method under certain conditions (Par. 5) no such restriction is present. To show the importance of this, the case of a flood wave in a river is considered. Such waves are quite long, so that for instance $\Delta x = 5$ km could be used. Assuming the depth h to be 5 m it is found that

$$\Delta t < \frac{5000}{7} \approx 700 \text{ s}$$

or approximately 10 min. As the "period" of a flood wave may be several days it is seen that a large number of steps must be taken in the explicit case. Using an implicit method fewer steps may be required, which, though more complicated individually may be more economical. An additional point is that Eq. (3) should also be valid for the smallest mesh Δx and the largest depth h. Sometimes the physical situation dictates the introduction of a very short or very deep channel reach, which would impose severe restrictions to the time step according to Eq. (3), unnecessary in the rest of the system. An implicit method does not require this and therefore is preferred.

2.2. *Difference equations*

The difference equations can be obtained most simply by considering a certain part of the channel or channel system and applying the principles of conservation of mass and momentum. A more formal derivation from the differential equations (1) and (2) is given in Par. 4. Physically a channel or channel system can be thought of as a series of small storage basins connected by conduits with friction and inertia. Such a storage basin (called a "node" of the network) is shown in Fig. 2.

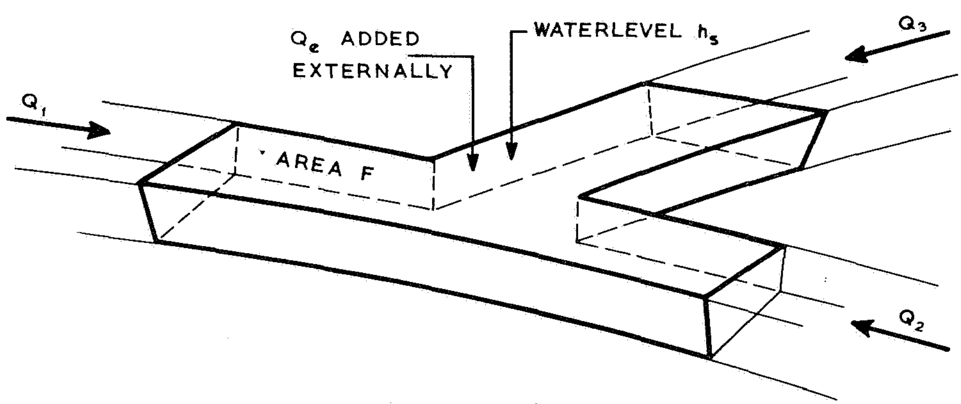


Fig. 2. Continuity in a node.

In unsteady flow the sum of the discharges will not be zero. Therefore during the time interval from t to $t + \Delta t$ a quantity of water

$$(Q_1 + Q_2 + Q_3 + Q_e)\Delta t$$

will flow into the node, where all discharges for the moment are counted positive when flowing into the node. The values have to be taken at some intermediate instant $t + \theta\Delta t$ (the role of the parameter θ is discussed below). This amount of water is stored by a rise of the water level from $h_s(t)$ to $h_s(t + \Delta t)$ over a storage area F:

$$F\{h_s(t + \Delta t) - h_s(t)\} = \theta\Delta t[Q_1 + Q_2 + Q_3 + Q_e]_{t+\Delta t} +$$
$$+ (1 - \theta)\Delta t[Q_1 + Q_2 + Q_3 + Q_e]_t$$

Generally:

$$F\,\frac{h_s(t + \Delta t) - h_s(t)}{\Delta t} = \theta[\sum_j Q_j + Q_e]_{t+\Delta t} + (1 - \theta)[\sum_j Q_j + Q_e]_t \qquad (4)$$

where the summation extends over all branches meeting at the node (for one single channel only two discharges occur).

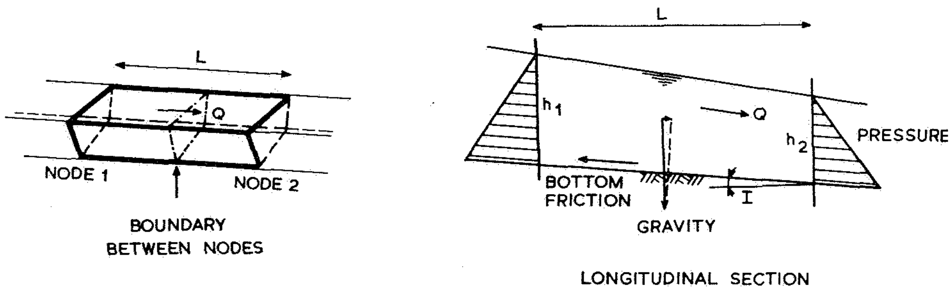


Fig. 3. Forces in a channel section.

To derive the momentum equation Newton's second law (force equals mass times acceleration) is used for a section between two nodes (fig. 3). The forces on the element of water are: pressure difference between the end sections, weight, and bottom friction (possibly also wind stress at the surface). The net force is

$$gA_s(h_1 - h_2) + LA_s g \sin I - Lg\,\frac{Q|Q|}{C^2RA_s}$$

which again has to be evaluated at the instant $t + \theta\Delta t$. The acceleration being given by the change in mean velocity during the interval Δt the momentum equation becomes (assuming A_s to be constant for the moment)

$$\underbrace{L\frac{Q(t+\Delta t)-Q(t)}{\Delta t}}_{\text{inertia}} = \theta\left[\underbrace{gA_s(h_1-h_2)}_{\text{pressure}} + \underbrace{LA_sgI}_{\text{gravity}} - \underbrace{L\frac{Q|Q|}{C^2RA_s}}_{\substack{\text{bottom}\\ \text{friction}}}\right]_{t+\Delta t} +$$

$$+ (1-\theta)\left[gA_s(h_1-h_2) + LA_sgI - L\frac{Q|Q|}{C^2RA_s}\right]_t$$

where the bottom slope I has been assumed to small that $\sin I \approx I$. Taking the variation of A_s and the flow of momentum through the end sections into account (cf. Par. 4) the equation is found to be somewhat more complicated:

$$\frac{L}{gA_s}\,\frac{Q(t+\Delta t)-Q(t)}{\Delta t} - \frac{2QLB}{gA_s^2}\,\frac{h(t+\Delta t)-h(t)}{\Delta t} +$$
$$+ \theta[(1-Fr^2)(h_2-h_1) + \xi Q|Q|]_{t+\Delta t} +$$
$$+ (1-\theta)[(1-Fr^2)(h_2-h_1) + \xi Q|Q|]_t - IL = 0 \qquad (5)$$

where ξ is an abbreviation for $LC^{-2}R^{-1}A_s^{-2}$ and B is the storage width. In this equation the average $(h_1 + h_2)/2$ is taken for the mean water level at either time level. The discharge Q is the discharge halfway the element, or, more correctly, at the boundary between the two adjacent nodes (cf. Par. 4).

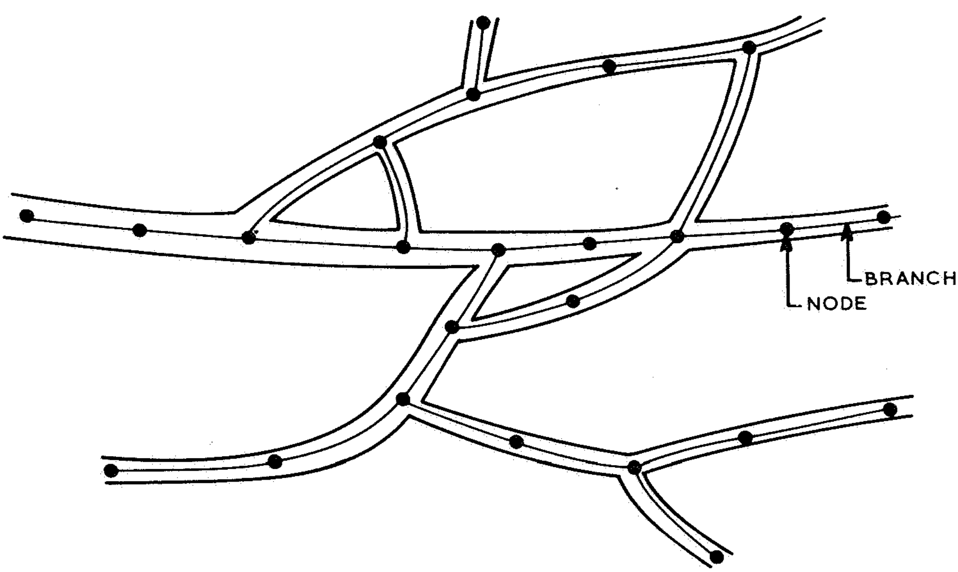


Fig. 4. Example of schematization.

Eqs. (4) and (5) can be written down for each node and branch in a channel system (Fig. 4). Unknowns are the water levels (depths) at each node and the discharges in each branch. The numbers of equations and unknowns consequently are equal and the system can be solved at each time level. In the derivation no assumptions have been made concerning the type of water-course involved. The equations therefore can be applied to open channels, rivers, partially-filled sewers, etc.

As the difference equations (4) and (5) are nonlinear there is no alternative to an iterative method of solution. By analogy to the Gauss-Seidel or successive relaxation method for systems of linear equations the method could work as follows. First an initial approximation is determined to start the process. In an unsteady flow situation the discharges and water levels at the preceding time level can be used very well for this purpose. Next one of the equations (4) or (5) is considered, say Eq. (5) for branch 2. All terms are evaluated from the initial estimates except those containing $Q_2(t + \Delta t)$. From the remaining equation a new approximation to $Q_2(t + \Delta t)$ can be solved. This value replaces the initial approximation, after which the process is carried out for a different branch. After each branch and node has been treated a step in the iteration process is completed. Such steps are then repeated until the approximations converge to a solution. For computational purposes some modifications to this process can be applied as described in Par. 4.

2.3. *Accuracy*

Unsteady flow can be considered as a combination of *waves* of different length, height and shape, propagating in different directions. To make a useful computation it is of great importance that these characteristics of waves are reproduced faithfully.

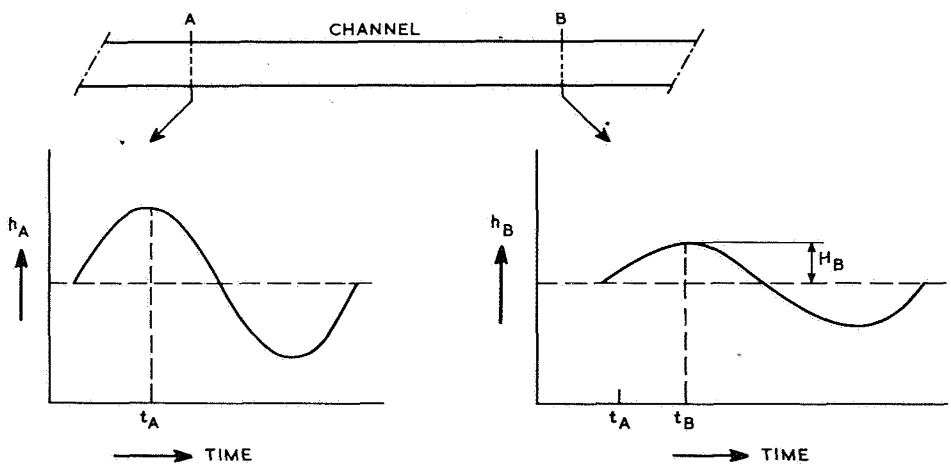


Fig. 5. Wave propagation.

Concentrating on one wave in a single channel for simplicity the following situation could be present physically (Fig. 5). The wave propagates at a certain velocity ($\sim \sqrt{gh}$) and it is damped due to bottom friction. During the passage of the wave from A to B (during a time $t_B - t_A$ equal to the distance $A - B$ divided by the velocity of propagation $\sqrt{gh}$) the amplitude decreases from H_A to H_B. In a numerical computation the result can be different, as shown in Fig. 6.

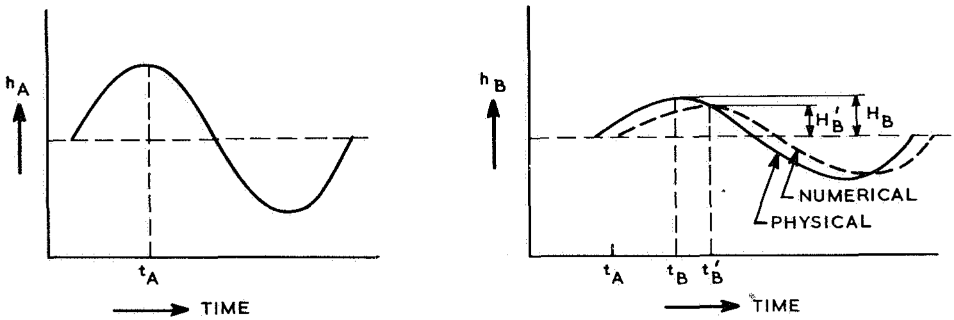


Fig. 6. Numerical wave propagation.

The wave computed numerically can arrive at an instant t'_B earlier or later than t_B. The velocity of propagation of the "numerical wave" is then too high or too low. Also the amplitude H'_B can differ from H_B, indicating that there is an additional (positive or negative) *numerical damping*. Neither of these two effects can be avoided as the differential equations are solved only approximately. The deviations however should be within reasonable limits. It might be required, e.g., that the velocity of propagation and the wave damping per unit of time should be within 5% of their real values. These limits must be considered in relation to the inaccuracy of the physical data, such as the bottom roughness.

In Par. 5 an analysis of these numerical effects is given from which rough estimates for the time step and the mesh size (length of branches) can be derived. General rules can be summarized as follows:

(i) branch lengths should not be larger than about 1/20 to 1/10 of the *relevant* wave lengths in the problem;

(ii) the closer the weighting parameter θ is to 0.5 (meaning a straightforward average between the two time levels), the greater is the accuracy. For reasons of stability, however, θ may not be smaller than 0.5.

Selecting a relevant wave length is not always a simple matter. In any case it is possible to consider the results afterwards to determine whether significant numerical effects could have occurred, in which case the computation can be repeated with greater accuracy.

3. Facilities

A computer program for a computational method as discussed in the preceding paragraph will need a considerable amount of data. To ensure a flexible use it is necessary that this amount is reduced to the minimum and that the computer takes care of the greatest possible part of the processing of data. With the present day computer technology there are numerous possibilities, such as the direct processing of data in graphical form. There will, however, always be a difficult link between the field measurements (e.g. of channel cross-sections) and the data for the computer. It is not the purpose of this section to go into this matter in great detail, but a few subjects of importance are discussed below.

3.1. Network layout

The layout of a channel network must be specified in a convenient form. For flexible use the numbers of nodes and branches should not be subject to restrictions. This can be accomplished by distinguishing between external numbers, specified by the user, and internal numbers, specified by the computer for its own use. All information can be derived from a table specifying the two nodes forming the limits of each branch. In addition a sign convention for the discharge must be adopted.

3.2. Channel geometry and resistance

Generally the data, such as the conveying cross-section A_s, width B etc. depend on the water level. For irregular cross-sections this information must be specified in tabular form. The minimal number of data is obtained by considering the equivalent (average) conveying width B_s for each branch as a function of depth and the storage area F (related to the average storage width B) for each node as a function of depth. From the former the conveying cross-section A_s and the hydraulic radius R can be derived (by the computer). It is noted that discontinuities can be present in these data (Fig. 7) which must be taken care of in the computer program.

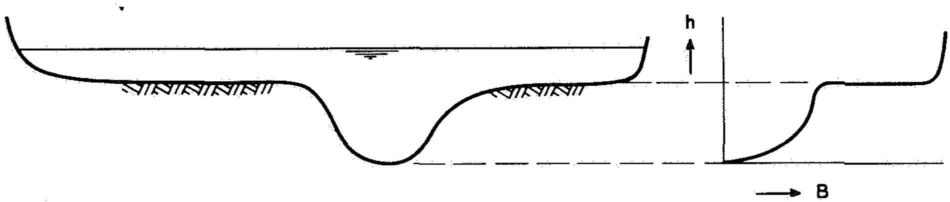


Fig. 7. Discontinuity in width.

If a frictional law is specified together with additional data such as the bottom roughness, the computer can also determine the resistance parameter. Generally this parameter will vary about exponentially with depth, as shown in Fig. 8. If the

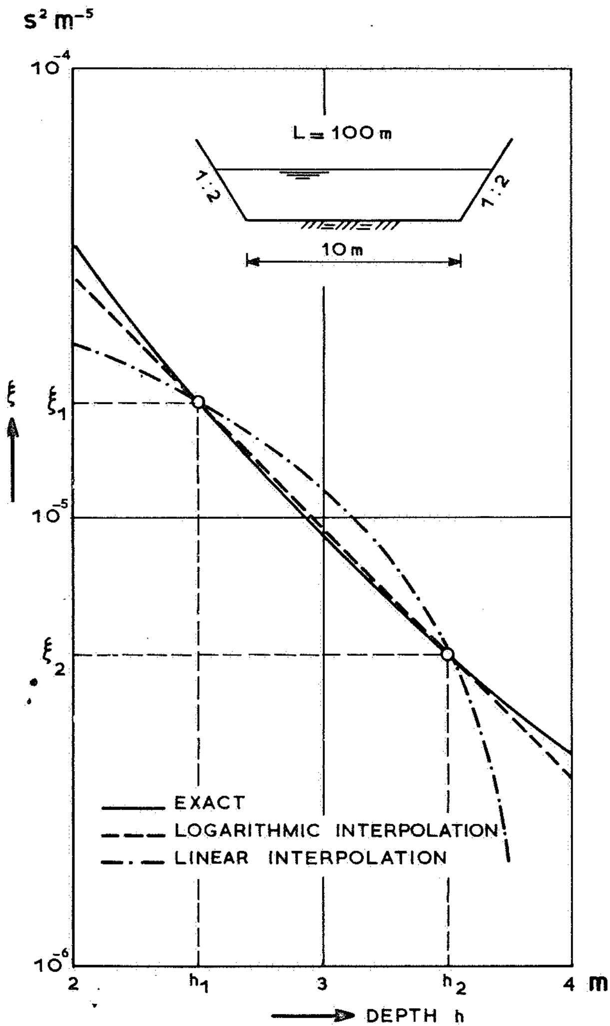


Fig. 8. Logarithmic interpolation of resistance factor.

choice is made not to compute the resistance parameter directly each time it is required, a table can be made in advance at fixed intervals. Then a logarithmic interpolation between the tabulated values should be used:

$$\ln \xi = \frac{(h_2 - h) \ln \xi_1 + (h - h_1) \ln \xi_2}{h_2 - h_1} \qquad h_1 < h < h_2$$

This also applies to the steady flow case (Par. 6). In the linearized process, described in Par. 4 $\xi(t + \theta \Delta t)$ is approximated by

$$\xi(t) + \theta \Delta h \frac{\mathrm{d}\xi}{\mathrm{d}h}(t)$$

where Δh is the increase in depth during one time step. It is seen that the slope $\mathrm{d}\xi/\mathrm{d}h$ should not be determined by straightforward differencing if fixed tabular values are used, as the slope can be very much out of range (dotted line). Instead a difference between close depths should be used, e.g.

$$\frac{\mathrm{d}\xi}{\mathrm{d}h} \approx \frac{\xi\{h(t) + \delta\} - \xi\{h(t)\}}{\delta}$$

where $\delta = 1$ cm.

3.3. Boundary conditions

Boundary conditions can be present in different forms. As an initial condition the simplest method is to take zero velocities and a horizontal water surface, or, as the next simple situation, steady flow. The latter can be determined by methods described in Par. 6. As a boundary condition a water level can be specified as a function of time at a node. In such a node the equation of continuity (4) is disregarded, as there will be an unknown discharge into or out of the system at that node. Boundary conditions for the discharge are taken care of by introducing the correct amount of water into a node. The most difficult boundary conditions are those relating discharge to the water level (weirs or pumping stations). These may also be present within the system, thus forming "internal" boundaries, which can be treated by introducing a special kind of branch where a relation between discharge and water levels (pump characteristic or weir formula) replaces Eq. (5). Also the situation in a different part of the system may be involved, e.g., in the case of a discharge regulation controlled by a water level at a remote station. In the case of "external" boundary conditions of this type, the program should be able to take care of an input Q_e as a function of the local water level.

3.4. Inflow

Especially in hydrological applications the rate of inflow Q_e at each node is of great importance. It can be of different types:

- rainfall runoff directly into the channel system, specified as a function of time at each location;
- water from external sources (e.g. polders) discharged into the system at a given (possibly varying) rate;
- water flowing into or out of the system by means of weirs, pumps, etc.; see section 3.3;
- groundwater flow into or out of drainage channels, determined by rainfall and

water levels at present and previous times. It can be determined from mathematical models of groundwater flow by coupling them to the channel-flow computation;

— evaporation, if of any importance.

It will be clear from this discussion that one computation requires a considerable amount of imput data. As soon as these have been collected, however, the entire system is represented by the information in, e.g., a punched-card deck, in which modifications can be introduced quite simply and a rerun on the computer is easily made. Actual computer costs often are unimportant compared to the preparation of data and the interpretation of results in an investigation.

*4. Difference equations

In this section a more fundamental approach is given to arrive at the difference equations (4) and (5). The idea of this derivation is not really different from that in Par. 2, but in this way a better interpretation of the relevant parameters, such as the storage area F is obtained. Also it will be shown that the equations formally are difference approximations to the differential equations (1) and (2) if a single channel is considered.

4.1. Equation of continuity

Consider the system of channels shown in Fig. 9.

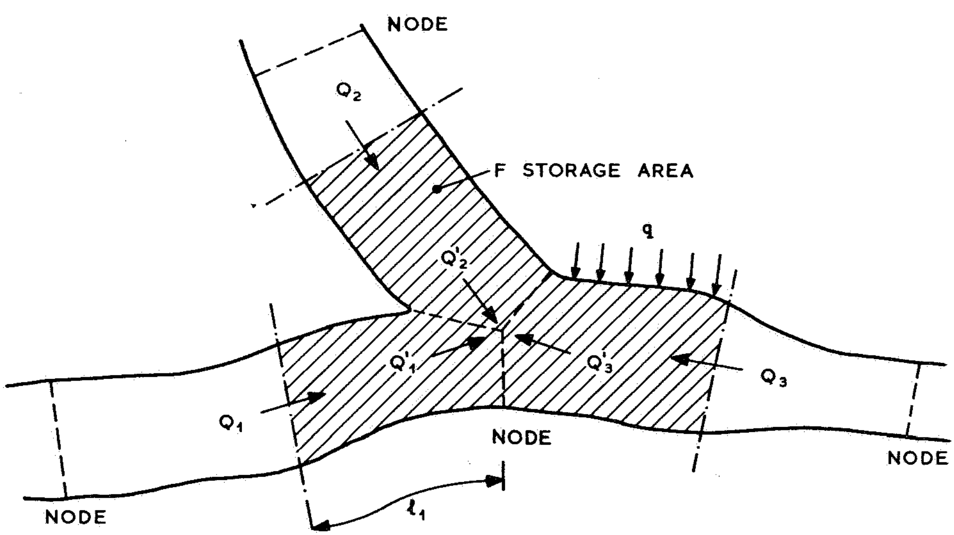


Fig. 9. Equation of continuity: definitions.

The equation of continuity (1), including lateral inflow q per unit of length is integrated over each branch from a certain limiting section (not necessarily halfway) to the node under consideration.

$$\int_0^{l_1} \left(\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q \right) \mathrm{d}x = 0$$

Carrying out the integration results in

$$\frac{\partial}{\partial t} \int_0^{l_1} A \,\mathrm{d}x + Q_1' - Q_1 - \int_0^{l_1} q \,\mathrm{d}x = 0 \tag{7}$$

where Q_1' is the discharge into the location called "node". Eq. (7) is now summed over all branches meeting at the node, i.e. over branches 1, 2, 3 in the example:

$$\frac{\partial}{\partial t} \sum_j \int_0^{l_j} A \,\mathrm{d}x + \sum_j Q_j' - \sum_j Q_j - \sum_j \int_0^{l_j} q \,\mathrm{d}x = 0 \tag{8}$$

In this equation $\sum Q_j'$ will be zero except if there is an inflow or outflow of water at the node (e.g., by a pumping station). The lateral inflow q represents rainfall, evaporation and ground water flow. The two quantities together make up the *net inflow* Q_e into the node, and this can be regarded as concentrated in the node itself.

$$\sum_j Q_j' - \sum_j \int_0^{l_j} q \,\mathrm{d}x = -Q_e$$

The first term of Eq. (8) represents the rate of change of the contents of water, present in the volume bounded by the channel sections (dash-dot in Fig. 9, see also Fig. 2). This rate of change can only be caused by a change of the water level h_s:

$$\frac{\partial}{\partial t} \sum_j \int_0^{l_j} A \,\mathrm{d}x = F \frac{\mathrm{d}h_s}{\mathrm{d}t}$$

The storage area F therefore is the total area at the water level within the boundaries of the node. Eq. (8) now becomes

$$F \frac{\mathrm{d}h_s}{\mathrm{d}t} = \sum_j Q_j + Q_e \tag{9}$$

A second integration is now carried out with respect to time from t to $t + \Delta t$; after division by Δt this gives

$$F\frac{h_s(t+\Delta t)-h_s(t)}{\Delta t}=\frac{1}{\Delta t}\int_t^{t+\Delta t}(\sum_j Q_j+Q_e)\,\mathrm{d}t$$

The right-hand member can be approximated by any weighted average of the values at t and $t+\Delta t$:

$$F\frac{h_s(t+\Delta t)-h_s(t)}{\Delta t}=\theta[\sum_j Q_j+Q_e]_{t+\Delta t}+(1-\theta)[\sum_j Q_j+Q_e]_t \tag{10}$$

where the greatest accuracy is obtained if $\theta=\frac{1}{2}$. Eq. (10) is identical to Eq. (4). The value of F to be used in the left-hand member should be evaluated at $t+\Delta t$.

4.2. *Momentum equation*

The momentum equation in finite-difference form is also obtained by integration,

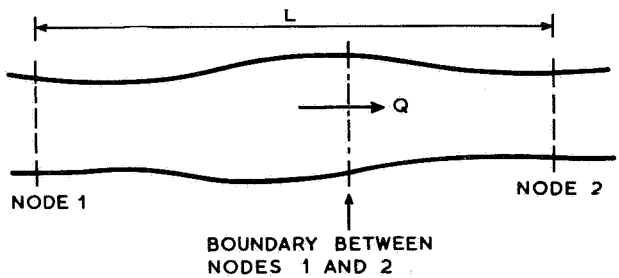


Fig. 10. Momentum equation, definitions

now over one branch (Fig. 10). Eq. (2) is first transformed because otherwise the second term would give difficulties. Assuming $\alpha'=1$, this term can be written as

$$\frac{\partial}{\partial x}(Q^2/A_s)\Rightarrow 2\frac{Q}{A_s}\frac{\partial Q}{\partial x}-\frac{Q^2}{A_s^2}\frac{\partial A_s}{\partial x}$$

The former term can be transformed into

$$-2\frac{Q}{A_s}\frac{\partial A}{\partial t}=-2\frac{QB}{A_s}\frac{\partial h}{\partial t}$$

using the equation of continuity (1) and defining B as the storage width (full width at the water level). Further

$$\frac{\partial A_s}{\partial x}=\frac{\partial}{\partial x}(B_s h)=B_s\frac{\partial h}{\partial x}+h\frac{\partial B_s}{\partial x}$$

where B_s is the conveying width. It is now assumed that the branch is more or less prismatical, in which case $\partial B_s/\partial x \approx 0$. This indicates that one branch should not include important variations in the cross-section. Then Eq. (2) becomes

$$\frac{\partial Q}{\partial t} - 2\frac{QB}{A_s}\frac{\partial h}{\partial t} + gA_s(1 - Fr^2)\frac{\partial h}{\partial x} - gA_sI + g\frac{Q|Q|}{C^2RA_s} = 0 \qquad (11)$$

where I is the bottom slope (positive if sloping downwards in the positive x-direction) and

$$Fr^2 = \frac{Q^2B_s}{gA_s^3}$$

Eq. 11 is integrated with respect to x over a full branch:

$$\frac{\partial}{\partial t}\int_0^L Q\mathrm{d}x - 2\frac{QB}{A_s}\frac{\partial}{\partial t}\int_0^L h\mathrm{d}x + gA_s(1 - Fr^2)(h_2 - h_1) - gI\int_0^L A_s\mathrm{d}x +$$

$$+ g\int_0^L \frac{Q|Q|}{C^2RA_s}\mathrm{d}x = 0 \qquad (12)$$

Note that the coefficients $2QB/A_s$ and $gA_s(1 - Fr^2)$ are taken outside the integration. They should be averaged over the length of the branch. The integrals are now evaluated as follows

$$\int_0^L Q\mathrm{d}x = LQ; \quad \int_0^L A_s\mathrm{d}x = LA_s$$

$$\int_0^L \frac{Q|Q|}{C^2RA_s}\mathrm{d}x = \frac{LQ|Q|}{C^2RA_s}$$

where the right-hand members are taken at the boundary between the adjacent nodes (dash-dot in Fig. 10). This is a good approximation if this boundary is halfway along the branch, if it is not the approximation is less accurate. For a good representation it is therefore necessary that

(i) the properties of one branch do not vary too much over its length;

(ii) the boundary between adjacent nodes should be placed at a representative cross-section, preferably halfway along the branch.

Finally, the following approximation is made:

$$\int_0^L h\mathrm{d}x = \tfrac{1}{2}L(h_1 + h_2)$$

Eq. (12) now becomes, after division by gA_s:

$$\frac{L}{gA_s}\frac{\mathrm{d}Q}{\mathrm{d}t} - \frac{QBL}{gA_s^2}\frac{\mathrm{d}}{\mathrm{d}t}(h_1 + h_2) +$$

$$+ (1 - Fr^2)(h_2 - h_1) - IL + \frac{LQ|Q|}{C^2RA_s^2} = 0 \tag{13}$$

In the same way as shown for the equation of continuity this equation is now integrated with respect to time. The non-derivative terms are evaluated as a weighted average at time $t + \theta\Delta t$:

$$\frac{L}{gA_s}\frac{Q(t+\Delta t) - Q(t)}{\Delta t} +$$

$$- \frac{QBL}{gA_s^2}\frac{h_1(t+\Delta t) + h_2(t+\Delta t) - h_1(t) - h_2(t)}{\Delta t} +$$

$$+ \theta[(1 - Fr^2)(h_2 - h_1) + \xi Q|Q|]_{t+\Delta t} +$$

$$+ (1 - \theta)[(1 - Fr^2)(h_2 - h_1) + \xi Q|Q|]_t - IL = 0 \tag{14}$$

which is identical to Eq. (5). Here again the coefficients of the first two terms should be taken at $t + \theta\Delta t$.

4.3. *Difference approximations*

If a single channel is considered, nodes can be distributed at certain (generally unequal) distances (Fig. 11). For convenience numbers are assigned to the nodes and

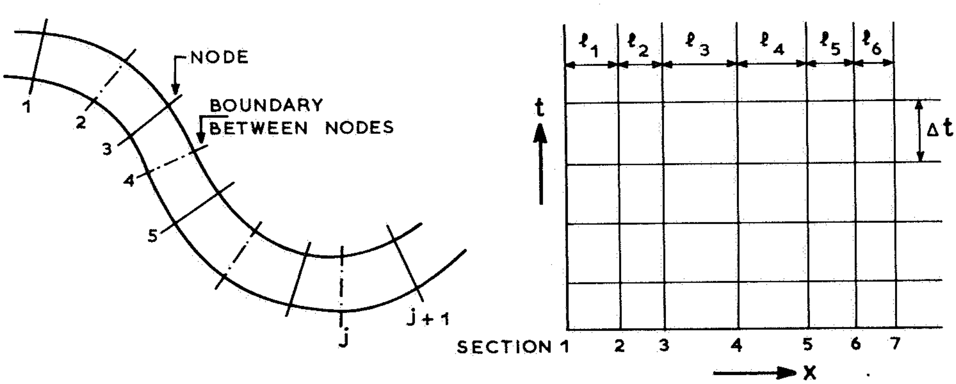


Fig. 11. Schematization of single channel.

their boundaries as shown in the figure. Eqs. (10) and (14) can now be interpreted as finite-difference approximations of the differential equations (1) and (11) relative to this set of cross-sections. For instance, considering node 3 the approximations read

$$\frac{\partial A}{\partial t} = B\frac{\partial h}{\partial t} \approx B_3(t + \theta\Delta t)\frac{h_3(t + \Delta t) - h_3(t)}{\Delta t}$$

$$\frac{\partial Q}{\partial x} \approx \theta\left[\frac{Q_4 - Q_2}{l_2 + l_3}\right]_{t+\Delta t} + (1 - \theta)\left[\frac{Q_4 - Q_2}{l_2 + l_3}\right]_t$$

Introducing this into Eq. (1) yields Eq. (10) with

$$F = (l_2 + l_3)B_3(t + \theta\Delta t)$$

It is seen that the x derivative is centered halfway between sections 2 and 4 which generally is not section 3. The two terms are, therefore, evaluated at different locations, which causes a low order of approximation. The highest order of accuracy is obtained if all l_i are equal and $\theta = \frac{1}{2}$ (see also Par. 5). A similar result is obtained for the momentum equation (11) by substituting

$$\frac{\partial Q}{\partial t} \approx \frac{Q_4(t + \Delta t) - Q_4(t)}{\Delta t}$$

$$\frac{\partial h}{\partial t} \approx \frac{h_5(t + \Delta t) + h_3(t + \Delta t) - h_5(t) - h_3(t)}{2\Delta t}$$

$$\frac{\partial h}{\partial x} \approx \theta\left[\frac{h_5 - h_3}{l_3 + l_4}\right]_{t+\Delta t} + (1 - \theta)\left[\frac{h_5 - h_3}{l_3 + l_4}\right]_t$$

Then Eq. (14) is found with $L = l_3 + l_4$. Again the time and spatial derivatives are not evaluated at the same location unless all l_i are equal.

4.4. *Linearization*

Solution of the equations (10) and (14) for each node and each branch is possible by an iterative technique as indicated in Par. 3. However, because the coefficients (F, A_s etc.) depend on the water levels, they have to be re-evaluated after each iteration. This can be avoided by linearizing the equations. In this way again ad approximation is introduced, but this is not more serious then the approximation already present in the difference equations as such. The linearization is accomplishen by defining

$$Q(t + \Delta t) = Q(t) + \Delta Q$$

$$h(t + \Delta t) = h(t) + \Delta h$$

and by neglecting all terms containing Δh and/or ΔQ to higher than first powers. The coefficients are evaluated by means of a Taylor series, e.g.

$$F(t+\theta\Delta t) = F\{h(t+\theta\Delta t)\} =$$

$$= F\{h(t)\} + \left[\frac{\mathrm{d}F}{\mathrm{d}h}\right]_t \{h(t+\theta\Delta t) - h(t)\} + \ldots =$$

$$= F(t) + \left[\frac{\mathrm{d}F}{\mathrm{d}h}\right]_t \{\theta\Delta h + \ldots\} + \ldots \approx$$

$$\approx F(t) + \theta\Delta h \left[\frac{\mathrm{d}F}{\mathrm{d}h}\right]_t$$

neglecting higher powers of Δh. It is seen that coefficients have to be evaluated at time t only. They do not change during the iteration, so they have to be determined only once every time-step. Introducing this kind of approximations systematically the following equations result

$$F\frac{\Delta h}{\Delta t} = \sum Q_j + \theta\sum \Delta Q_j + Q_e + \frac{\mathrm{d}Q_e}{\mathrm{d}h}\theta\Delta h \tag{15}$$

$$\frac{L}{gA_s}\frac{\Delta Q}{\Delta t} - \frac{QBL}{gA_s^2}\frac{\Delta h_1 + \Delta h_2}{\Delta t} + (1 - Fr^2)(h_2 - h_1) + \xi Q|Q| - IL +$$
$$+ \theta(1 - Fr^2)(\Delta h_2 - \Delta h_1) + 2\theta\xi|Q|\Delta Q +$$
$$- \theta(h_2 - h_1)\left\{\frac{\partial Fr^2}{\partial Q}\Delta Q + \frac{1}{2}\frac{\partial Fr^2}{\partial h}(\Delta h_1 + \Delta h_2)\right\} +$$
$$+ \tfrac{1}{2}\theta\frac{\mathrm{d}\xi}{\mathrm{d}h}Q|Q|(\Delta h_1 + \Delta h_2) = 0 \tag{16}$$

All coefficients (not containing Δ) are now taken at time t. In Eq. 15 it has been assumed that Q_e depends on h; if it does not and is just a function of time, Q_e should be specified at $t + \theta\Delta t$ and $\mathrm{d}Q_e/\mathrm{d}h = 0$. The resulting equations involve the same variables as the original ones, but they are now linear in Δh and ΔQ, which facilitates the solution.

4.5. *Solution of the difference equations*

The first choice to be made when solving systems of linear equations is that between direct and iterative methods. It should be realized

(i) that the number of unknowns can be rather large (say several hundreds)
(ii) that each equation does not involve all unknowns but only a few of them (eq.

of continuity: 1 + number of connected branches, momentum eq.: 3), so that the matrix of coefficients is sparse.

The fact that the situation at time t is a good initial approximation to that at $t + \Delta t$ has been used already in the process of linearizing the equations.

Direct solution will not take into account the sparseness of the matrix if the nonzero elements are not located systematically. Secondly, iterative methods for any matrix are more efficient if the number of iterations required is small relative to the dimension of the matrix. This is even more so if the matrix is sparse, because in iterative methods it is very well possible to use only the nonzero elements. Finally for large matrices iterative methods are generally preferred, although the idea of what is large is shifting (upward). From these considerations a choice can be made for an iterative method in the present case. Indeed it turns out that the number of iterations is small (in the order of 5–10). The method applied is the Gauss-Seidel method of successive relaxation: any value, as soon as computed, is taken into account for the subsequent computations. This choice is rather heuristic: there are different iterative methods in existence, which can show a faster convergence. These methods, however, require an "acceleration parameter" which governs the convergence and which must be chosen according to the properties (especially the eigenvalues) of the matrix. As this is almost impossible for any but very special matrices, the advantage of such methods cannot be utilized.

Finally, it is noted that the size of the system of equations can be reduced significantly by eliminating the changes in the water level Δh using Eq. (15). Substituting this into Eq. (16), a system is obtained in which only the values ΔQ occur. It can be made plausible that the convergence is not worse than for the original system. The amount of work therefore is reduced.

*5. Numerical effects

5.1. General

A closer investigation of the stability and accuracy of the numerical method cannot be done for the general case of a network of channels. For this purpose a single channel as shown in Fig. 11 is considered. The entire analysis is a local one, i.e., it is assumed that local conditions at the point considered prevail in the entire network. If local properties are satisfactory, it is assumed that the complete system also works well.

First some of the relevant notions are reviewed. For a general theory cf. Richtmyer and Morton (1967). For simplicity a further restriction is made by writing the momentum equation as

$$\frac{\partial Q}{\partial t} + gA_s \frac{\partial h_s}{\partial x} + rQ = 0 \qquad (17)$$

The equation of continuity is repeated for convenience

$$B\frac{\partial h_s}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{18}$$

The coefficients A_s, r and B are assumed to be constant (locally).

A difference approximation of these equations should satisfy the following requirements.

(i) *Consistency:* if the mesh width Δx and the time step Δt vanish the difference equations should approach the differential equations. The deviation between the differential and difference equations is called the *truncation error*. For consistency, therefore, the truncation error should vanish when Δx and Δt vanish.

(ii) *Stability:* inevitably errors are introduced into the computation by round-off and by truncation. A method is called stable if these errors do not grow with time.

(iii) *Convergence:* even if the difference equations approach the differential equations for vanishing Δx and Δt (consistency), the solutions of both do not necessarily do so. If they do, the method is called convergent. Then the *discretization error* between the solutions of the difference and differential equations vanishes. The "equivalence theorem" by Lax (cf. Richtmyer and Morton, 1967) states that consistency and stability are sufficient for convergence. This means that a separate analysis of convergence is generally not necessary.

5.2. *Consistency*

The difference equations corresponding to the simplified equations (17) and (18) are (Fig. 11)

$$\frac{Q_j^{n+1} - Q_j^n}{\Delta t} + \theta g A_s\left[\frac{h_{j+1}^{n+1} - h_{j-1}^{n+1}}{L_j} + rQ_j^{n+1}\right] + $$

$$+ (1-\theta)\left[gA_s\frac{h_{j+1}^n - h_{j-1}^n}{L_j} + rQ_j^n\right] = 0 \tag{19}$$

$$B\frac{h_{j+1}^{n+1} - h_{j+1}^n}{\Delta t} + \theta\frac{Q_{j+2}^{n+1} - Q_j^{n+1}}{l_j + l_{j+1}} + (1-\theta)\frac{Q_{j+2}^n - Q_j^n}{l_j + l_{j+1}} = 0 \tag{20}$$

where the superscript n denotes the time level $n\,\Delta t$. The consistency can be investigated by expanding each term into a Taylor series with respect to the point $(x_j, t + \theta\Delta t)$ for Eq. (19) and $(x_{j+1}, t + \theta\Delta t)$ for Eq. (20). This yields for the two equations

$$\frac{\partial Q}{\partial t} + (\tfrac{1}{2} - \theta)\Delta t\frac{\partial^2 Q}{\partial t^2} + gA_s\left\{\frac{\partial h}{\partial x} + \tfrac{1}{2}(l_j - l_{j-1})\frac{\partial^2 h}{\partial x^2}\right\} + rQ + \ldots = 0$$

$$B\left\{\frac{\partial h}{\partial t} + (\tfrac{1}{2} - \theta)\Delta t\frac{\partial^2 h}{\partial t^2}\right\} + \frac{\partial Q}{\partial x} + \tfrac{1}{2}(l_{j+1} - l_j)\frac{\partial^2 Q}{\partial x^2} + \ldots = 0$$

where higher order terms are neglected. It is seen that the truncation error contains *first order* terms in Δt and Δx ($= l_j + l_{j+1}$) unless $\theta = \frac{1}{2}$ and $l_j = l_{j+1}$. Only if the latter two conditions are satisfied is there second order accuracy. It is seen that always the truncation error vanishes if Δx and $\Delta t \to 0$, so the method is consistent.

5.3. *Stability*

Stability according to its definition is related to error growth. Assuming errors to be present in Q_j and h_{j+1} at time t, the difference equations determine the magnitude of these errors at the next time level. If the equations are linear, as they are in this example, the errors satisfy the same difference equations as the values of Q and h themselves. Eqs. (19) and (20) can therefore also be looked upon as error-propagation equations. Assume now, according to Von Neumann (Richtmyer and Morton, 1967) that the errors can be developed into a Fourier series. Each term looks like

$$Q_j^n = \bar{Q}^n \mathrm{e}^{ikx_j}$$

$$h_j^n = \bar{h}^n \mathrm{e}^{ikx_j}$$

where complex notation is used for convenience. At the time level $n + 1$ similar expressions will be valid. From the difference equations it is then found that (locally)

$$\frac{\bar{Q}^{n+1} - \bar{Q}^n}{\Delta t} + gA_s\{\theta\bar{h}^{n+1} + (1 - \theta)\bar{h}^n\}\frac{\mathrm{e}^{ikl_j} - \mathrm{e}^{-ikl_{j-1}}}{l_j + l_{j-1}} +$$

$$+ r\{\theta\bar{Q}^{n+1} + (1 - \theta)\bar{Q}^n\} = 0$$

$$B\frac{\bar{h}^{n+1} - \bar{h}^n}{\Delta t} + \{\theta\bar{Q}^{n+1} + (1 - \theta)\bar{Q}^n\}\frac{\mathrm{e}^{ikl_{j+1}} - \mathrm{e}^{-ikl_j}}{l_j + l_{j+1}} = 0$$

or

$$\begin{pmatrix} 1 + \theta r\Delta t & \theta gA_s\Delta t a \\ \theta\Delta t\dfrac{b}{B} & 1 \end{pmatrix}\begin{pmatrix} \bar{Q}^{n+1} \\ \bar{h}^{n+1} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 - (1 - \theta)r\Delta t & -gA_s\Delta t(1 - \theta)a \\ -(1 - \theta)\Delta t\dfrac{b}{B} & 1 \end{pmatrix}\begin{pmatrix} \bar{Q}^n \\ \bar{h}^n \end{pmatrix}$$

where

$$a = \frac{\mathrm{e}^{ikl_j} - \mathrm{e}^{-ikl_{j-1}}}{l_j + l_{j-1}} \text{ and } b = \frac{\mathrm{e}^{ikl_{j+1}} - \mathrm{e}^{-ikl_j}}{l_j + l_{j+1}}$$

Now the magnitude of the errors cannot grow if the eigenvalues of the matrix

$$\begin{pmatrix} 1+\theta r\Delta t & \theta\Delta t g A_s a \\ \theta\Delta t\dfrac{b}{B} & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1-(1-\theta)r\Delta t & -(1-\theta)\Delta t g A_s a \\ -(1-\theta)\Delta t\dfrac{b}{B} & 1 \end{pmatrix}$$

do not exceed unity. This is therefore the condition for stability. These eigenvalues λ are defined by the determinantal equation

$$\begin{vmatrix} 1-\lambda-r\Delta t(\theta\lambda+1-\theta) & -gA_s a\Delta t(\theta\lambda+1-\theta) \\ -\Delta t\dfrac{b}{B}(\theta\lambda+1-\theta) & 1-\lambda \end{vmatrix} = 0$$

or, introducing $p=(\lambda-1)(\theta\lambda+1-\theta)^{-1}$ as a variable:

$$\begin{vmatrix} p+r\Delta t & gA_s a\Delta t \\ \Delta t\, b/B & p \end{vmatrix} = 0$$

from which

$$p_{1,2} = -\tfrac{1}{2}r\Delta t \pm \sqrt{(\tfrac{1}{2}r\Delta t)^2 - c^2 ab\Delta t^2} \tag{22}$$

where $c=(gA_s/B)^{\frac{1}{2}}$ is the velocity of propagation for long surface waves. With some patience it can be shown that $|\lambda| \leqq 1$ if

$$\operatorname{Re} p \leqq (\theta-\tfrac{1}{2})\,|p|^2$$

If $l_j = l_{j+1} = \Delta x$

$$p_{1,2} = -\tfrac{1}{2}r\Delta t \pm i\sqrt{\left(c\frac{\Delta t}{\Delta x}\right)^2 \sin^2 k\Delta x - (\tfrac{1}{2}r\Delta t)^2} \tag{23}$$

It may be safely assumed that the expression under the square root is positive. Then $\operatorname{Re} p$ is seen to be always negative. Stability is then ensured if

$$\theta \geqq \tfrac{1}{2} \tag{24}$$

This is not a quite sharp condition but it is a sufficient one. The general case $l_j \neq l_{j+1}$ is difficult to analyse. On the other hand the stability analysis as a whole is only approximative for the general non-linear equations. The criterion (24) has proved to be sufficient in most cases. Attention is drawn to the fact that the stability is only marginal if $\theta = \frac{1}{2}$ for maximal accuracy.

5.4. *Wave propagation*

The resistance term in the differential equations, better approximations being lacking, can be assumed to represent the frictional damping satisfactorily. In numerical calculations additional numerical damping can originate from quite different causes. It will occur *in excess* to the physical damping of a wave. For the numerical analysis the latter can, therefore, be disregarded (except possibly for the case of flood waves).

The wave propagation by the numerical method is to be compared to the desired behaviour which is assumed to be well represented by the differential equations (as far as the linear properties are concerned, expressed by Eqs. (17) and (18)). If the initial condition

$$\begin{aligned} Q(x, 0) &= \bar{Q}\, e^{ikx} \\ h(x, 0) &= \bar{h}\, e^{ikx} \end{aligned} \tag{25}$$

is introduced the solution will have the form

$$\begin{aligned} Q(x, t) &= \bar{Q}\, e^{i(kx - \omega t)} \\ h(x, t) &= \bar{h}\, e^{i(kx - \omega t)} \end{aligned}$$

where ω is determined by substitution of these expressions into Eqs. (17) and (18).

$$\begin{aligned} &- i\omega\bar{Q} + gA_s ik\bar{h} = 0 \\ &- i\omega B\bar{h} + ik\bar{Q} = 0 \end{aligned}$$

There is a solution only if

$$- \omega^2 B + gA_s k^2 = 0$$

or

$$\omega = \pm\, ck \quad \text{with} \quad c = (gA_s/B)^{\frac{1}{2}}$$

The solution represents two waves running with a velocity c in opposite directions without damping. Concentrating on the forward running wave it is found that after one wave period $T = 2\pi/\omega$:

$$Q = \bar{Q}\, e^{ikx - 2\pi i} \tag{26}$$

and similarly for the water level.

If the same initial condition (25) is applied to the difference equations (19) and (20), a similar solution is found, for after one time step

$$\frac{e^{-i\omega\Delta t} - 1}{\Delta t}\bar{Q} + (\theta\, e^{-i\omega\Delta t} + 1 - \theta)\frac{gA_s}{\Delta x} i\bar{h} \sin k\Delta x = 0$$

$$B\frac{e^{-i\omega\Delta t} - 1}{\Delta t}\bar{h} + (\theta\, e^{-i\omega\Delta t} + 1 - \theta)\frac{\bar{Q}}{\Delta x} \sin k\Delta x = 0$$

where for simplicity $l_j = \Delta x$ has been taken as constant. From these equations the factor $e^{-i\omega\Delta t}$ can be determined and by comparison it is seen to be identical to λ in the analysis of stability. The numerical solution, therefore, in each time step is multiplied by a factor λ. To cover a wave period T there are $T/\Delta t$ time steps, so after one period

$$Q_{\text{num}} = \bar{Q}\lambda^{T/\Delta t}\, e^{ikx} \tag{27}$$

with

$$\lambda = \frac{1 + (1-\theta)ic\dfrac{\Delta t}{\Delta x}\sin k\Delta x}{1 - \theta ic\dfrac{\Delta t}{\Delta x}\sin k\Delta x} \tag{28}$$

This should be compared to the "physical" solution (26). Formally Eq. (27) can be written as

$$Q_{\text{num}} = \bar{Q}D\, e^{ikx - c_r 2\pi i}$$

where D is the numerical damping factor per wave period and c_r is the relative velocity of propagation, i.e. the ratio between the velocities of the numerical and physical solutions (these concepts have been introduced by Leendertse, 1967). By comparison with Eq. (27) it is seen that

$$D = |\lambda|^{T/\Delta t} \tag{29}$$

and

$$c_r = -\frac{T}{\Delta t}\arg(\lambda)/2\pi \tag{30}$$

Both quantities should ideally be unity. If $D < 1$ a numerical damping is present. The numerical wave runs too fast or too slow if $c_r > 1$ or $c_r < 1$ respectively. The number of time steps $T/\Delta t$ can be written as

$$T/\Delta t = \frac{2\pi}{\omega\Delta t} = 2\pi\left(k\Delta x . c\frac{\Delta t}{\Delta x}\right)^{-1}$$

In combination with Eq. (28) it is seen that both D and c_r are determined by the three parameters θ, $\mu = c\Delta t/\Delta x$ and $k\Delta x = 2\pi\Delta x/l$ if l is the wave length considered. The relationships are illustrated in Fig. 12 for $\theta = 0.55$ (satisfying the stability condition and close to 0.5 for accuracy).

These figures bring out an important point. As discussed in Par. 2 one of the advantages of the implicit method is that the parameter $\mu = c\Delta t/\Delta x$ is not restricted as in explicit methods. The price paid for this, however, is a decrease of the accuracy.

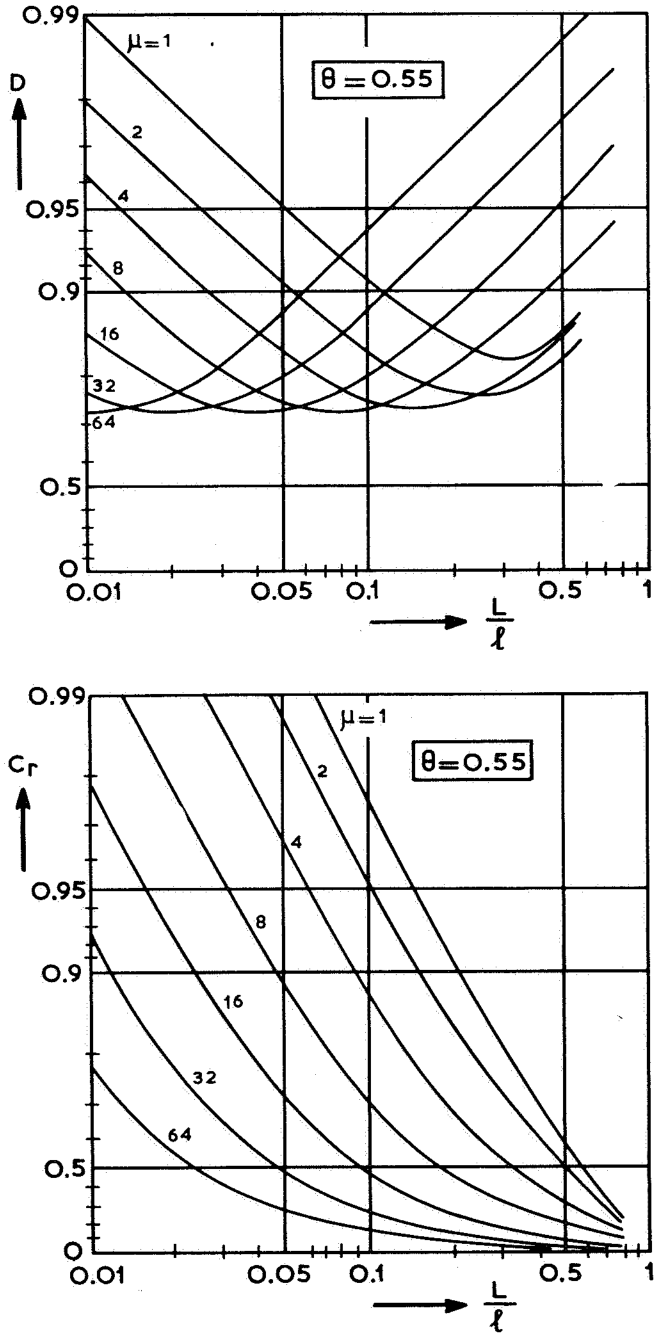


Fig. 12. Numerical effects.
L = branch length, l = wave length, $\mu = 2c\Delta t/L$.

This is especially so for the velocity of propagation. It is therefore not always possible to utilize the advantage. Still, for certain applications it is valid, e.g. if very large wave lengths are considered (flood waves in rivers). Then L/l will be small and μ can be taken quite large.

The procedure to determine the branch length L (which equals twice the mesh width Δx) and the time step Δt is as follows.

(i) Determine the relevant wave length in the problem, i.e. the length of the basic wave or any of its harmonics if the shape of the wave is also important.

(ii) Determine the number of wave periods it is running before reaching boundaries.

(iii) Set conditions for acceptable damping and propagation speed. If the wave is running freely for a long time (item ii), a smaller damping per period will be allowed than for a short time.

(iv) Find a combination of L/l and $\mu = 2c\Delta t/L$ from Fig. 12, which meets the requirements. There usually are several possibilities. Then try to have L/l and μ as large as possible to minimize the amount of work.

(v) From these values the order of magnitudes of L and Δt can be derived.

An example is given in Chapter V. Although it has been assumed in the theoretical analysis that all branch lengths are equal there are reasons to assume that the same results apply to unequal branches showing the same *average* length.

6. Computational methods for steady flow

Setting up special computational methods for steady flow would not seem to be necessary as it is a special case of unsteady flow. An unsteady flow situation with boundary conditions kept constant in time will generally approach steady conditions in the long run. Actually this can be considered as an iterative process for computing steady flow, each time step forming one iteration. However, more efficient methods can be found. Historically these were developed before unsteady cases were attacked. Reference can be made to the classic work by Hardy Cross (1936) which also forms the basis of the method discussed below. A large number of research papers has appeared since but the basic method has hardly been changed.

6.1. *Equations*

In the steady-flow case a similar schematization of nodes and branches is used as shown in Par. 2. Nodes will be present at all junctions and important points of inflow. If the resistance factors ξ depend on the water level (see below) also intermediate points will be required for accuracy. The equations can be derived from those for unsteady flow (4) and (5) by assuming the situations at t and $t + \Delta t$ to be identical. From the equation of continuity (4) this gives

$$\sum_j Q_j = Q_e \tag{31}$$

at each node, meaning that the discharges in the branches at a node just balance the inflow.

The momentum equation along a branch can be derived from Eq. (5):

$$(1 - Fr^2)(h_2 - h_1) - IL + \xi Q|Q| = 0 \tag{32}$$

For many applications the Froude number Fr will be quite small. Generally, therefore, the following equation is used

$$h_{s2} - h_{s1} = -\xi Q|Q| \tag{33}$$

where water levels instead of depths have been introduced. Eqs. (31) and (33) correspond to Kirchhoff's laws in electrical networks, modified for the quadratic resistance.

If there are m nodes and n branches Eqs. (31) and (33) form a system of $m + n$ equations in the m unknown water levels and n unknown discharges. More precisely, if the water level is specified as a boundary condition at p nodes, the equation of continuity (31) is omitted at those nodes and there remain $m + n - p$ equations in $m + n - p$ unknowns. The system is non-linear in general, because

(i) the discharge occurs quadratically in Eq. (33);

(ii) the resistance coefficient ξ may depend on the water level and/or the discharge,

(iii) the external inflow Q_e may depend on the water level (in the case of a pump or weir).

The solution therefore has to be determined iteratively.

6.2. Iterative solution

The equations can be attacked in several ways. In all cases an initial approximation is guessed which is improved systematically until successive approximations agree sufficiently closely. This can be done directly with the given system of equations (31) and (33). However, it is possible to eliminate either the discharges or the water-levels, which gives a smaller number of equations to be solved. In the former case the discharge Q of each branch is solved from Eq. (33) and substituted into the corresponding Eqs. (31). There remains a system of $m - p$ equations in $m - p$ unknowns. An advantage is that a variation of the resistance factor ξ with the waterlevel can be treated without difficulty. A disadvantage is that in each equation only neighbouring nodes occur. It takes therefore a number of iterations before the influence of a correction to a certain node reaches the entire network.

If the waterlevels are eliminated the concept of *meshes* or loops is used. It is then noted that a network contains $n - m + p$ independent meshes (Fig. 13) i.e., closed loops in which each branch occurs at least once. It is not proved here that the number is exactly $n - m + p$; the proof can be given by induction. Any mesh above this number will be dependent, which means that its mesh equation (34) (see below) can

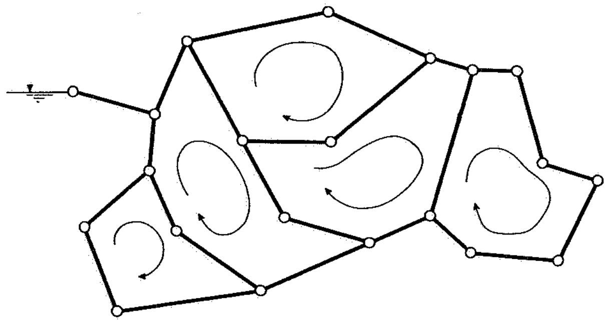


Fig. 13. Elementary meshes.

be found as a combination of the mesh equations for some of the independent meshes. If there is more than one given water level, open loops can be formed running between them. It is noted that the choice of the meshes is not unique; in Fig. 13 "elementary" meshes are shown, but a different possibility is shown in Fig. 14.

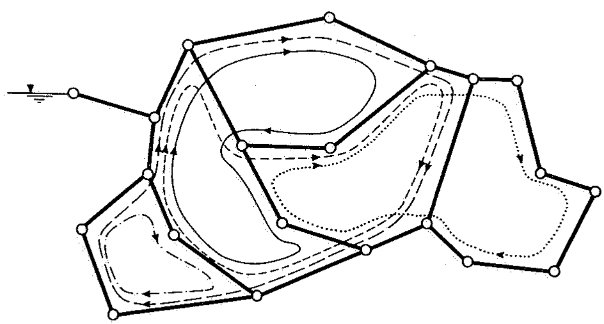


Fig. 14. Non-elementary meshes.

Adding Eq. (33) for each branch along a closed loop must give zero as the water levels at the starting and finishing points are the same. Doing so along an open mesh will result in the difference between the given water levels at the end points. The mesh equations therefore read

$$\sum_j \xi_j Q_j |Q_j| = a_k \text{ for mesh } k \tag{34}$$

where a_k is zero for a closed mesh and equal to the difference between the given waterlevels for an open mesh. The idea of the mesh method is that the discharge in each branch can be thought to be built up from loop discharges in each mesh, the number of which is $n - m + p$. The water levels do not occur explicitly in Eq. (34) if the resistance factors ξ_j are constant. The mesh equations then constitute a system of $n - m + p$ equations in $n - m + p$ unknowns. Unless the network has a great number of interconnections, the number of meshes $(n - m + p)$ is much smaller than the number of nodes $(m - p)$. This is especially so if nodes have to be distributed along long branches because of accuracy, or because of lateral inflow. This is often the case in networks of open channels. For such situations the mesh iteration method therefore has the advantage of a smaller number of equations over the node-iteration method discussed before.

In practice the mesh method starts by the construction of an initial approximation $Q_j^{(0)}$ to the discharges, satisfying the equation of continuity (31). These initial guesses will not satisfy the mesh equations. Therefore corrections q_k in each mesh k are sought, such that

$$\sum_j \xi_j(Q_j^{(0)} + q_k)|Q_j^{(0)} + q_k| = a_k$$

Linearizing (assuming q_k to be small), this gives

$$\sum_j \xi_j Q_j^{(0)}|Q_j^{(0)}| + 2q_k \sum_j \xi_j |Q_j^{(0)}| = a_k$$

or

$$q_k = \frac{a_k - \sum_j \xi_j Q_j^{(0)}|Q_j^{(0)}|}{2\sum_j \xi_j|Q_j^{(0)}|}$$

The numerator is the residual when substituting the initial approximation into the mesh equation. As soon as q_k has been computed all discharges in the corresponding mesh are modified with it. Then the next mesh is treated. Due to the linearization and to the interactions between meshes the solution will not yet be obtained when all meshes have been treated. The process is then repeated until convergence is obtained. This is a method of *successive relaxation*. In a manual computation (Hardy Cross) one would first pick out the meshes with the largest residuals for correction. In an automatic computation there is no point in doing this because the computation of the residuals is about as complicated as the determination of the correction itself. It is advantageous, however, to correct meshes only until the correction becomes less

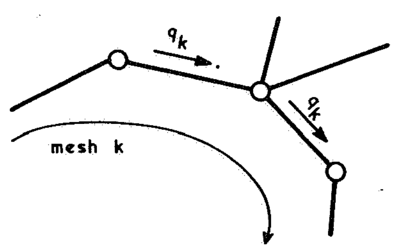


Fig. 15. Mesh correction and continuity.

than a certain limit. When all meshes have reached this limit it is lowered and the iteration goes on. This process is repeated until the final convergence limit is reached. It is noted that the corrections q_k do not modify the equations of continuity, as in each node two branches will be affected, one flowing into and one out of the node (Fig. 15). An advantage of this method is the rapid spreading of the influence of an individual correction if non-elementary meshes are chosen (Fig. 14).

So far the resistance factors ξ_j have been assumed to be constant. In open channels they generally depend on the waterlevels. Often, however, the variation is only slight within the range of levels met during the iteration. Therefore the resistance factors can be kept constant during a number of steps, after which they are readjusted and the iterations are continued. Only during the readjustments is it necessary to compute all waterlevels. If the resistance factors show a more important variation with the water level, this technique may cause a slow convergence, as each readjustment disturbs the converged solution. In such cases the node iteration may be more efficient, although a general comparison of this advantage with the consequences of a larger number of equations is difficult. A similar technique can be applied if the external inflow Q_e depends on the local water level (see also section 7.3).

7. FACILITIES

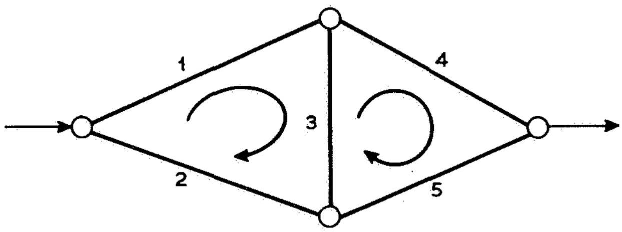
For a convenient use of a steady flow computation along the lines discussed above, a flexible computer program is of great importance, maybe even more than in the case of unsteady flow. The remarks made in Par. 3 about the coding for the network layout, the computer part in data processing and the calculation of resistance factors also apply here. In addition a few points are discussed specifically concerning the steady flow case.

7.1. *Initial approximation*

The computation requires an initial approximation for the discharges, satisfying the equation of continuity. Such an approximation can easily be made manually by running through the network from a certain point and adding all external discharges that are met. This is, however, a rather inefficient method especially for large networks. A computer method is possible which uses the information gathered during the construction of optimal meshes (Par. 8). In this process a number of branches is removed from the network, such that a "tree" structure is obtained (i.e. a network without any closed loops) containing all nodes. The removed branches are selected such that they can be expected to carry relatively little flow. In such a structure the flow is uniquely determined by the inflow at each node, which constitutes an initial approximation. The flow in the removed branches is assumed to be zero initially. This will be a reasonable starting approximation.

7.2. Choice of meshes

As has been noted in Par. 6 the choice of the meshes is not unique. On the other hand it is not arbitrary as it has been found to influence the convergence of the process in a decisive way. A simple example will clarify this (Fig. 16). If elementary



BRANCH	ξ	$Q^{(o)}$
1	10	2
2	1	-8
3	1000	0.35
4	1	1.65
5	10	-8.35

(ARBITRARY UNITS)

Fig. 16. Illustration of convergence.

meshes are chosen as shown in the figure, the convergence is as shown in the first columns of the following table. The convergence is seen to be slow, which is caused by the common branch 3 which has a large resistance coefficient. Balancing one mesh has the effect that the other is disturbed, and conversely. If one of the elementary meshes is replaced by the mesh 3 consisting of branches 1, 4, 5 and 2 the convergence is as shown in the second part of the table. The convergence now is much more rapid because the common branches do not have a great influence.

iteration	elementary meshes corrections		non elementary meshes corrections	
	mesh 1	mesh 2	mesh 1	mesh 3
1	−0.131	1.218	−0.131	3.240
2	0.501	0.222	−0.518	0.259
3	0.346	0.369	−0.171	0.085
4	0.341	0.272	−0.019	−0.010
5	0.287	0.254	−0.0014	−0.0007
6	0.252	0.215	convergence	
7	0.214	0.184		
8	0.180	0.155		
9	0.150	0.129		
10	0.123	0.106		

On the basis of this and similar effects, strategies have been developed to make an *optimal* choice for the meshes, i.e. a choice which results in the fastest convergence (or, more precisely, in the smallest amount of work to reach a certain accuracy). These methods usually are designed so as to avoid "heavy" branches (branches with high resistance factors) occurring in several meshes. A rather successful strategy has been given by Travers (1967). It is discussed more closely in Par. 8 where it is also shown that the important point is not the resistance coefficient ξ of a branch but the product $\xi|Q|$. A method is, therefore, proposed which consists of four steps:

1. determination of optimal meshes based on the ξ-factors; connected with this is an initial approximation of the discharges (par. 7.1);
2. iteration until the discharges have converged to the correct order of magnitude;
3. determination of a new set of optimal meshes, based on the $\xi|Q|$ factors;
4. continuation of the iteration until final convergence.

7.3. *Inflow depending on waterlevel*

In the mesh-iteration method as such it is not possible to have external inflows Q_e depending on the local water level, because it would not be possible to satisfy the equation of continuity at the relevant node in the initial approximation, the water level (and therefore the inflow Q_e) being unknown. For a number of common cases, however, an approximate method is possible as follows. A pumping-station is shown as an example but a similar method is possible in other cases (Fig. 17).

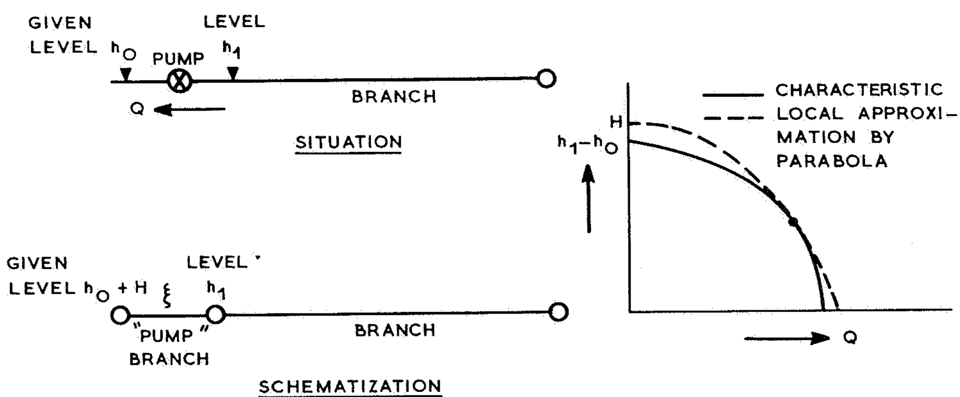


Fig. 17. Pumping station as boundary condition.

The pump characteristic can be written as

$$h_1 - h_0 = f(Q)$$

where f denotes an empirical function. In the neighbourhood of the expected (guessed) solution Q_0 for the discharge this function can be approximated by a parabola as follows:

$$h_1 - h_0 = H - \xi Q^2$$

such that at Q_0 the value of $h_1 - h_0$ and the slope of the curve are correct:

$$H - \xi Q_0^2 = f(Q_0)$$

$$-2\xi Q_0 = f'(Q_0)$$

from which H and ξ can be determined. The pump can now be replaced by an imaginary branch with a resistance factor ξ, if the given water level is increased by H (Fig. 17). This branch can be treated in the same way as the normal ones. In this way an open mesh will be needed in addition to the existing meshes. After some iterations a better approximation for the discharge through the pump will be known. The coefficients H and ξ can then be improved (e.g. at the same time as the readjustment of water-level-dependent ξ-values in the rest of the network). This process can be repeated until convergence is reached.

*8. Convergence and optimal meshes

The choice of the meshes has a great influence on the convergence of the iteration. This is because the meshes determine the structure of the system of equations to be solved. In this paragraph an investigation of this structure is given, aimed at formulating criteria for the choice of the meshes.

8.1. *Structure of the equations*

To formalize the notation the *incidence matrix* with elements c_{ij} is introduced

$$c_{ij} = \begin{cases} +1 \text{ if branch } j \text{ is connected to node } i \text{ with its positive direction pointing away from } i; \\ 0 \text{ if branch } j \text{ is not connected to node } i; \\ -1 \text{ if branch } j \text{ is connected to node } i \text{ with its positive direction pointing towards } i \end{cases}$$

As mentioned before, the choice of the positive direction is arbitrary. With this matrix it is possible to formulate Eqs. (31) and (33) as follows (note that s and e are no number subscripts, but abbreviations of surface and external respectively):

$$\sum_{i=1}^{m} c_{ij} h_{si} = \xi_j Q_j |Q_j| \qquad (j = 1, \ldots, n) \tag{35}$$

$$\sum_{j=1}^{n} c_{ij} Q_j = Q_{ei} \qquad (i = 1, \ldots, m - p) \tag{36}$$

A second incidence matrix with element b_{kj} describes the composition of meshes (in which an arbitrary positive direction is defined):

$$b_{kj} = \begin{cases} +1 \text{ if branch } j \text{ belongs to mesh } k, \text{ positive directions coinciding} \\ 0 \text{ if branch } j \text{ does not belong to mesh } k \\ -1 \text{ if branch } j \text{ belongs to mesh } k, \text{ positive directions opposite.} \end{cases}$$

Then by multiplication of Eq. (35) with b_{kj} and summation over j (i.e. by summation of Eq. (35) over all branches belonging to mesh k) it is found that

$$\sum_{j=1}^{n} b_{kj} \sum_{i=1}^{m} c_{ij} h_{si} = \sum_{j=1}^{n} b_{kj} \xi_j Q_j |Q_j| \qquad (k = 1, \ldots, n - m + p)$$

The left-hand member gives by reversing the operations:

$$\sum_{i=1}^{m} h_{si} \sum_{j=1}^{n} b_{kj} c_{ij} = a_k$$

where use has been made of the constraints valid for the matrix b_{kj}, viz.:

(i) for any node in a closed mesh and for any node except the end-nodes for an openmesh *two* of the connected branches will belong to a mesh k passing through the node; therefore, taking care of the signs:

$$\sum_{j=1}^{n} b_{kj} c_{ij} = 0 \tag{37}$$

(ii) for the end-points of open meshes only one branch will belong to the open mesh k:

$$\sum_{j=1}^{n} b_{kj} c_{ij} = \pm 1 \tag{38}$$

It follows that the mesh equations read

$$\sum_{j=1}^{n} b_{kj} \xi_j Q_j |Q_j| = a_k \qquad (k = 1, \ldots, n - m + p) \tag{39}$$

It is noted that $b_{kj} Q_j \geqq 0$ if the discharge Q_j flows in the positive direction of loop k, irrespective of the positive direction of branch j.

To obtain the solution of the equations (39) and (36) the discharges, if satisfying the equation of continuity (36), can be composed from mesh-discharges q_k:

$$Q_j = Q_{j0} + \sum_{k=1}^{n-m+p} b_{kj} q_k \tag{40}$$

Here for the purpose of theoretical analysis Q_{j0} is chosen as the *solution* of the problem, which of course in practice is not known. The values Q_{j0}, forming the true solution, satisfy the equation of continuity. So does the approximate value (40), as found by substituting it into Eq. (36) and taking the relations (37) into account. This formally describes the fact that (except for end-nodes of open meshes) a mesh-flow q_k always enters *and* leaves a node.

The iteration process described in Par. 6 now consists in finding an initial approximation (corresponding to certain non-zero values of q_k in Eq. (40)) and correcting each mesh successively by balancing Eq. (39) which was not satisfied initially. The purpose is to determine the solution of the system

$$\sum_{j=1}^{n} b_{kj}\xi_j\{Q_{j0} + \sum_{i=1}^{n-m+p} b_{ij}q_i\}|Q_{j0} + \sum_{i=1}^{n-m+p} b_{ij}q_i| = a_k \qquad (k = 1, \dots, n-m+p) \tag{41}$$

The obvious solution is $q_i = 0$, but the purpose here is to see whether the iteration process will converge to this solution and, if so, how rapidly.

8.2. *Convergence*

The system of equations (41), being non-linear, does not lend itself to theoretical analysis. If it is assumed that the mesh-flows q_k are small, the equations can be linearized. The following analysis, therefore, is valid once the process is approaching the solution; it is an analysis of *asymptotic* convergence. Linearizing Eq. (41) results in

$$2\sum_{j=1}^{n} b_{kj}\xi_j|Q_{j0}|\sum_{i=1}^{n-m+p} b_{ij}q_i = a_k - \sum_{j=1}^{n} b_{kj}\xi_j Q_{j0}|Q_{j0}| = 0 \tag{42}$$

or

$$\sum_{i=1}^{n-m+p} d_{ki}q_i = 0 \qquad (k = 1, \dots, n-m+p) \tag{43}$$

where

$$d_{ki} = 2\sum_{j=1}^{n} b_{kj}b_{ij}\xi_j|Q_{j0}| \tag{44}$$

The latter equation (except for the signs) sums the contributions $\xi_j|Q_{j0}|$ over those branches common to meshes k and i.

The method of successive substitution employed for the iteration amounts to the Gauss-Seidel method when applied to the linearized system (43). The convergence of this method has been the subject of many theoretical investigations (Varga, 1963). One of the results is the following theorem: for a symmetric matrix with positive diagonal elements the Gauss-Seidel method converges if, and only if, the matrix is positive-definite, i.e.:

$$S = \sum_{i=1}^{n-m+p}\sum_{k=1}^{n-m+p} d_{ki}x_k x_i \geqq 0 \text{ for any vector with components } x_k \tag{45}$$

From the definition (44) of d_{ki} it is easily seen that the matrix is symmetric and that the diagonal elements d_{kk} are positive. Then

$$S = \sum_{i=1}^{n-m+p} \sum_{k=1}^{n-m+p} \sum_{j=1}^{n} 2b_{kj}b_{ij}\xi_j |Q_{j0}| x_k x_i =$$

$$= 2 \sum_{j=1}^{n} \xi_j |Q_{j0}| \sum_{i=1}^{n-m+p} b_{ij}x_i \sum_{k=1}^{n-m+p} b_{kj}x_k =$$

$$= 2 \sum_{j=1}^{n} \xi_j |Q_{j0}| \{ \sum_{i=1}^{n-m+p} b_{ij}x_i \}^2 \geqq 0$$

which proves that the matrix is indeed positive definite and *the mesh-iteration method is convergent.*

8.3. Rate of convergence

For some special types of matrices quantitative estimates of the convergence of the Gauss-Seidel method can be given. This is the case for matrices with positive diagonal and negative off-diagonal elements and matrices having the so-called "property A" (Varga, 1963). For such matrices the analysis leads to a sufficient condition for convergence, requiring that the diagonal elements of the matrix (in absolute value) exceed the sum of the other elements in the corresponding row (in absolute values). The more the diagonal elements dominate, the better is the convergence.

Unfortunately it can be shown that neither of the above-mentioned special cases applies to the present system. Still it has been noticed in numerical analysis that diagonal dominance also for other types of matrices often results in rapid convergence. The principle therefore is applied to the matrix d_{ki} although there is *no theoretical basis* for this.

Recalling the definition of the matrix elements d_{ki} (44) it is seen that the diagonal elements sum the "weights" $\xi_j |Q_{j0}|$ over the branches of each mesh and the off-diagonal elements (disregarding the signs) do so over those branches *common* to two meshes. Diagonal dominance can now be obtained by avoiding, as far as possible, the occurrence of branches with important "weights" $\xi_j |Q_{j0}|$ in more than one mesh. They will occur then in the diagonal elements only. This of course is not a strict criterion, but considering the lack of theoretical basis it does not seem useful to formulate a closer one. Attention is drawn to the fact that it is not the resistance factor ξ of a branch which is important, but the product $\xi |Q|$. It is, therefore, not possible to apply the criterion beforehand; first a number of iterations are required to have a reasonable picture of the discharges. This leads to the procedure outlined in Par. 7.2.

8.4. Optimizing the meshes

From the preceding discussion it is clear that a really optimal choice of the meshes is still impossible as a firm criterion is lacking. A number of publications has been devoted however to procedures of finding "good" meshes. One of the most satisfactory methods is that described by Travers (1967) which more or less corresponds to the remarks of section 8.3. The method has been given for the resistance factor ξ but it can as well be applied taking the correct weights $\xi|Q|$ into account once the discharges have been estimated.

The idea of the method is to form a (not necessarily connected if there are open meshes) tree structure (containing no loops) involving all nodes and using branches with weights as small as possible. This tree is also utilized to find an initial approximation to the discharges (which only make sense if the weights ξ are used). Each branch added to the tree will cause exactly one mesh to be formed. As such a branch has a relatively high weight the criterion for optimal meshes is satisfied to some extent (there are, however, examples in which the resulting meshes clearly are not optimal). These branches, each of which defines a mesh, can be called *primary* branches. Their number can be shown to be $n - m + p$ which is exactly the number of meshes required.

The tree structure is formed in two steps. Firstly all branches are arranged in order of increasing "weight". Then a tree is built up from branches in this order. As each branch is investigated there are four possibilities:

(i) neither of its nodes so far belongs to a tree. The branch then is the start of a new tree;
(ii) *one* node belongs to a tree; the branch then is added to that tree;
(iii) *both* nodes belong to the same tree; adding the branch then would cause the appearance of a mesh: it is a primary branch and it is not added to the tree;
(iv) the two nodes are in different trees; the two trees are then combined into one.

In this way a number of trees is formed which gradually "grow" by branches being added. Ultimately they will have joined together to the required structure containing all nodes and all branches except the primary ones. The latter now define the meshes in a unique way. Open meshes can be treated by stating in advance that the end-nodes already belong to a tree.

8.5. Illustration

Different choices of meshes have been applied to the network shown in fig. 23 of Chapter V. The rate of convergence is shown in fig. 18 where the largest error in Eq. (39) for any mesh is shown. Three cases are illustrated:

(i) an arbitrary choice of meshes;

(ii) a choice by Travers' method using ξ as weight;

(iii) the same using $\xi|Q|$ as weight, the discharge being known in this case from the preceding results; in practice this should be preceded by iteration with meshes according to (i) or (ii).

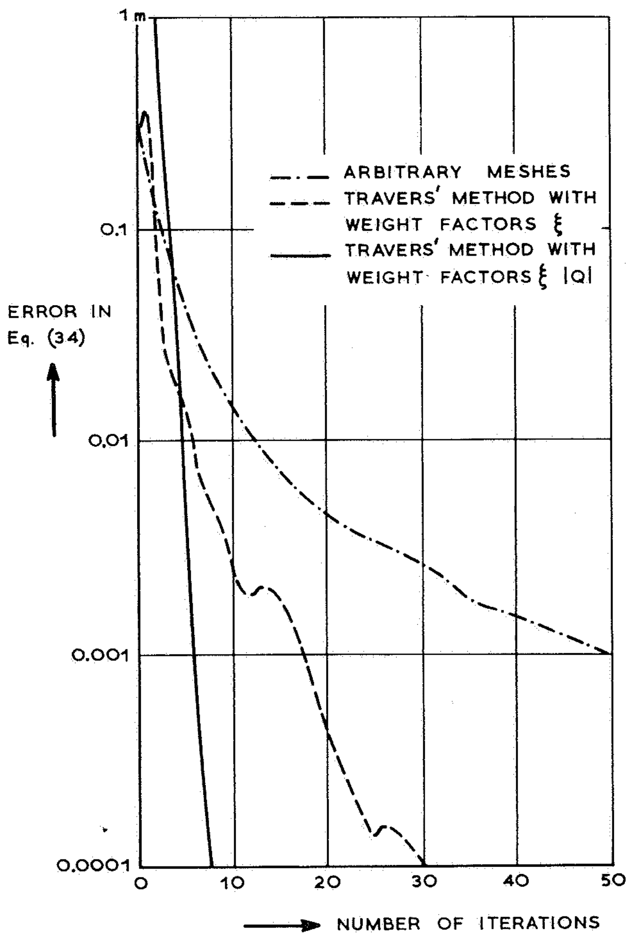


Fig. 18. Convergence of mesh-iteration method

It is seen that a very considerable difference in the rate of convergence is obtained. The arbitrary choice is not necessarily a bad one: by chance it might be as good as or even better than the "optimal" one. It is however probable that it is far from optimal. The use of Travers' method is seen to result in an important improvement, but the use of the correct weights $\xi|Q|$ is still considerably more efficient. It can be concluded that especially for large networks in which convergence may be slow anyway, it is worthwhile to spend some effort in defining "optimal" meshes and that Travers' method, preferably using $\xi|Q|$ as weight factors, gives very satisfactory results.

Acknowledgements

Much of the credit for the work described in this contribution goes to Th. J. G. P. Meijer and B. H. J. van Zanten, who took a great part in the development, programming and application of the computational methods.

NOTATION

a_k	difference of end levels in an open mesh
A	full cross-sectional area
A_s	conveying cross-sectional area
b_{kj}	incidence matrix of branches and meshes
B	storage width
B_s	conveying width
C	Chézy coefficient
c	velocity of propagation
c_r	relative velocity of propagation
c_{ij}	incidence matrix of nodes and branches
d_{ki}	coefficients of linearized system of mesh equations
D	damping factor
F	storage area
Fr	Froude number
g	acceleration due to gravity
h	depth
h_s	water level
I	bottom slope
k	wave number
$l_{1,2}$	lengths of branches connected to node
l	wave length
L	branch length
q	lateral inflow
q_k	mesh-correction discharge
Q	discharge
Q_e	external inflow
r	linearized friction parameter
R	hydraulic radius

t	time
T	wave period
α'	coefficient for non-uniform distribution of velocity
x	location
Δh	change in depth during one time step
ΔQ	change in discharge during one time step
Δt	time step
Δx	mesh width
λ	eigenvalue
θ	weighting parameter in difference scheme
ω	angular frequency
ξ	resistance factor

REFERENCES

BALTZER, R. A. and C. LAI (1968) – Computer simulation of unsteady flow in waterways, J. ASCE *94*, HY4, 1083–1117.

HARDY CROSS (1936) – Analysis of flow in networks of conduits or conductors, Univ. Illinois Eng. Exp. Station, Bulletin 286.

ISAACSON, E., J. J. STOKER and A. TROESCH (1958) – Numerical solution of flow problems in rivers, J. ASCE *84*, HY5, paper 1810.

LEENDERTSE, J. J. (1967) – Aspects of a computational model for long-period water-wave-propagation, RAND Memorandum RM 5294–PR, also Thesis Delft 1967.

MEIJER, TH. J. G. P., C. B. VREUGDENHIL and M. DE VRIES (1965) – A method of computation for non-stationary flow in open channel networks, IAHR Congress Leningrad, also Delft Hydraulics Laboratory Publ. no. 34.

RICHTMYER, R. D. and K. W. MORTON (1967) – Difference methods for initial value problems, Interscience Publ., New York.

TRAVERS, K. (1967) – The mesh method in gas network analysis, Gas Journal *332*, 167–174.

VARGA, R. S. (1963) – Matrix iterative analysis, Prentice Hall, New York.

VREUGDENHIL, C. B. (1970) – Computation of gravity currents in estuaries, Delft Hydraulics Laboratory Publ. no. 86.

SAMENVATTING

REKENMETHODES VOOR STROMING IN OPEN WATERLOPEN

C. B. VREUGDENHIL

Bij een onderzoek van waterbeheersing, en op langere termijn, vooral ook bij een verantwoord waterbeheersings-beleid, is een betrouwbaar instrument voor het vaststellen van de waterbeweging onmisbaar. Deze bijdrage beschrijft methodes voor de berekening van permanente of niet-permanente stromingen, in het bijzonder de stroming in open waterlopen (kanalen, rivieren enz.). Voor andere situaties bestaan gelijksoortige methodes die echter buiten het bestek van dit artikel vallen. Bij deze berekeningen zijn twee zaken van fundamenteel belang: ten eerste een op het doel afgestemde beschrijving van de verschijnselen in wiskundige termen en ten tweede een inzicht in de eigenschappen van het numerieke proces dat wordt toegepast voor de oplossing van de wiskundige vergelijkingen. Het laatste is essentieel om vast te stellen of men inderdaad de gewenste vergelijkingen oplost dan wel een door numerieke oorzaken misvormd beeld daarvan.

Voor de beschrijving van de fysische verschijnselen staan de differentiaalvergelijkingen voor continuiteit en krachten ter beschikking zoals afgeleid in de bijdrage van Allersma (hoofdstuk II). Toepassing op kanaalsecties gedurende bepaalde tijdintervallen resulteert in algebraische vergelijkingen die de toestand van een kanaalsectie bepalen in relatie tot de naburige kanaalsecties en tot de toestand op een voorafgaand tijdsniveau, rekening houdend met de stroming van water van een kanaalsectie naar een andere en met de krachten die de secties op elkaar uitoefenen. Deze vergelijkingen lenen zich goed voor oplossing met behulp van een rekentuig. Globaal gezien komt deze aanpak erop neer dat een kanaal of netwerk van kanalen wordt geschematiseerd in *knooppunten*, die voorgesteld kunnen worden als reservoirs voor de berging van water, en *takken* waarin de op het water werkende krachten in rekening worden gebracht, nl. de zwaartekracht, drukverschillen, wrijvings- en traagheidskrachten (zie fig. 2, 3 en 4). Deze schematisering geldt zowel voor niet-permanente (beschreven in Par. 2) als permanente stromings-situaties (Par. 6), hoewel de fijnheid van schematisering uit nauwkeurigheidsoverwegingen voor beide gevallen niet gelijk hoeft te zijn. Uit een gedetailleerd onderzoek van de schematisering (Par. 4) blijkt dat de kanaalvakken (takken) zoveel mogelijk een constant dwarsprofiel moeten hebben over hun lengte. Bij sterke profielwijzigingen (evenals bij splitsingen en punten waar een belangrijke water-toevoer optreedt) moeten dus knooppunten tussengevoegd worden. Overigens kan het dwarsprofiel zelf een willekeurige vorm hebben als functie van de waterdiepte. Formeel blijken de geschematiseerde vergelijkingen overeen te komen met differentie-benaderingen van de oorspronkelijke differentiaalvergelijkingen, hetgeen waardevolle informatie oplevert met betrekking tot de nauwkeurigheid.

Voor niet-permanente stroming wordt een impliciete rekenmethode gebruikt, d.w.z. een methode waarbij de te berekenen waterstanden en debieten op een zeker tijdsniveau mede bepaald worden door de (eveneens onbekende) naburige waarden op hetzelfde tijdsniveau (Par. 4). Hoewel dit het oplossen van een stelsel niet-lineaire vergelijkingen met zich mee brengt heeft de methode het grote voordeel van onbeperkte stabiliteit, hetgeen betekent dat fouten in de berekening onder geen enkele omstandigheid kunnen aangroeien, welke keuze ook wordt gedaan voor de taklengte en de tijdstap. Dit houdt echter, zoals in Par. 5 wordt besproken, niet in dat die keuze dan willekeurig is. Het blijkt dat grote onnauwkeurigheden (te sterke demping of onjuiste voortplantingssnelheid van een golf) kunnen ontstaan als een onjuiste keuze wordt gedaan. De taklengte en de tijdstap moeten worden gekozen in relatie tot golflengte en periode van de golf die berekend moet worden. Voor het speciale geval van een enkelvoudig kanaal kan deze keuze zelfs kwantitatief worden gemaakt (Fig. 12). Er kan worden verondersteld dat deze schatting ook in meer algemene gevallen van toepassing zal zijn.

Voor permanente stroming wordt een iteratie-methode beschreven die bekend is uit de literatuur en terug gaat op Hardy Cross. De methode is gebaseerd op de aanwezigheid van mazen in een netwerk. Rondlopende correctie-debieten in de mazen worden bepaald zodanig dat uiteindelijk aan de vergelijkingen wordt voldaan (Par. 6).

Een numerieke moeilijkheid bij deze mazenmethode is het feit dat de keuze van de mazen de convergentiesnelheid van het proces sterk kan beinvloeden. In Par. 8 wordt daarom getracht langs theoretische weg tot een methode te komen om de mazen optimaal te kiezen, d.w.z. op een zodanige wijze dat de oplossing van het probleem met een minimale hoeveelheid werk bereikt wordt. Hoewel een afgeronde theorie niet mogelijk blijkt kan wel een richtlijn worden afgeleid. Deze blijkt ongeveer in overeenstemming te zijn met een in de literatuur bekende optimaliseringsmethode voor de maaskeuze, zij het dat het daarin toegepaste criterium wat gewijzigd dient te worden (Fig. 18).

Bij de toepassing van deze rekenmethodes spelen een aantal praktische zaken een rol die in Par. 3 en 7 worden besproken. Het betreft de wijze van schematisering, vastlegging van de gegevens, behandeling van verschillende typen randvoorwaarden en watertoevoer naar het netwerk en de keuze van een beginschatting bij de berekening van een permanente situatie. Verdere praktische toepassingen worden besproken in de bijdrage van Veeningen (hoofdstuk V).

Deze bijdrage is zodanig samengesteld dat Par. 4, 5 en 8 bij een eerste kennismaking met de stof kunnen worden overgeslagen. Deze paragrafen geven detailleringen van de afleiding en de numerieke behandeling van de wiskundige vergelijkingen.

IV. TRANSPORT OF POLLUTANTS OR HEAT IN A SYSTEM OF CHANNELS

J. C. W. BERKHOFF

1. Introduction

As far as the transport of pollutants is concerned, this contribution will discuss in a more mathematical way the basic concepts which are given by Allersma. Attention will be paid mainly to the derivation, validity and boundary conditions of the one-dimensional dispersion equation, to the dispersion coefficient, and to numerical methods for solving the steady or unsteady problem. It is assumed in these methods that there exists complete mixing of pollutants or heat at the confluence of channels. In the case of transport of pollutant, the unknown function will be the concentration of the substance, where as in the case of transport of heat the difference between the water temperature and the background temperature (i.e., the temperature of the water in the absence of sources of heat) is the unknown function.

2. Mathematical method

2.1. *The one-dimensional dispersion equation*

In this section a formal derivation of the dispersion equation will be given. For simplicity a channel with a rectangular cross-section is assumed (see fig. 1), and the basic equation is the three-dimensional mass transport equation:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}(wc) + \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z} + \frac{c}{\gamma} = 0 \qquad (1)$$

$c = c(x, y, z, t)$	concentration
$u, v, w(x, y, z, t)$	velocity components in x, y and z directions respectively
x, y	horizontal coordinates
z	vertical coordinate
t	time
T_x, T_y, T_z	transport components, due to turbulence, in x, y and z direction respectively
γ	relaxation time, i.e. the time in which the concentration would have been reduced by a factor of e in the absence of advection or dispersion.

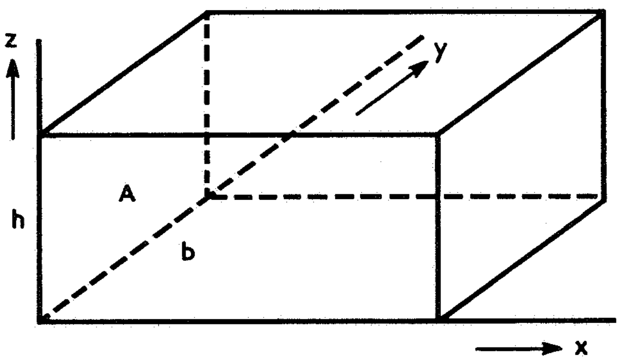


Fig. 1. *Rectangular Channel.*

Integrate Equation (1) over the cross-sectional area $A = hb$. Consideration of each term separately gives:

$$\int_0^h\int_0^b \frac{\partial c}{\partial t}\,\mathrm{d}y\,\mathrm{d}z = \frac{\partial}{\partial t}\int_0^h\int_0^b c\,\mathrm{d}y\,\mathrm{d}z - \left[\int_0^b c\,\mathrm{d}y\right]_{z=h}\frac{\partial h}{\partial t}$$

$$\int_0^h\int_0^b \frac{\partial}{\partial x}(uc)\,\mathrm{d}y\,\mathrm{d}z = \frac{\partial}{\partial x}\int_0^h\int_0^b uc\,\mathrm{d}y\,\mathrm{d}z - \left[\int_0^b uc\,\mathrm{d}y\right]_{z=h}\frac{\partial h}{\partial x}$$

$$\int_0^h\int_0^b \frac{\partial}{\partial y}(vc)\,\mathrm{d}y\,\mathrm{d}z = \int_0^h [vc]_{y=0}^{y=b}\,\mathrm{d}z = 0 \quad \text{because } v=0 \text{ for } y=b \text{ and } y=0$$

$$\int_0^h\int_0^b \frac{\partial}{\partial z}(wc)\,\mathrm{d}y\,\mathrm{d}z = \int_0^b [wc]_{z=0}^{z=h}\,dy = \int_0^b [wc]_{z=h}\,\mathrm{d}y \quad \text{because } w=0 \text{ for } z=0$$

$$\int_0^h\int_0^b \frac{\partial T_x}{\partial x}\,\mathrm{d}y\,\mathrm{d}z = \frac{\partial}{\partial x}\int_0^h\int_0^b T_x\,\mathrm{d}y\,\mathrm{d}z - \left[\int_0^b T_x\,\mathrm{d}y\right]\frac{\partial h}{\partial x}$$

$$\int_0^h\int_0^b \frac{\partial T_y}{\partial y}\,\mathrm{d}y\,\mathrm{d}z = \int_0^h [T_y]_{y=0}^{y=b}\,\mathrm{d}z = 0$$

$$\int_0^h\int_0^b \frac{\partial T_z}{\partial z}\,\mathrm{d}y\,\mathrm{d}z = \int_0^b [T_z]_{z=0}^{z=h}\,\mathrm{d}y = \int_0^b [T_z]_{z=h}\,\mathrm{d}y \quad \text{because there is no mass-}$$

transport across the walls.

$$\int_0^h \int_0^b \frac{c}{\gamma} \, dy \, dz = \frac{1}{\gamma} \int_0^h \int_0^b c \, dy \, dz$$ if the coefficient γ does not depend on y and z.

By defining the mean value of some function f by:

$$\bar{f} = \frac{1}{A} \int_0^h \int_0^b f \, dy \, dz$$

Equation (1) becomes:

$$\frac{\partial}{\partial t}(A\bar{c}) + \frac{\partial}{\partial x}(A\overline{uc}) + \frac{\partial}{\partial x}(A\overline{T_x}) + \frac{\bar{c}}{\gamma} A +$$

$$- \left\{ \left[\int_0^b c \, dy \right]_{z=h} \frac{\partial h}{\partial t} + \left[\int_0^b uc \, dy \right]_{z=h} \frac{\partial h}{\partial x} - \int_0^b [wc]_{z=h} \, dy \right\}$$

$$- \left\{ \left[\int_0^b T_x \, dy \right]_{z=h} \frac{\partial h}{\partial x} - \int_0^b [T_z]_{z=h} \, dy \right\} = 0 \tag{2}$$

The equation of the free surface is: $z = h(x, t)$.
This yields the free surface condition:

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} - w = 0 \text{ for } z = h \tag{3}$$

In (2) the first term between curly brackets can be changed into:

$$\int_0^b \left[c \left(\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} - w \right) \right]_{z=h} dy$$

This term vanishes because of (3). There is also no transport of mass across the free surface.

$$T_x \frac{\partial h}{\partial x} - T_z = 0 \text{ for } z = h$$

so the second term between curly brackets also vanishes. In the case of heat transport the decay in the channel in general will be zero ($\gamma = \infty$), but there is now a transport of heat across the free surface to the atmosphere, which is assumed to be proportional to the difference in temperature of the water and the background temperature. In both cases the following equation remains:

$$\frac{\partial}{\partial t}(A\bar{c}) + \frac{\partial}{\partial x}(A\overline{uc}) + \frac{\partial}{\partial x}(A\overline{T}_x) + \frac{\bar{c}A}{\gamma} = 0 \tag{4}$$

The velocity u and concentration c are now split up in the following way:

$$u(x, y, z, t) = \bar{u}(x, t) + u'(x, y, z, t) \text{ and}$$

$$c(x, y, z, t) = \bar{c}(x, t) + c'(x, y, z, t)$$

From this is derived the expression:

$$\overline{uc} = \overline{(\bar{u} + u')(\bar{c} + c')} = \bar{u}\bar{c} + \overline{u'c'}$$

Substitution into Equation (4) gives:

$$\frac{\partial}{\partial t}(A\bar{c}) + \frac{\partial}{\partial x}(A\bar{u}\bar{c}) + \frac{\partial}{\partial x}(A\overline{T}_x + A\overline{u'c'}) + \frac{\bar{c}A}{\gamma} = 0$$

In general, the transport $\overline{T}_x$ is negligible with respect to $\overline{u'c'}$.

Under certain conditions (see Appendix A) it is possible to derive that the transport $A\overline{u'c'}$ is proportional to the gradient of the mean concentration. Thus:

$$A\overline{u'c'} = -AD\frac{\partial \bar{c}}{\partial x} \tag{5}$$

in which D is the dispersion coefficient, the equation becomes:

$$\frac{\partial}{\partial t}(A\bar{c}) + \frac{\partial}{\partial x}(A\bar{u}\bar{c}) - \frac{\partial}{\partial x}\left(AD\frac{\partial \bar{c}}{\partial x}\right) + \frac{\bar{c}A}{\gamma} = 0 \tag{6}$$

There is also the eqùation of continuity for the water

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x}(A\bar{u}) = 0 \tag{7}$$

Combination of Equation (6) and (7) gives:

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} - \frac{1}{A}\frac{\partial}{\partial x}\left(AD\frac{\partial \bar{c}}{\partial x}\right) + \frac{\bar{c}}{\gamma} = 0 \tag{8}$$

2.2. *Validity of the dispersion model*

Equation (5) in section 2.1 means that the transport which is caused by the non-uniformity of the velocity field over the cross-section, can be described as a diffusive mass transport. This sounds reasonable when looking at the behaviour of a tracer cloud originating from a plane source in an infinitely wide channel flow. Figure 2 shows this qualitatively.

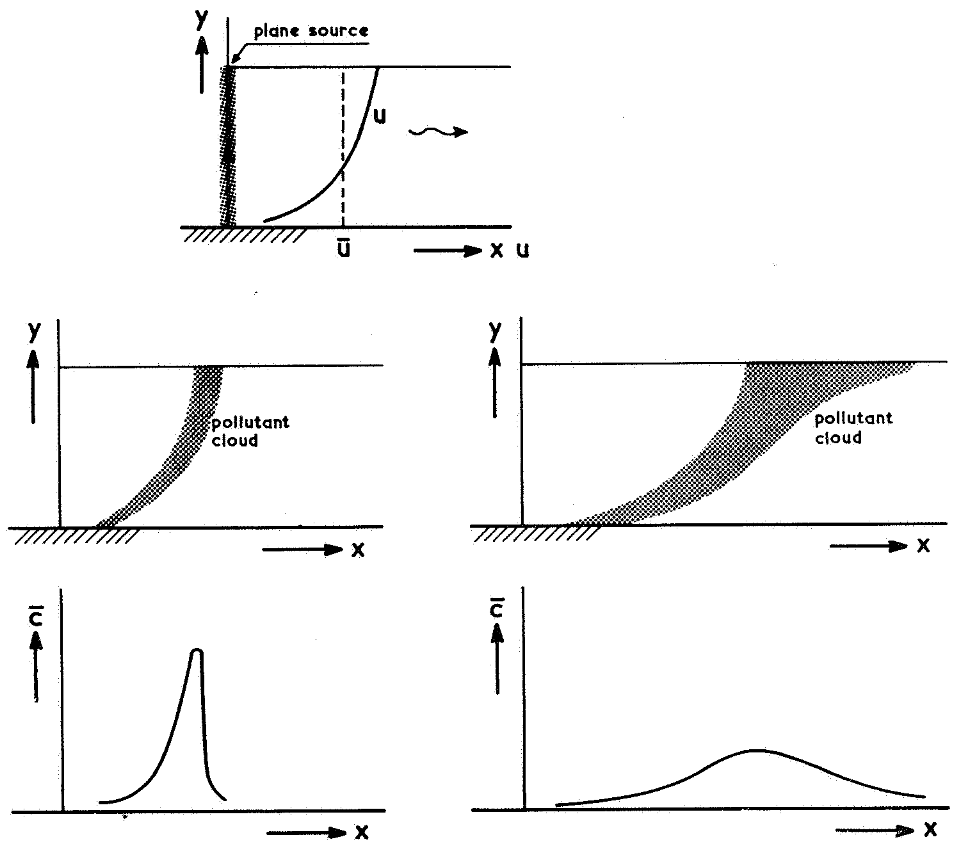


Fig. 2. Dispersion Mechanism (after Fischer, 1966).

Initially the convective motion is the dominant transport mechanism. The shape of the pollutant cloud quickly becomes similar to that of the velocity profile. Vertical diffusion, due to turbulence, slowly destroys the induced vertical concentration gradients. A short time after insertion of the pollutant a plot of the cross-sectional mean concentration will show a skewed curve, but after a certain period of time (Taylor, 1954; Fischer, 1966) the longitudinal distribution of mean concentration gets less skewed and converges to a "Gaussian" distribution. The similarity with a real diffusive transport mechanism is evident, so it is plausible to describe the phenomenon (after a certain period of time) mathematically just like a diffusion proces: the transport of substance is proportional to the concentration gradient.

2.3. *The longitudinal dispersion coefficient*

Not only the non-uniformity of the velocity in a *vertical* direction but also, and sometimes much more importantly, in a lateral direction contributes to the phenomenon of dispersion (Fischer, 1966). In a one-dimensional dispersion model this lateral contribution has to be taken into account in the dispersion coefficient which can also include the effect of tides and temporary storage. In general, the dispersion coefficient depends on the shape of the cross-section, the water depth and the mean velocity. In Appendix A a derivation of a formula for the dispersion coefficient for flow in channels with a constant cross-section is given, so here it will be sufficient to give some results of this derivation for different flows of special type

(a) Steady pipe flow (Taylor, 1954)

$$D = 10.1 r u_* \tag{9}$$

r: radius of pipe
u_*: shear velocity $(\tau_b/\rho)^{1/2}$

(b) Steady open channel flow, homogeneous fluid, infinitely wide channel, constant depth (Elder, 1959)

$$D = 5.9 h u_*$$

h: water depth

(c) Steady open channel flow, homogeneous fluid, lateral effect upon velocity distribution (Fischer, 1967)

$$D = -\frac{1}{A}\int_0^b q\left[\int_0^z \frac{1}{\varepsilon_z h}\left(\int_0^z q\,\mathrm{d}z\right)\mathrm{d}z\right]\mathrm{d}z \tag{11}$$

q: rate of flow per unit of width (m^2/s)
b: width
A: cross-sectional area
ε_z: coefficient of lateral turbulent diffusion

with $\varepsilon_z = 0.23 h u_*$

For practical conditions Equation (11) gives

$$D = (100 \text{ to } 600) h u_*$$

2.4. *Boundary conditions*

Equation (8) is of parabolic type. To solve this equation an initial condition and two boundary conditions are required. The initial condition describes the concentration distribution over the whole area at the initial time $t = 0$

$$c(x, 0) = f(x)$$

Some examples of boundary conditions are:

(a) $c(a, t) = c_0(t)$ — The concentration at the place $x = a$ is a specified function of time.

(b) $\dfrac{\partial c}{\partial x} = 0$ at $x = a$ — There is no dispersive transport at the boundary $x = a$.

(c) $\dfrac{\partial^2 c}{\partial x^2} = 0$ at $x = a$ — At the boundary $x = a$ the concentration is a linear function of x. This approximation may be applied, for instance, at the outflow into the sea under ebb conditions (Shamir, 1966).

(d) $\lim\limits_{x \to \infty} c(x, t) = 0$ — At infinity the concentration must vanish.

It is also possible to formulate boundary conditions in terms of mass-flux, for instance:

(e) $$\lim_{x \downarrow 0} \left[Auc - AD \frac{\partial c}{\partial x} \right] - \lim_{x \uparrow 0} \left[Auc - AD \frac{\partial c}{\partial x} \right] = \emptyset$$

$\emptyset$ is the supply of mass per unit of time at the place $x = 0$ (kg/s).

2.5. *An analytical solution*

To illustrate the mechanisme of dispersion an analytical solution of Equation (8) is given. It is assumed that there is no dissipation of pollutant ($\gamma = \infty$). From the time $t = 0$ there is a constant supply $\emptyset$ of pollutant at the place $x = 0$. The channel is uniform and infinite to both sides of the source. The velocity u and dispersion coefficient D are constants in time and place. The following conditions hold:
At time $t = 0$ there is no pollutant in the channel

$$c(x, 0) = 0$$

At infinity the concentration is zero.

$$\lim_{|x| \to \infty} c(x, t) = 0$$

At the source

$$\lim_{x \downarrow 0} AD \frac{\partial c}{\partial x} - \lim_{x \uparrow 0} AD \frac{\partial c}{\partial x} = \phi$$

because the concentration is continuous at $x = 0$.

The solution in this case is:

$$c(\alpha, \beta) = \frac{\phi}{2u}\left[e^{4\alpha}\left\{\operatorname{erf}\left(\beta + \frac{\alpha}{\beta}\right) - \operatorname{sign}(\alpha)\right\} + \operatorname{erf}\left(\beta - \frac{\alpha}{\beta}\right) + \operatorname{sign}(\alpha)\right]$$

in which

$$\alpha = \frac{u}{4D}x; \quad \beta = \frac{u}{2}\sqrt{\frac{t}{D}}$$

$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-x^2}\,dx; \quad \operatorname{sign}(\alpha) = \begin{cases} +1 & \alpha > 0 \\ -1 & \alpha < 0 \end{cases}$$

Figure 3 shows the concentration distribution in the channel for different times. There is almost no diffusion in an upstream direction, although, of course, this depends upon the magnitude of u and D. Downstreams of the source the concentration converges to the stationary concentration $\phi/(Au)$.

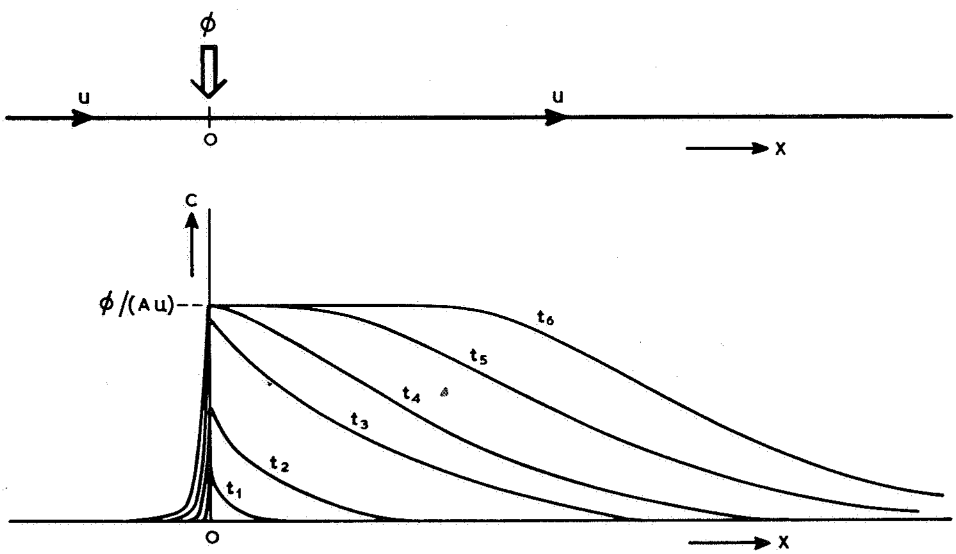


Fig. 3. Concentration Distribution in the Channel.

3. Numerical methods

3.1. *The unsteady-state problem*

3.1.1. The unsteady-state equation

The problem is to know the concentration of a pollutant as a function of time and place if there is a time-dependent supply of pollutant at some places in a network of channels. The motion of water as a function of time and place is known either by computations or by measurements. In each branch the concentration must satisfy Equation (8). At each junction of branches (node) a continuity equation must hold. In the nodes, which are boundary points of the network, boundary conditions can be imposed. All these equations together are sufficient to compute the concentration as a function of time and place over the network (Delft Hydraulics Laboratory, 1969).

3.2.1. Difference-equation for a branch

A finite differences method is chosen to solve equation (8) for each branch. It is an implicit method combined with a splitting method (Stone and Brian, 1963). Its applicability was studied by Siemons (Delft Hydraulics Laboratory, 1970). The term $\frac{\partial c}{\partial t} + \frac{c}{\gamma}$ is split up over the locations $j - 1$, j an $j + 1$ with the weighting factors 1/6, 2/3 and 1/6 respectively (see fig. 4).

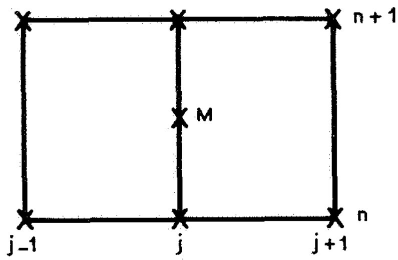


Fig. 4. Difference Scheme.

The derivative with respect to t is approximated by a central difference formula with respect to M and the derivatives with respect to x by central differences. The following implicit difference-equation arises if the quantities U, A, D and γ are independent of x.

$$A_1 C_{j+1}^{n+1} + A_2 C_j^{n+1} + A_2 C_{j-1}^{n+1} = E_j^n \qquad (12)$$

with $A_1 = 2 + \tau + 3\mu - 3\lambda$

$A_2 = 8 + 4\tau + 6\lambda$

$A_3 = 2 + \tau - 3\mu - 3\lambda$

$$E_j^n = (2 - \tau - 3\mu + 3\lambda)C_{j+1}^n + (8 - 4\tau - 6\lambda)C_j^n + (2 - \tau + 3\mu + 3\lambda)C_{j-1}^n$$

in which $\mu = \frac{u\Delta t}{\Delta x}$, $\lambda = \frac{2D\Delta t}{\Delta x^2}$ and $\tau = \frac{\Delta t}{\gamma}$

The notation C_j^n is to be understood as $C(j\Delta x, n\Delta t)$

Δx: step size in x-direction
Δt: time step
L: length of branch ($J\Delta x = L$)
$j = 1, 2, \ldots, J - 1$
$n = 0, 1, 2, \ldots$

The choice of this specific finite difference method is determined by considerations of accuracy and numerical stability, an aspect which is dealt within Paragraph 3.1.4. Difference Equation (12) gives rise to $J - 1$ linear equations in the $J + 1$ unknown concentrations $C_0^{n+1}, C_1^{n+1}, \ldots, C_J^{n+1}$. The concentrations of the nodes at the level $n + 1$ (C_0^{n+1} and C_J^{n+1}) are computed from the continuity equations for internal nodes. The boundary conditions for the boundary nodes of the network give the supplementary equations.

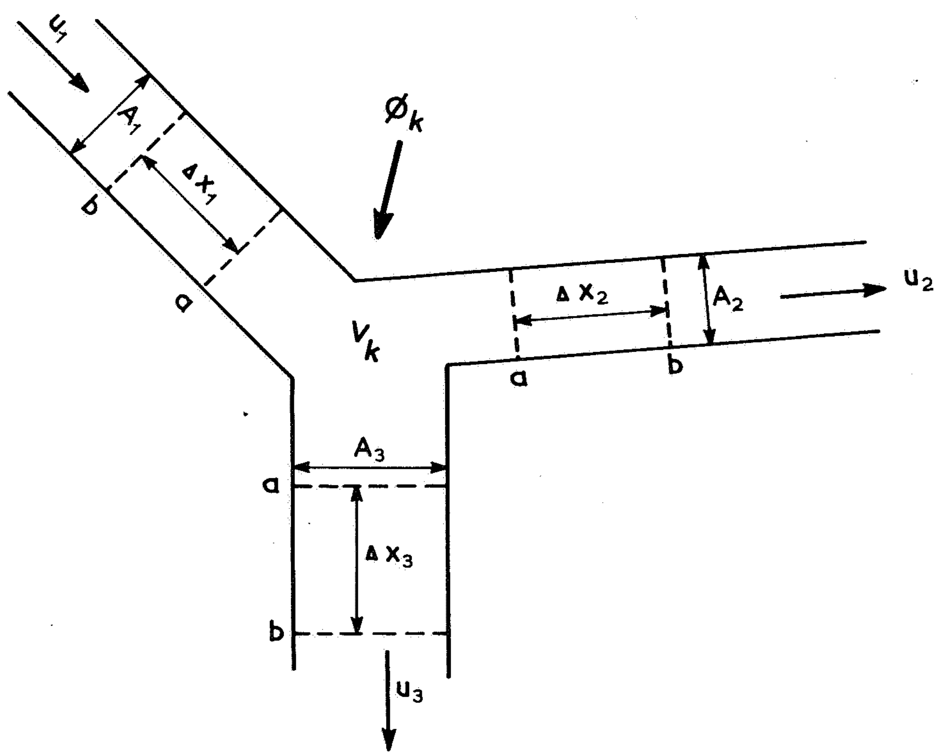


Fig. 5. Scheme of a Node.

3.1.3. Contuinity equation for a node

The continuity condition for node k with a volume V_k reads (see fig. 5):

$$\frac{\partial V_k C_k}{\partial t} + \sum_{i=1}^{n_k} l_i \left[Auc - AD\frac{\partial c}{\partial x}\right]_a + \frac{V_k C_k}{\gamma} = \phi_k \tag{13}$$

in which l_i: indicator for positive direction in $_i$-th branch of node k (see fig. 6)
n_k: number of branches connected in node k
C_k: concentration in node k
V_k: volume of node k
ϕ_k: supply of mass per unit of time into node k

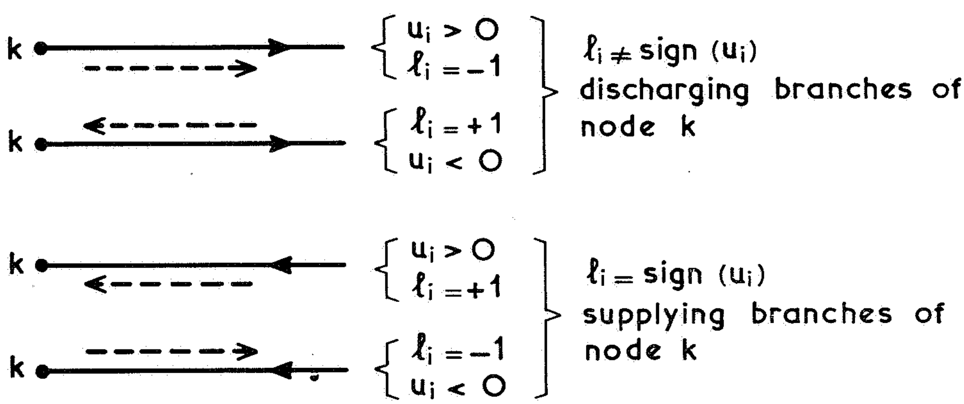


Fig. 6. Definition of Discharging and Supplying Branches.

A complete mixing of the pollutants at the node is assumed. Replace $\frac{\partial V_k C_k}{\partial t}$ by the difference formula $\frac{(V_k C_k)^{n+1} - (V_k C_k)^n}{\Delta t}$ and $\left(\frac{\partial c}{\partial x}\right)_a$ by $\frac{C_b^n - C_k^n}{2\Delta x}$.

The explicit expression for the concentration in node k at the time level $n + 1$ becomes:

$$C_k^{n+1} = \frac{V_k^{n+1}}{V_k^n} C_k^n + \frac{\Delta t}{V_k^{n+1}} \sum_{i=1}^{n_k} l_i \left[u^n A^n C_a^n - A^n D^n \frac{(C_b^n - C_k^n)}{2\Delta x}\right] +$$
$$+ \frac{\phi_k \Delta t}{V_k^{n+1}} - \frac{C_k^n \Delta t}{\gamma}$$

in which C_a: concentration in point a of the branch
C_b: concentration in point b of the branch (see fig. 5).

3.1.4. Accuracy and numerical stability

The accuracy of a numerical method for solving a differential equation can be investigated by means of the propagation factor, which is the ratio between one term of the Fourier series of the numerical solution and the corresponding term of the analytical solution. The two components of the complex propagation factor are in this case after N time steps:

The damping factor

$$\beta = R^n \, \mathrm{e}^{D\omega^2 N\Delta t}$$

in which

$$R = \left[\frac{1 - (\lambda + \frac{2}{3}) \sin^2(\frac{1}{2}\omega\Delta x) + \dfrac{\mu^2}{4} \sin^2(\omega\Delta x)}{1 + (\lambda - \frac{2}{3}) \sin^2(\frac{1}{2}\omega\Delta x) + \dfrac{\mu^2}{4} \sin^2(\omega\Delta x)} \right]^{1/2}$$

and the velocity factor

$$\alpha = \varphi / \omega u \Delta t$$

in which

$$\varphi = \arctan \left\{ \frac{\dfrac{\mu}{2} \sin \omega\Delta x}{1 - (\lambda + \frac{2}{3}) \sin^2(\frac{1}{2}\omega\Delta x)} \right\} + \arctan \left\{ \frac{\dfrac{\mu}{2} \sin \omega\Delta x}{1 + (\lambda - \frac{2}{3}) \sin^2(\frac{1}{2}\omega\Delta x)} \right\}$$

ω: wave number of the Fourier term.

A comparison of these factor for different finite-difference schemes, taking N such that the amplitude of the analytical solution has been decreased by a factor e (when $N = 1/D\omega^2\Delta t$), shows the more accurate process of the implicit splitting method for higher values of the parameter μ (see fig. 7 and 8). The explicit method gives

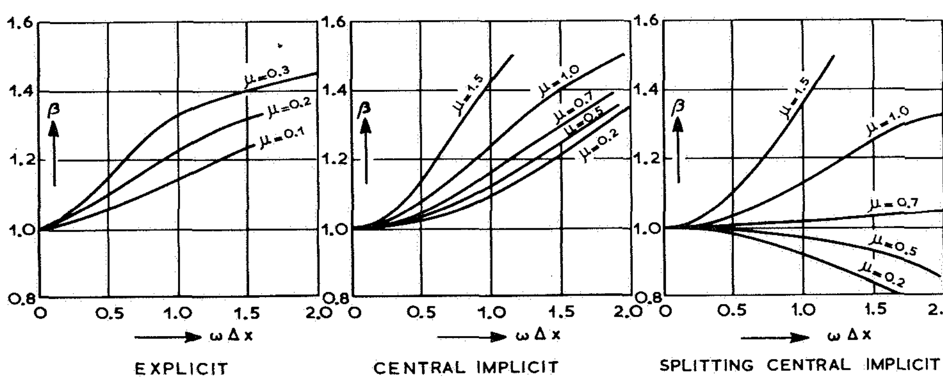


Fig. 7. Damping Factor.

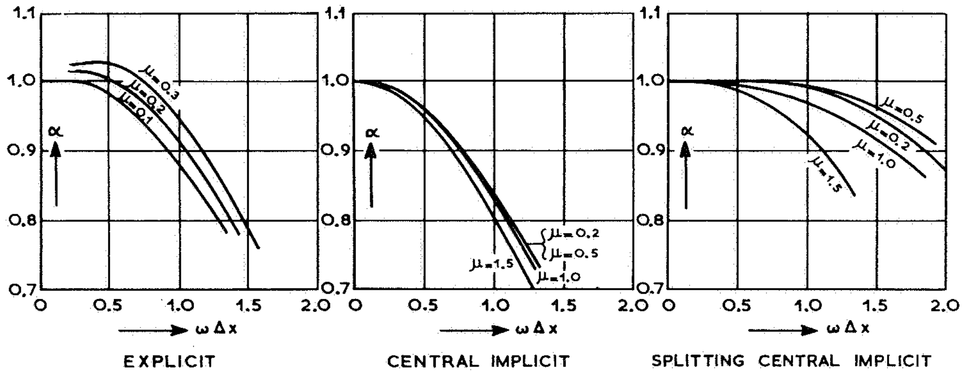


Fig. 8. Velocity Factor.

directly the value of C_j at the time level $n + 1$ from the three values C_{j-1}, C_j and C_{j+1} at the time level n. It is a fast method of computation but has the disadvantage that instability can occur. The implicit method is a splitting method with weighting factors 0, 1 and 0 for the place levels $j - 1$, j and $j + 1$ respectively. The accuracy of the node equation depends in a large measure on the accuracy of the computed values in the adjoining branches, and especially in the branches which supply water to the node.

The implicit method for solving the branch equation is stable in all circumstances. The explicit node equation, however, is stable within certain values of the parameters μ and λ. To get an approximation of these bounds of μ and λ, a simplified case can be given. Consider two half-infinite branches connected in the point $x = 0$ (see fig. 9).

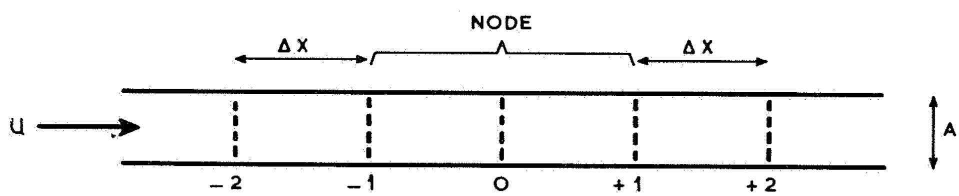


Fig. 9. Two Half-infinite Branches.

The values of u, A and D are assumed constant in both branches. There is no dissipation of pollutant ($\gamma = \infty$). At infinity the concentration must be zero. The step sizes in the branches are equal (Δx). The node equation in this simplified case is:

$$C_0^{n+1} = C_0^n\left[1 - \frac{Au\Delta t}{V_0}\left(\frac{C_1^n - C_{-1}^n}{C_0^n}\right) + \frac{AD\Delta t}{2V_0\Delta x}\left(\frac{C_2^n - 2C_0^n + C_{-2}^n}{C_0^n}\right)\right]$$

in which $V_0 = 2A\Delta x$.

Consider only the solutions of the homogeneous difference equations of the branches:

$$A_1 C_{j+1}^{n+1} + A_2 C_j^{n+1} + A_3 C_{j-1}^{n+1} = 0$$

These solutions are

$$C_j^n = C_0^n r_1^j \text{ for } j > 0$$

$$C_j^n = C_0^n r_2^j \text{ for } j < 0$$

r_1 and r_2 are solutions of the equation:

$$A_1 r^2 + A_2 r + A_3 = 0$$

with $|r_1| < 1 < |r_2|$.

Then

$$C_0^{n+1} = C_0^n \left[1 - \frac{u\Delta t}{2\Delta x}\left(r_1 - \frac{1}{r_2}\right) + \frac{D\Delta t}{4\Delta x^2}\left(r_1^2 - 2 + \frac{1}{r_2^2}\right)\right]$$

The contribution of these solutions to the concentration in the node will not increase if the expression between brackets has an absolute magnitude smaller then unity.

This leads to the requirement:

$$\left|1 - \frac{\mu}{2}\left(r_1 - \frac{1}{r_2}\right) + \frac{\lambda}{8}\left(r_1^2 - 2 + \frac{1}{r_2^2}\right)\right| < 1 \tag{14}$$

with

$$r_{1,2} = \frac{4 + 3\lambda \mp \sqrt{9\mu^2 + 36\lambda + 12}}{-2 + 3(\lambda - \mu)}$$

In the $\mu - \lambda$ field areas can be indicated in which the node equation gives unstable, oscillating or damped solutions (see fig. 10):

3.1.5. General remarks

(a) To compute the phenomenon of diffusion against the direction of the velocity, small step sizes in x and t are required for accurate results. When the dispersion coefficient is small the diffusion in an upstream direction is negligible, and it is then possible to solve Equation (8) with the boundary conditions: $c(0, t) = c_k$ and $\left(\frac{\partial^2 c}{\partial x^2}\right)_m = 0$ with:

m: the end of the downstream branch

k: the upstream node

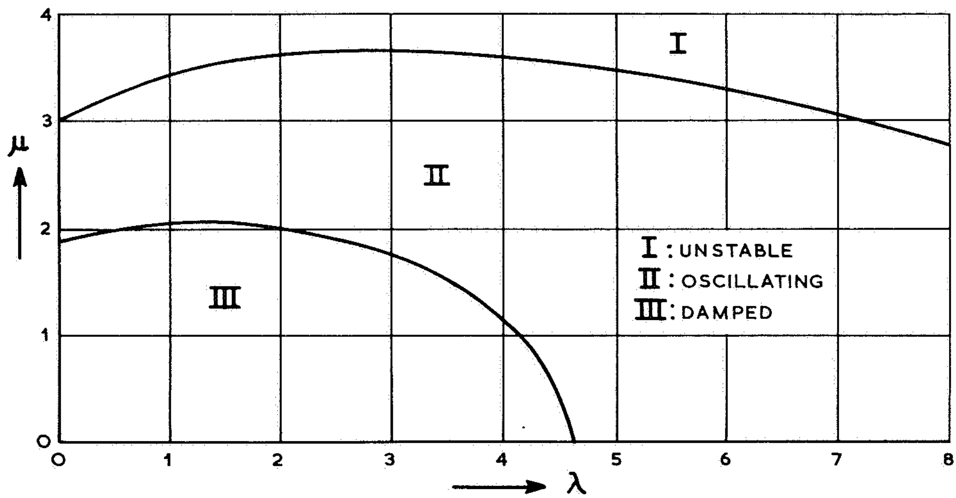


Fig. 10. Stability Areas.

which gives less severe requirements for the step sizes Δx and Δt. The definition of the dispersion coefficient in an upstream direction is otherwise disputable.

(b) In the case of recirculation of pollutant, there is a supply of pollutant ϕ into a node, which may be a function of the concentration of pollutant in other nodal points.

(c) In contrast to the computational method for the movement of water, where the length of a branch is also the step size in the space direction, in the case of dispersion computations more steps must be taken in the spatial direction in each branch to follow the dispersion phenomenon in the branch correctly. This requires more storage in the computer and also more computing time.

3.2. *The steady-state problem*

3.2.1. The steady-state equation

When the supply of pollutant is constant in time, as also is the velocity in the branches, the concentration distribution will not alter after a certain period of time. If the interest is only in this steady concentration distribution, then there must be a solution of the steady-state equation

$$u \frac{\mathrm{d}c}{\mathrm{d}x} - D \frac{\mathrm{d}^2 c}{\mathrm{d}x^2} + \frac{c}{\gamma} = 0 \tag{15}$$

in each branch, and the node equation

$$\sum_{i=1}^{n_k} l_i \left[Auc - AD \frac{\mathrm{d}c}{\mathrm{d}x} \right]_k - Q_k c_k + M_k = 0 \tag{16}$$

where

l_i: indicator of pos. direction (see fig. 10)
n_k: number of branches connected in node k
Q_k: rate of flow which is withdrawn ($Q_k > 0$) from the node k
M_k: mass-supply into node k ($M_k > 0$)

for each node (Delft Hydraulics Laboratory, 1971). The rate of outflow Q_k can be expressed in terms of flows in the branches:

Write

$$Q_t = \sum_{i=1}^{n_k} l_i A_i U_i$$

then

$$\begin{aligned} Q_k &= -Q_t \quad \text{if } Q_t < 0 \\ Q_k &= 0 \qquad \text{if } Q_t \geqq 0 \end{aligned} \tag{17}$$

3.2.2. Method of solution in the case of no dispersion ($D = 0$)

The branch equation is now:

$$u \frac{\mathrm{d}c}{\mathrm{d}x} + \frac{c}{\gamma} = 0 \tag{18}$$

with the boundary condition $c = c_a$ at the upstream end of the branch. The node equation becomes

$$Q_d c_k + M_s - Q_k c_k + M_k = 0 \tag{19}$$

$$Q_d = \sum_{i=1}^{n_k} l_i A_i u_i \qquad \text{(outflow of water through the discharging branches)}$$

$$l_i \neq \operatorname{sign}(u_i)$$

$$M_s = \sum_{i=1}^{n_k} l_i A_i u_i c_b \qquad \text{(inflow of mass through the supplying branches)}$$

$$l_i = \operatorname{sign}(u_i)$$

c_b: the concentration at the downstream end of the supplying branch.

If the concentrations c_b are known, the concentration c_k can be computed with

$$c_k = \frac{M_k + M_s}{Q_k - Q_d} \tag{20}$$

The solutions of Equation (18) for each branch give the values c_b. These solutions are:

$$c = c_a \, \mathrm{e}^{-\frac{1}{\gamma}\left|\frac{x}{u}\right|} \tag{21}$$

c_a: the concentration at the upstream end of the branch.

Then

$$c_b = c_a \, \mathrm{e}^{-\frac{1}{\gamma}\left|\frac{L}{u}\right|} \tag{22}$$

$|L|$: length of the branch.

Starting at a node which has only discharging branches, it is possible to compute the concentrations in all nodes. Recirculation of pollutant can be handled by means of an iteration process.

3.2.3. Method of solution neglecting upstream dispersion

Solving Equation (15) for each branch with the boundary conditions

$c = c_a$ at the upstream end of the branch

and

$c = 0$ at infinity downstreams

neglects automatically the dispersion from upstream direction. The solution in this case is:

$$c(x) = c_a \, \mathrm{e}^{r|x|} \tag{23}$$

in which

$$r = \frac{u - \sqrt{u^2 + 4D/\gamma}}{2D} \quad \text{if } u > 0$$

$$r = -\frac{u + \sqrt{u^2 + 4D/\gamma}}{2D} \quad \text{if } u < 0$$

The node equation is:

$$\sum_{i=1}^{n_k} l_i \left[Auc - AD \frac{\mathrm{d}c}{\mathrm{d}x} \right]_b + \sum_{i=1}^{n_k} l_i \left[Auc - AD \frac{\mathrm{d}c}{\mathrm{d}x} \right]_k - Q_k c_k + M_k = 0 \tag{24}$$

$l_i = \mathrm{sign}(u_i)$ $\qquad\qquad$ $l_i \neq \mathrm{sign}(u_i)$

Index b means: at the downstream end of a supplying branch.

From solution (23) there can be derived

$$l_i\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_b = c_a r\, \mathrm{e}^{r|L|} \qquad l_i = \mathrm{sign}(u_i)$$

$$l_i\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_k = -\, c_k r \qquad l_i \neq \mathrm{sign}(u_i)$$

$$c_b = c_a\, \mathrm{e}^{r|L|} \qquad l_i = \mathrm{sign}(u_i)$$

The expression for concentration c_k becomes

$$c_k = \frac{M_k + M_s'}{Q_k - Q_d'} \tag{25}$$

in which

$$Q_d' = \sum_{i=1}^{n_k} l_i A_i u_i + A_i D_i r_i$$

$$l_i \neq \mathrm{sign}(u_i)$$

$$M_s' = \sum_{i=1}^{n_k} l_i A_i u_i - A_i D_i r_i\, c_a\, \mathrm{e}^{r_i L_i}$$

$$l_i = \mathrm{sign}(u_i)$$

3.2.4. Solution method with full dispersion

The boundary conditions of Equation (15) are in this case:

$$c = c_{\mathrm{I}} \ \text{ for } \ x = 0$$

$$c = c_{\mathrm{II}} \ \text{ for } \ x = L$$

The solution of the equation with these boundary conditions is:

$$c(x) = K_1\, \mathrm{e}^{r_1 x} + K_2\, \mathrm{e}^{r_2 x} \tag{26}$$

in which

$$K_1 = \frac{c_{\mathrm{I}}\, \mathrm{e}^{r_2 L} - c_{\mathrm{II}}}{\mathrm{e}^{r_2 L} - \mathrm{e}^{r_1 L}}$$

$$K_2 = \frac{c_{\mathrm{I}}\, \mathrm{e}^{r_2 L} - c_{\mathrm{II}}}{\mathrm{e}^{r_2 L} - \mathrm{e}^{r_1 L}}$$

$$r_{1,2} = \frac{u \pm \sqrt{u^2 + 4D/\gamma}}{2D}$$

From (26) it can be derived that

$$\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_{x=0} = \frac{c_{\mathrm{I}}(r_1\,\mathrm{e}^{r_2L} - r_2\,\mathrm{e}^{r_1L}) + c_{\mathrm{II}}(r_2 - r_1)}{\mathrm{e}^{r_2L} - \mathrm{e}^{r_1L}}$$
$$\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_{x=L} = \frac{c_{\mathrm{I}}(r_1 - r_2)\,\mathrm{e}^{(r_1+r_2)L} + c_{\mathrm{II}}(r_2\,\mathrm{e}^{r_2L} - r_1\,\mathrm{e}^{r_1L})}{\mathrm{e}^{r_2L} - \mathrm{e}^{r_1L}} \tag{28}$$

In node k Equation (16) must hold. For the i-th branch of node k these can be written

$$\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_k = \left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_{x=L}; \quad c_{\mathrm{II}} = c_k \text{ and}$$

c_{I} is the concentration in the node at the other end of the branch if $l_i = +1$;

$$\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_k = \left(\frac{\mathrm{d}c}{\mathrm{d}x}\right)_{x=0}; \quad c_{\mathrm{I}} = c_k \text{ and}$$

c_{II} is the concentration in the node at the other end of the branch if $l_i = -1$.

With the expressions (27) and (28) node Equation (16) gives rise to a set of K linear equations in the K unknown concentrations in the K nodes. This set of linear equations can be solved, e.g., with the well-known Gauss-Seidel iteration method.

4. Programme facilities

4.1. *Input data*

To fix the configuration of the system of channels, the branches and nodes must be assigned a number, which can be done in an arbitrary way. For each branch the input data is: branch number, first node number of the branch, second node number and further quantities such as length, width, depth, velocity (in steady problems), dispersion factor, etc. The positive direction in a branch will be from the first-mentioned node number to the second-mentioned number. To compute the dispersion phenomenon, the movement of water in the whole system as a function of time must be known, which means that for unsteady problems the values of the water velocities in each branch at each time step are required. The supply and discharge of mass at each node as a function of time must be given, and the concentration distribution over the whole system at the beginning of the computation is also essential.

4.2. *Output data*

The computed values of the concentration at the nodes are output data in the case of a steady problem. For unsteady problems it is possible to give the node concentrations and the concentration distribution in each branch after one or more time steps.

4.3. Recirculation

If the supply of mass in a node depends on the concentration at another node, there occurs the effect of recirculation. The programmes for steady state can handle this effect by iterating the computation, while the program for unsteady state takes care of this effect automatically.

REFERENCES

DELFT HYDRUALICS LABORATORY	Verspreiding van afvalstoffen in netwerkstelsels. Speurwerkverslag, S146–I, 1969.
DELFT HYDRAULICS LABORATORY	Numerical Methods for the solution of diffusion-advection equations W.L. Publ., No 88, 1970
DELFT HYDRAULICS LABORATORY	Verspreiding van afvalstoffen of warmte; permanente en niet-permanente rekenmethodes. Speurwerkverslag, S146–II, 1971.
ELDER, J. W.	The dispersion of marked fluid in turbulent shear flow. Journal of Fluid Mech., Vol. 5, No. 4, May 1959.
FISCHER, H. B.	Longitudinal dispersion in laboratory and natural streams. Ph.D. Thesis, Cal. Inst. Techn., Pasadena, Cal., 1966.
FISCHER, H. B.	Analytic prediction of longitudinal dispersion coëfficients in natural streams. Proceedings of the Twelfth Congress of the I.A.H.R., Fort Collins, Colorado, 1967.
SHAMIR, U. Y. AND HARLEMAN, D. R. F.	Numerical and analytical solutions of dispersion problems in homogeneous and layered aquifers. Hydr. Lab., Report No. 89, Mass. Inst. of Techn., Mass., 1966
STONE, H. L. AND BRIAN, P. L. T.	Numerical solution of convective transport problems. A.I.Ch.E. Journal, Vol. 9, No. 5, 1963.

APPENDIX A

ESTIMATE OF THE MAGNITUDE OF THE LONGITUDINAL DISPERSION COEFFICIENT D

The dispersion coefficient depends on the shape of the cross-section, the water depth and the velocity profile. If all these quantities are known, it is possible after some assumptions to make an estimate of the magnitude of D. Starting again with

Equation (1), in which $\gamma = \infty$ is assumed, the turbulent transport components T_x, T_y and T_z can be defined as:

$$T_x = -\varepsilon_x \frac{\partial c}{\partial x}, \quad T_y = -\varepsilon_y \frac{\partial c}{\partial y} \quad \text{and} \quad T_z = -\varepsilon_z \frac{\partial c}{\partial z}$$

in which ε_x, ε_y and ε_z are turbulent diffusion coefficients in x, y and z directions respectively. Some assumptions have now to be made.

Assumption I: neglect T_x

Integration of (1) over the cross-sectional area A gives the equation (see Equation (4))

$$\frac{\partial(A\bar{c})}{\partial t} + \frac{\partial}{\partial x}(A\overline{uc}) = 0 \qquad \text{(A1)}$$

Assumption II: the cross-sectional area A is constant. Equation (A1) becomes:

$$\frac{\partial \bar{c}}{\partial t} + \frac{\partial}{\partial x}(\overline{uc}) = 0 \qquad \text{(A2)}$$

Return to Equation (1). Combination with the equation of continuity for the water

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

gives

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right)$$

Assumption III: $v = w = 0$

Thus

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial z} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right) \qquad \text{(A3)}$$

Write $c = \bar{c} + c'$ and $u = \bar{u} + u'$ and substitute these expressions in (A3).

$$\frac{\partial \bar{c}}{\partial t} + \frac{\partial c'}{\partial t} + (\bar{u} + u')\left(\frac{\partial \bar{c}}{\partial x} + \frac{\partial c'}{\partial x}\right) = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c'}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c'}{\partial z}\right)$$

or with equation (A2)

$$-\frac{\partial}{\partial x}(\overline{uc}) + \frac{\partial c'}{\partial t} + \bar{u}\frac{\partial c'}{\partial x} + \bar{u}\frac{\partial \bar{c}}{\partial x} + u'\frac{\partial \bar{c}}{\partial x} + u'\frac{\partial c'}{\partial x} =$$

$$= \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c'}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c'}{\partial x}\right)$$

Assumption IV: u does not depend upon x

$$\frac{\partial \bar{u}}{\partial x} = 0, \quad \frac{\partial u'}{\partial x} = 0$$

With this assumption and $\overline{uc} = \bar{u}\bar{c} + u'c'$ the equation becomes:

$$\overline{-\,u'\frac{\partial c'}{\partial x}} + \frac{\partial c'}{\partial t} + \bar{u}\frac{\partial c'}{\partial x} + u'\frac{\partial \bar{c}}{\partial x} + u'\frac{\partial c'}{\partial x} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c'}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c'}{\partial z}\right)$$

Assumption V: after a sufficiently long period

$$c' \ll \bar{c} \text{ and hence } \frac{\partial c'}{\partial x} \ll \frac{\partial \bar{c}}{\partial x}$$

Assumption VI: a spectator moving with the cloud observes variations in time which are small compared with variations in x

$$\frac{\partial c'}{\partial t} + \bar{u}\frac{\partial c'}{\partial x} = 0$$

The following equation remains:

$$u'\frac{\partial \bar{c}}{\partial x} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c'}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c'}{\partial z}\right) \tag{A4}$$

If u', ε_y and ε_z are known as functions of y and z, c' can be derived from (A4) to obtain a solution of the character

$$c' = f(y, z)\frac{\partial \bar{c}}{\partial x} \tag{A5}$$

Combination of (A5) with Equation (5), in which $T_x = 0$, gives the result:

$$D = -\frac{1}{A}\int\int_h u'f \, \mathrm{d}y \, \mathrm{d}z \tag{A6}$$

Because of the assumptions V and VI, this expression does not keep close to the source. In the case that the non-uniformity of the velocity in a lateral direction gives the most important contribution to the dispersion coefficient, the formula of Fischer can be derived as follows.

Equation (A4) becomes

$$u' \frac{\partial \bar{c}}{\partial x} = \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c'}{\partial z}\right)$$

Integration over the vertical with the assumption that $\frac{\partial c'}{\partial z}$ is independent of y gives the equation

$$q \frac{\partial \bar{c}}{\partial x} = \frac{\partial}{\partial z}\left(\varepsilon_z h \frac{\partial c'}{\partial z}\right) \tag{A7}$$

in which

$$q = \int_0^h u' \, \mathrm{d}y$$

Integration of equation (A7) over z twice gives

$$c' = \int_0^z \frac{1}{\varepsilon_z h}\left\{\int_0^z q \, \mathrm{d}z\right\} \mathrm{d}z \frac{\partial \bar{c}}{\partial x}$$

So in this case the function f is given by

$$f(y, z) = \int_0^z \frac{1}{\varepsilon_z h}\left\{\int_0^z q \, \mathrm{d}z\right\} \mathrm{d}z$$

and with (A6) the expression (11) given in Paragraph 2.3 can easily be found.

NOTATION

		dimension
A	cross-sectional area	$[L^2]$
$A_1, A_2, A_3.$	coefficients	—
b	width	$[L]$
C	numerical solution of the differential equation (concentration)	$[ML^{-3}]$
c	concentration	$[ML^{-3}]$
$c_{\mathrm{I}}, c_{\mathrm{II}}$	concentrations at the ends of a branch	$[ML^{-3}]$
D	dispersion coefficient	$[L^2s^{-1}]$
E	coefficient	—
f	some function	var.
h	water depth	$[L]$
J	total step sizes in a branch ($J = L/\Delta x$)	—
K	total number of nodes	—
K_1, K_2	integration constants	$[ML^{-3}]$

L	length of a branch	[L]
l_i	indicator of positive direction	—
M_k	mass-supply from outside into node k	$[Ms^{-1}]$
M_s, M_s'	inflow of mass into a node through the supplying branches	$[Ms^{-1}]$
N	total time steps	—
n_k	number of branches connected in node k	—
Q_d, Q_d'	outflow of water through the discharging branches	$[L^3s^{-1}]$
Q_k	outflow of water from node k	$[L^3s^{-1}]$
Q_t	sum of the flows through the branches which are connected in a node	$[L^3s^{-1}]$
R	absolute value of the numerical solution	$[ML^{-3}]$
r, r_1, r_2	coefficients	var.
T_x, T_y, T_z	transport components, due to turbulence, in x, y and z-directions respectively	$[ML^{-1}s^{-1}]$
t	time	[s]
u	velocity in x-direction	$[Ls^{-1}]$
u_*	shear velocity	$[Ls^{-1}]$
v	velocity in y-direction	$[Ls^{-1}]$
w	velocity in z-direction	$[Ls^{-1}]$
x	horizontal coordinate	[L]
y	vertical coordinate	[L]
z	lateral coordinate	[L]
α	velocity factor	—
β	damping factor	—
γ	relaxation time	[s]
Δx	step size in x-direction	[L]
Δt	time step	[s]
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	turbulence diffusion coefficients	$[L^2s^{-1}]$
ϕ	supply of mass per unit of time	$[Ms^{-1}]$
φ	phase of the numerical solution	—
λ	parameter $\left(\dfrac{2D\Delta t}{\Delta x^2}\right)$ in the difference equation	—
μ	parameter $\left(\dfrac{u\Delta t}{\Delta x}\right)$ in the difference equation	—
ω	wave number	$[L^{-1}]$
ρ	density	$[ML^{-3}]$
τ	parameter $\left(\dfrac{\Delta t}{\gamma}\right)$ in the difference equation	—
τ_b	turbulent shear stress at the bottom	$[ML^{-1}s^{-2}]$

superscript:	$-$	cross-sectional mean
	$'$	deviation from the cross-sectional mean
	n	at the time $t = n\Delta t$
subscript:	i	i-th branch
	j	at the place $x = j\Delta x$
	k	k-th node
	a, b	at the places $x = a$, $x = b$ respectively

SAMENVATTING

BEREKENING VAN VERSPREIDING VAN AFVALSTOFFEN OF WARMTE IN OPEN WATERLOPEN

J. C. W. BERKHOFF

Het handhaven van een goede kwaliteit van het water in open waterlopen wordt hoe langer en meer een grote zorg voor de waterbeheerder. Wiskundige modellen die een inzicht geven in de verspreiding van afvalstoffen of warmte kunnen goede hulpmiddelen zijn bij eventueel te nemen maatregelen. Deze bijdrage beschrijft een aantal methoden voor de berekening van de verspreiding in permanente en niet-permanente situaties, waarbij een waterbeweging volledig bekend wordt verondersteld. Een tweetal onderwerpen wordt behandeld nl. de beschrijving van het verspreidingsmechanisme (dispersie) en de berekeningstechniek met de daarin aanwezige numerieke aspecten. De door Allersma op globale wijze en in par. 2.1 op wat formelere wijze afgeleide differentiaalvergelijking beschrijft het fysische verschijnsel zowel voor afvalstoffen als voor warmte voor eendimensionale stroming. De daarin voorkomende dispersiecoëfficient D is in 't algemeen afhankelijk van de waterdiepte, stroomsnelheid en vorm van het dwarsoppervlak van het kanaal. In de literatuur zijn voor verschillende gevallen formules voor de dispersiecoëfficient afgeleid. In appendix A wordt een formule voor D gegeven geldend in stromingen bij constant dwarsprofiel. De behandelde berekeningsmethode is toegespitst op de berekening van de verspreiding in een stelsel van open waterlopen, geschematiseerd tot takken en knopen, waarbij knopen gelegd moeten worden op die plaatsen waar sterke profielwijzigingen, splitsingen of lozingen optreden. In tegenstelling met de berekening van de waterbeweging moet in een tak ook rekenpunten worden meegenomen om een juiste weergave van het verspreidingsmechanisme te verkrijgen. Aangenomen is dat bij een samenloop van waterlopen volledige menging optreedt. Par. 3.1 beschrijft de numerieke oplossingstechniek van de geldende niet-permanente differentiaalvergelijking. Voor de rekenpunten in een tak wordt een impliciete rekenmethode gebruikt en voor de knooppunten een expliciete. Het voordeel van deze werkwijze is dat de hoeveelheid rekenwerk binnen de perken blijft en de eis voor numerieke stabiliteit niet zo streng is als bij volledig expliciet rekenen. De stapgroottes in een tak en de tijdstap moeten zodanig gekozen worden dat een redelijke nauwkeurigheid verkregen wordt. Enkele rekenmethodes voor permanente toestanden worden beschreven in par. 3.2. Indien de coëfficienten in de differentiaalvergelijking constant zijn over en tak is een analytische uitdrukking voor het concentratieverloop in de tak te geven.

Voor de concentraties in de knooppunten is een stelsel lineaire vergelijkingen op te stellen, welke op iteratieve wijze wordt opgelost. Eventuele optredende recirculatie kan bij alle rekenmethodes op betrekkelijk eenvoudige wijze worden ingebracht.

V. PRACTICAL APPLICATIONS OF COMPUTATIONS FOR CHANNEL NETWORKS

C. VEENINGEN

1. Introduction

The foregoing Chapters have shown that a mathematical model can be an important tool in the management of complex systems of drainage canals in polder areas or other catchment areas. However, the method of computation can be applied in a much wider field.

Although uptil now the main field of application has been the computation of open-channel networks, in this Chapter a concise review is given of applications in other fields of water management, on the basis of examples from the experience of the Delft Hydraulics Laboratory.

Further, also illustrated with examples, an insight will be given into the set-up of a mathematical model, the data which are necessary, and the results to be obtained from the computations.

2. Summary of applications

2.1. Open-channel networks

A. Problems concerning water management of polder areas

In Dutch circumstances many applications can be found concerning management of water levels and water quality in systems of drainage canals in polders or in larger areas where many polders are combined under a water board, a so-called "Waterschap", which is charged with the management of a network of drainage canals with pumping-stations, sluices, etc. Of course, the computational method is applicable to catchment areas in general.

Problems which can arise in the management of polder systems, and which lend themselves to computations, are, for example:

(a) Optimal choice of the location and capacity of pumping-stations and sluices. The computation is not an optimization in the sense that from an input of a programme of demands and possibilities the most favourable design can be obtained directly from the computation. In principle, such optimization is possible, and in fact has been accomplished already for a simple network for the design of a sewage system (De Vries, 1969). For complicated networks the application is restricted for the time being by the necessary computer time. Optimization methods are not considered in this paper.

Each computation must start from a given network and well-known boundary conditions, and the computation will produce the discharges and water levels in the various parts of the network. This means that an investigation must be set up in such a way that from the results of computations of several possible solutions, an optimal choice can be made, if necessary by interpolation between investigated situations.

(b) Computation of the necessary capacity of canals, investigation of the influence of civil works which lead to changes in the cross-sectional areas of canals, blocking of canals, or the influence which structures in the canals have on the discharge capacity of the whole system. At the same time the effect of compensation by improvements elsewhere in the system can be studied. Here also the set-up as described above must be followed to get an "optimal" solution.

(c) Effect of dike-collapse in a canal along a polder.

(d) Dispersion of salt or pollutants in an open-channel system, and computations to decide what measures must be taken in case of a calamity such as a sudden discharge of poisonous matter.

From some of the applications presented above, examples are given in Paragraph 3.

B. Computation of cooling-water circuits

Both the water levels and discharges, as well as the temperature of the cooling water at various points of the circuit, are subject of computation. Of course, a cooling-water problem may be part of a polder system, and as such it should be added to the group of problems mentioned under A.

An example of a computation of a cooling-water circuit is the investigation for the electrical power station to be designed near the mouth of the River IJssel (Ketelmeer Centrale). First a mathematical model is made of the present situation (Fig. 1a) in order to verify the response of the model. The areas, represented by branches in the model, are shown in this Figure. The verification was made with a view to the distribution of discharges over the various branches and the slope of the water level at various conditions. A detailed description of the study is beyond the scope of this Chapter, but this model is interesting to show how an area with two-dimensional flow can be simulated by a model with one-dimensional branches. After verification of the model, many layouts of cooling-water circuits, for various locations of the power station, were taken into consideration. Figure 1b shows an example of a possible future situation in which a reservoir for potable water is integrated in the layout.

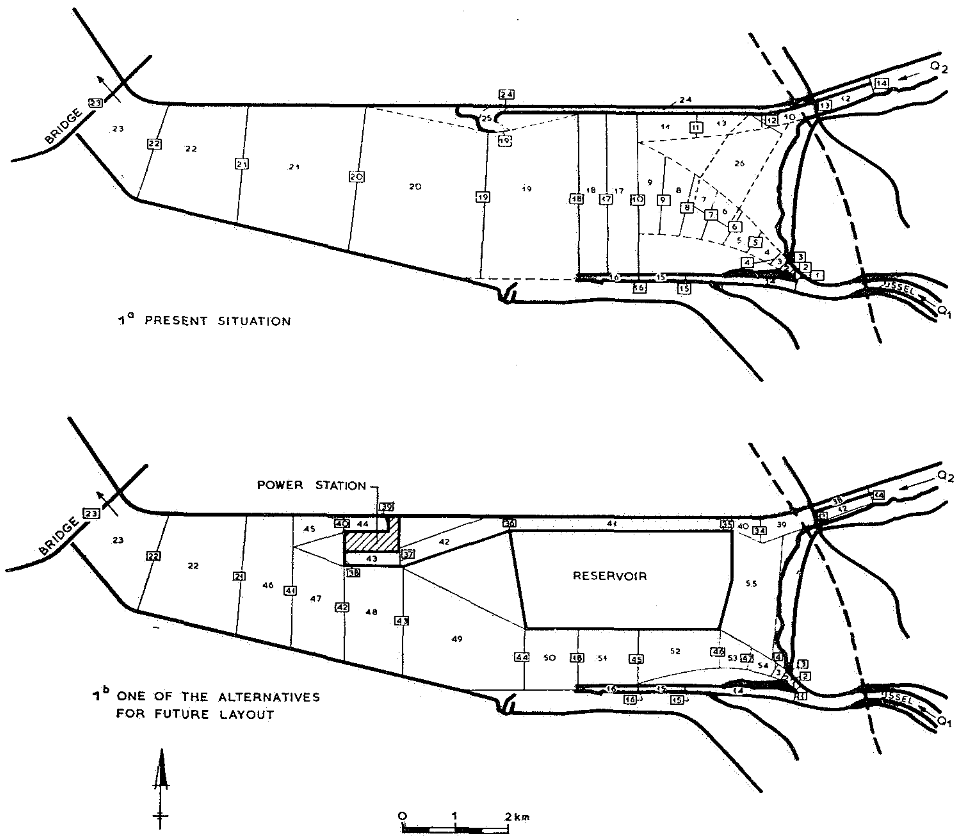


Fig. 1. Schematizations for computations of Ketelmeer Power-Station.

2.2. *Floods in creeks and rivers*

Besides applications for channel networks, the method is also suitable for the computation of water levels and discharges in creeks and rivers.

Especially the computation of non-steady flow is important in this respect, as computation of steady flow will generally not give problems, unless the river is very long so that the use of a computer might facilitate the work to be done, or if bifurcations of the rivers make iteration necessary.

A frequently-occurring problem is the propagation of a flood. An example is the computation of the propagation of floods in a creek in the eastern part of the Netherlands, caused by discharge from rain water overflows of a small municipality. The schematization of the creek downstream from the point of discharge is shown in Figure 2.

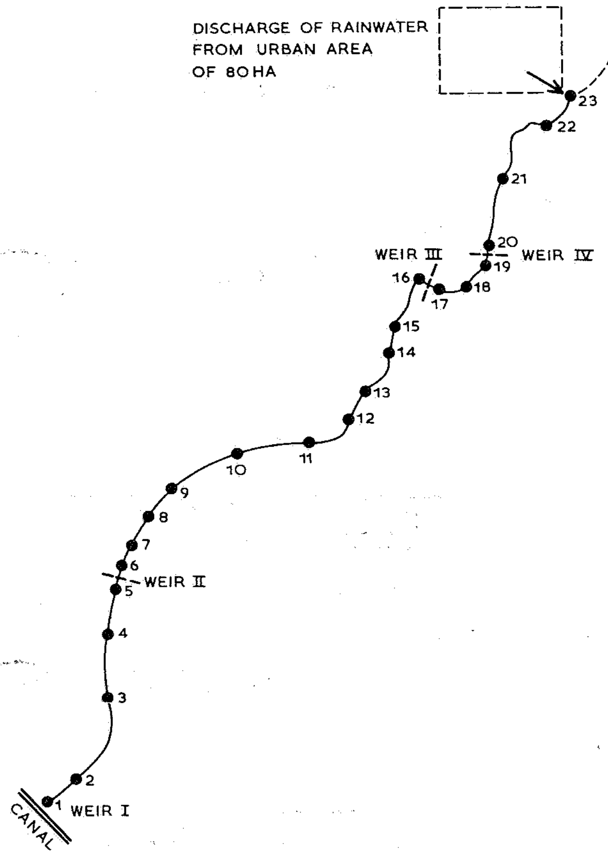


Fig. 2. Schematization of creek with weirs.

The length of the schematized part is 11.5 km, and the difference in bottom level between the two ends is about 5 m. To reduce the slope of the water surface several weirs are installed in the water-course. At the crests of the weirs the flow can be critical or sub-critical, dependent on the height of the downstream water level. If critical flow occurs, this means that in the computation the water-course upstream from the weir can be separated from the downstream part, as the water levels on both sides of the weir are independent of each other. The discharge of the upstream part is determined by the crest height of the weir. As an upstream boundary condition of the downstream part, the discharge over the weir, calculated as a function of time, is introduced. During increase of the water level downstream from the weir, the flow will eventually become sub-critical, so that both parts of the water-course must be coupled again. The possibility for this procedure can be inserted into the computer programme.

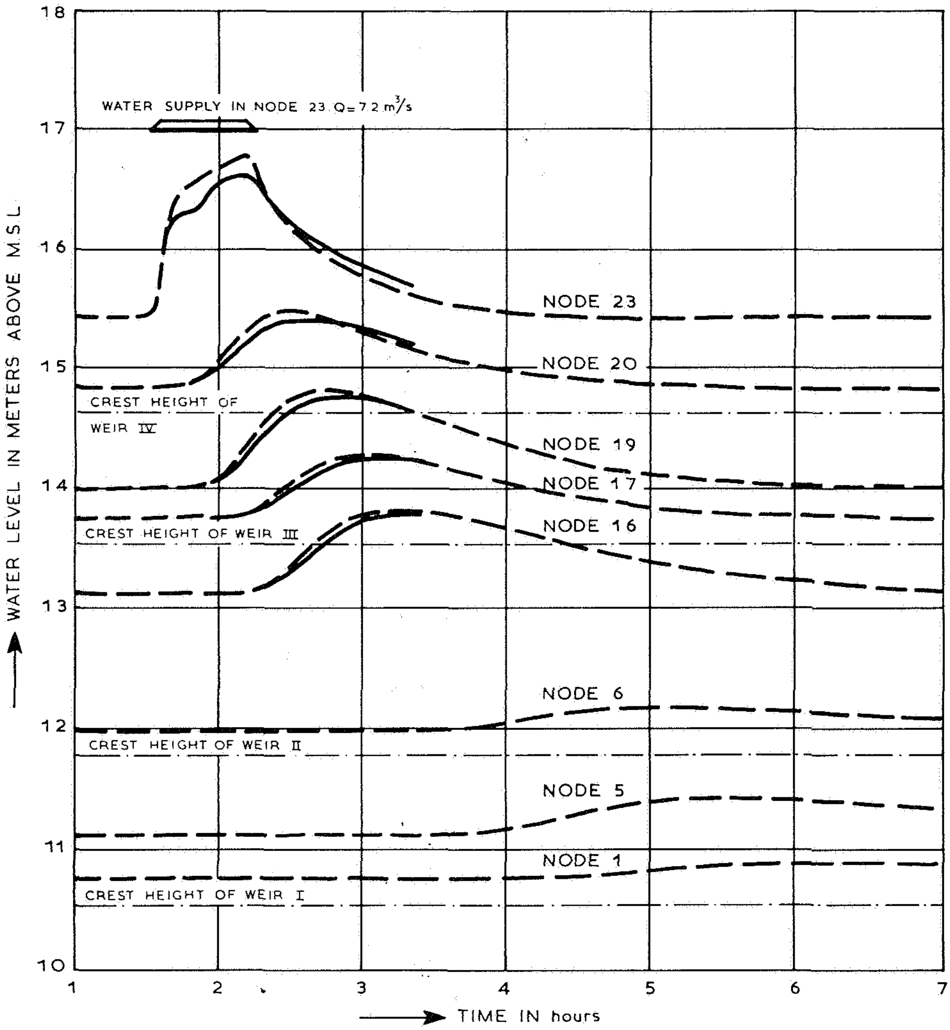


Fig. 3. Calculated flood in creek.

The results of a computation for a discharge of 90 liter/sec/ha from an urban area of 80 ha, due to a 45-minutes storm, is shown in Figure 3. The storm discharge is superimposed on some steady-state discharge from the agricultural areas. For several points of the water-course the water levels are given as a function of time. The flattening of the wave as it travels in a downstream direction can be clearly distinguished.

Most of the computations have been made for the water-course downstream from

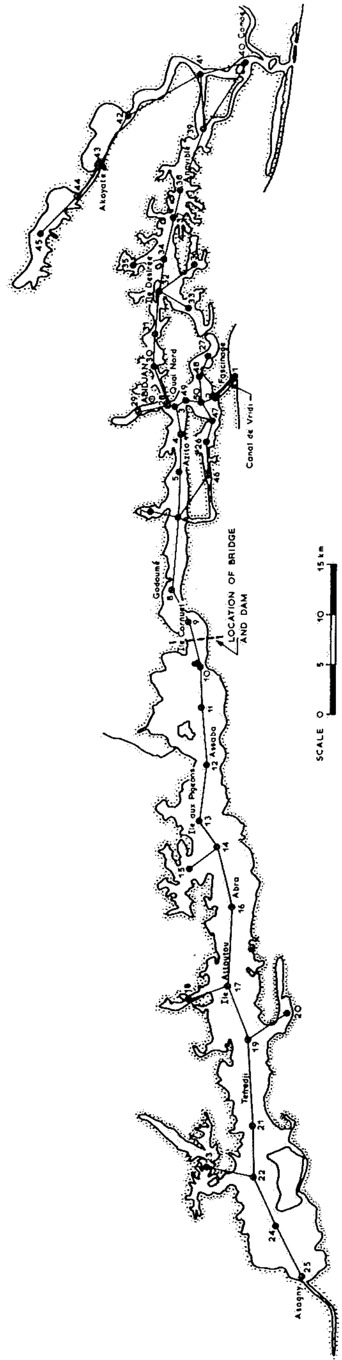


Fig. 4. Ebrié Lagoon, Ivory Coast.

the municipal overflows. In addition, a computation has been made in which the steep part of the water-course upstream from the overflows, with a length of a few kilometers, has been taken into account. Due to storage in this area, some reduction of the maximum water levels can be observed (non-interrupted lines). This example shows, however, that also a simplified system sometimes gives satisfactory results.

Computation of high-water waves on rivers is a matter of up-scaling. In such a case it will be clear that much attention should be paid to the collection of sufficiently detailed data on the dimensions of the cross-sections and the hydraulic resistance of the river sections.

2.3. *Tidal movement in estuaries and rivers*

Instead of using the characteristics of a pumping-station, sluice or weir as a boundary condition for a computation, such a boundary condition can also be formed by a tidal curve only.

Consequently with the aid of a computation of open-channel networks the propagation of a tidal movement into an estuary or river can be calculated, as well as the influence by future structures or other projects on this tidal propagation. This means a considerable extension of the possible applications.

The investigations for the Harbour of Abidjan, Ivory Coast, can serve as an example of the use of the computer programme in solving these problems.

Abidjan is situated on a large lagoon of about 600 km^2, connected with the Atlantic Ocean by a canal. the Canal de Vridi (Fig. 4). The hydraulic conditions in the lagoon are determined by the discharge of the rivers debouching into it, and by the tidal movement. Computations were made for two purposes:

1. Determination of the boundary conditions for a hydraulic (physical) model. To determine the influence of a proposed deepening of the canal entrance, causing lower current velocities in the entrance and consequently a dangerous situation regarding bar formation, a model investigation was carried out in a movable-bed model. The computations had to produce the discharges in the canal in the future situation, which discharges would be introduced as boundary conditions in the model. For this purpose the canal and the lagoon were schematized according to Figure 4. First, a verification was made for the existing situation, using prototype data concerning the discharge in the canal. Also the variations of the water levels at several points in the lagoon were known. In spite of the incompleteness of data of the very large area, a reasonable similitude of measured and calculated discharges was obtained (Fig. 5).

   Next the discharges as a function of time were calculated, for the future situation.
2. Investigation of the influence exercised on the discharge of the Canal de Vridi by the construction of a dam and connecting bridge in the lagoon (see Figs. 4 and 6).

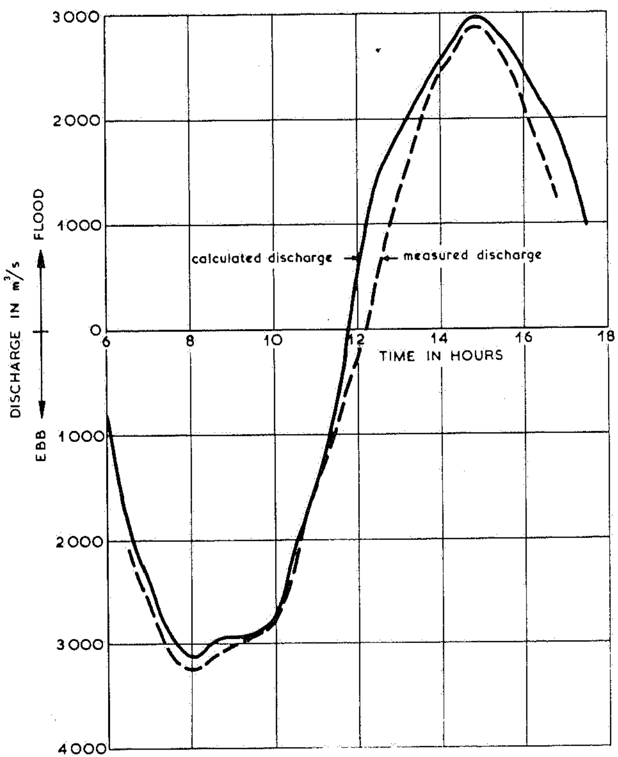


Fig. 5. Discharges in Canal de Vridi.

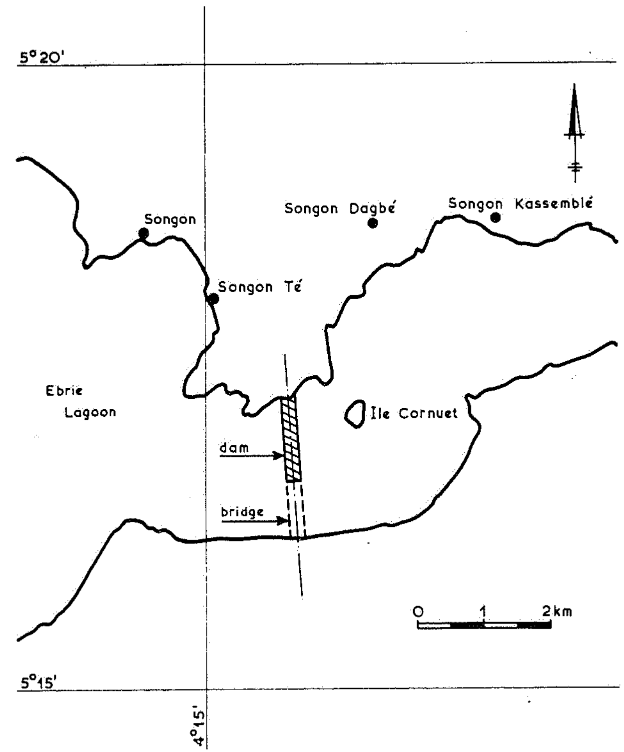


Fig. 6. Situation of bridge and dam.

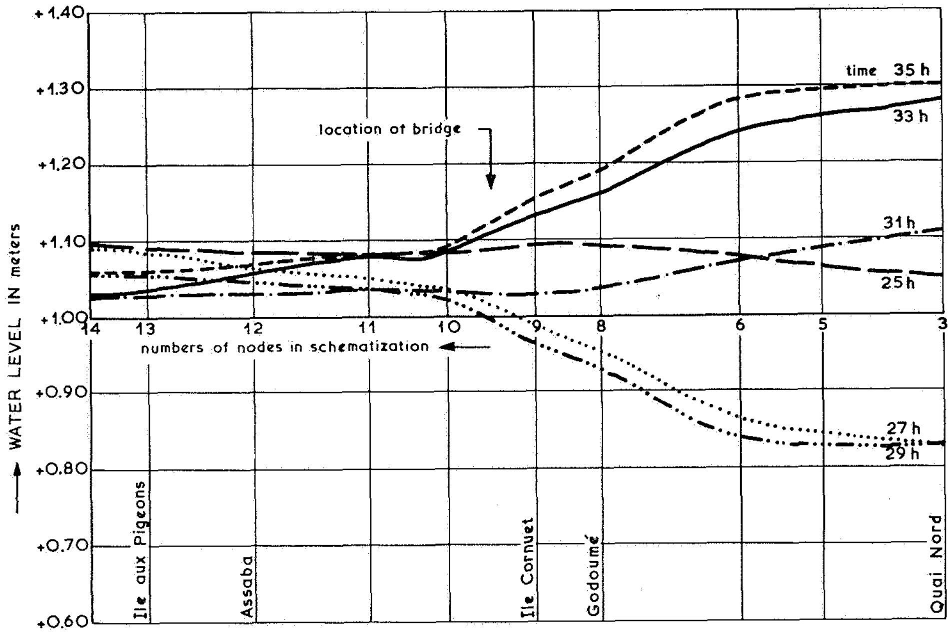


Fig. 7. Water levels in lagoon for present situation.

Structures in the lagoon can be translated into a hydraulic resistance for the branch concerned. To obtain a deeper understanding of the consequences of such structures at the location concerned, a computation was carried out for a situation in which the lagoon would be completely blocked at the location of the bridge. Figures 7 and 8 show the water levels in the lagoon at various times, without and with the dam respectively. Figure 9 show the discharges in the canal in both situations.

It appeared that the construction of the dam yields an increasing discharge in the canal. This can be explained by a closer consideration of the water levels in the lagoon at various times. From Figure 7 it can be concluded that the changes in the water level in the lagoon can be considered partially as a standing wave, with a node near the future dam. After blocking the lagoon at this location, a ventral segment appears near the dam, resulting in an increase in the magnitude of inflow and outflow in the canal. It was shown by further computations that these phenomena also appear, although less pronouncedly, if part of the dam would be replaced by a bridge, thus causing a partial reflection of the tidal wave.

An advantage of the computations is that in a simple way the computation of several extreme situations enables a good insight into some phenomena to be obtained.

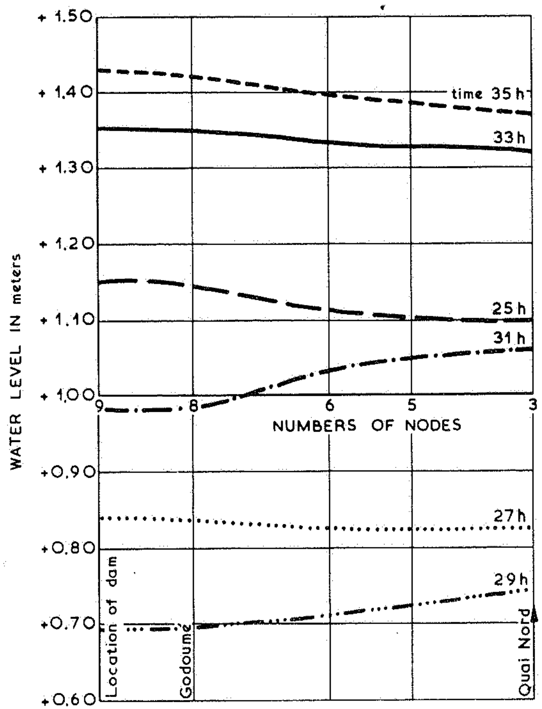


Fig. 8. Water levels in lagoon for hypothetical situation with dam.

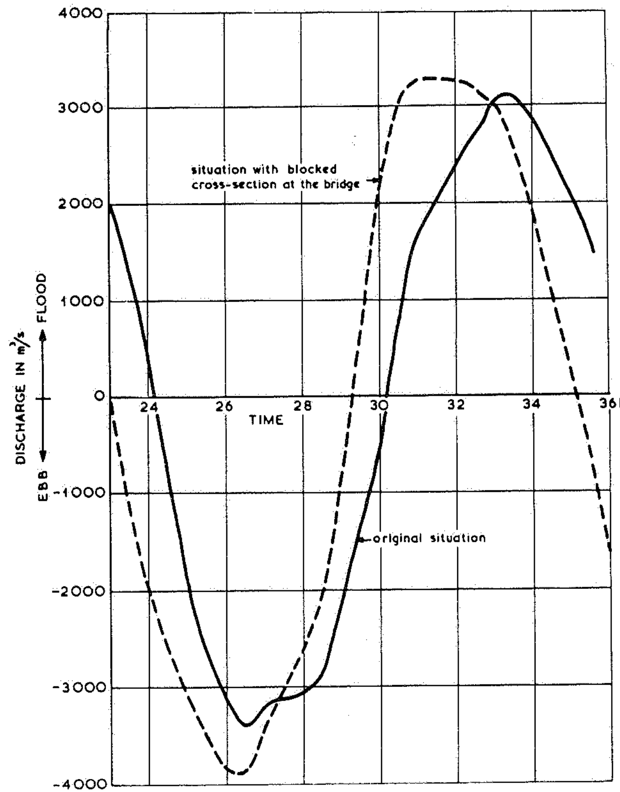


Fig. 9. Discharges in canal for situations with and without dam.

2.4. *Sewage systems*

The techniques of computations as developed for open channels can be applied in the field of sanitary engineering by computing the water movement in networks of sewage conduits. This leads to the need to introduce the possibility for computation of closed conduits, whether they be empty, partially filled, or completely filled. Special provisions in the computer programme are necessary, especially in connection with the possibility of conduits which are emptied or nearly emptied during the period of computation, due to the progressive increase of hydraulic resistance of an emptying conduit. In that case iteration via corrections of water levels might cause problems for the convergence of the computation. Further provisions to be introduced into the programme are lateral overflows, pumps, and conduits which are connected at one point at different heights, as is common practice in sewage networks.

Again, the computations can be made for steady and non-steady flow, so that in addition to determining the capacity of a sewage system, also the time can be determined during which superfluous water is present in an urban area, as well as the location of that area, in case rainfall exceeds the discharge and storage capacities of the system.

2.5. *Coupling of open-channel flow and ground-water movement*

The supply of water from outside a network, e.g., due to rainfall, must be introduced as an input in the nodes of the network. It is not necessary that such an input is independent of time; it may be given as a function of time. However, it is not the rainfall itself but the net supply to the system which forms the input, so that losses and retardation in the ground should be estimated. In calculating discharge and storage of excessive rainfalls in Dutch polder areas, this aspect generally does not form a major problem. Usually in such cases a period of heavy rainfall, lasting several days during the winter months, is decisive for a determination of the required discharge capacity. At the beginning of the rain period the soil will often be saturated to a high degree, so that mainly surface flow occurs, and the retardation will be of minor importance.

In these circumstances the assumptions to be made for the discrepancy between rainfall and runoff (i.e., input into the nodes) will have little influence on the result of the computation, as has been shown, for example, in a computation made for the Duurswold Water Board, in the province of Groningen (Waterschap Duurswold). In one computation the 3-days' rain period used for the design of the channel system was represented by a rectangular diagram; in another case, the same amount of precipitation was assumed to reach the open canals in the form of a trapezoid diagram, linearly increasing to its maximum in 1.5 days and linearly decreasing after the third day (see Figs. 10 and 11). In Figures 12 and 13 for both cases the calculated water levels at various points of the system are represented as functions of time. It

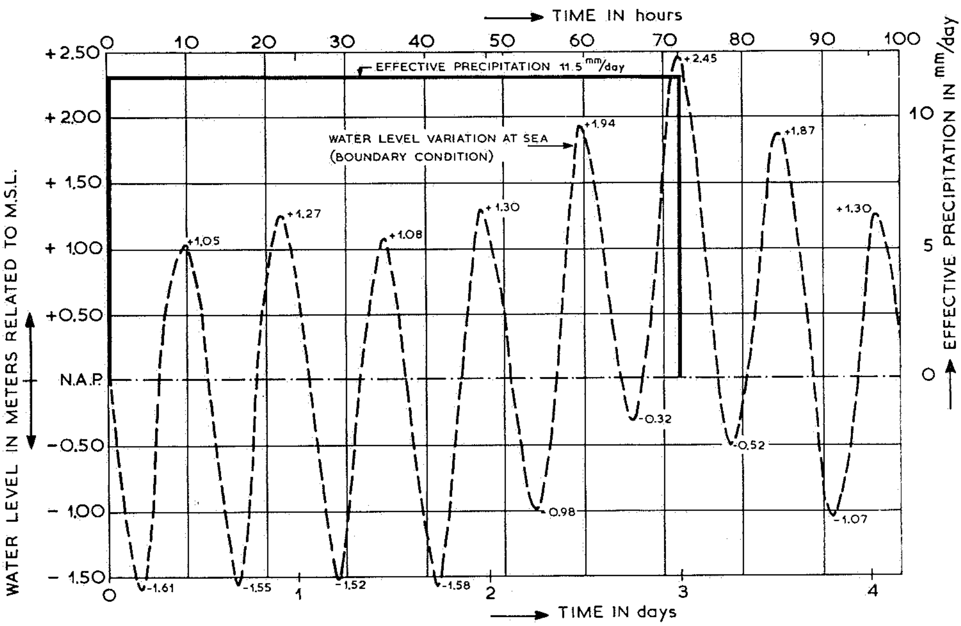


Fig. 10. Input and boundary conditions computation I.

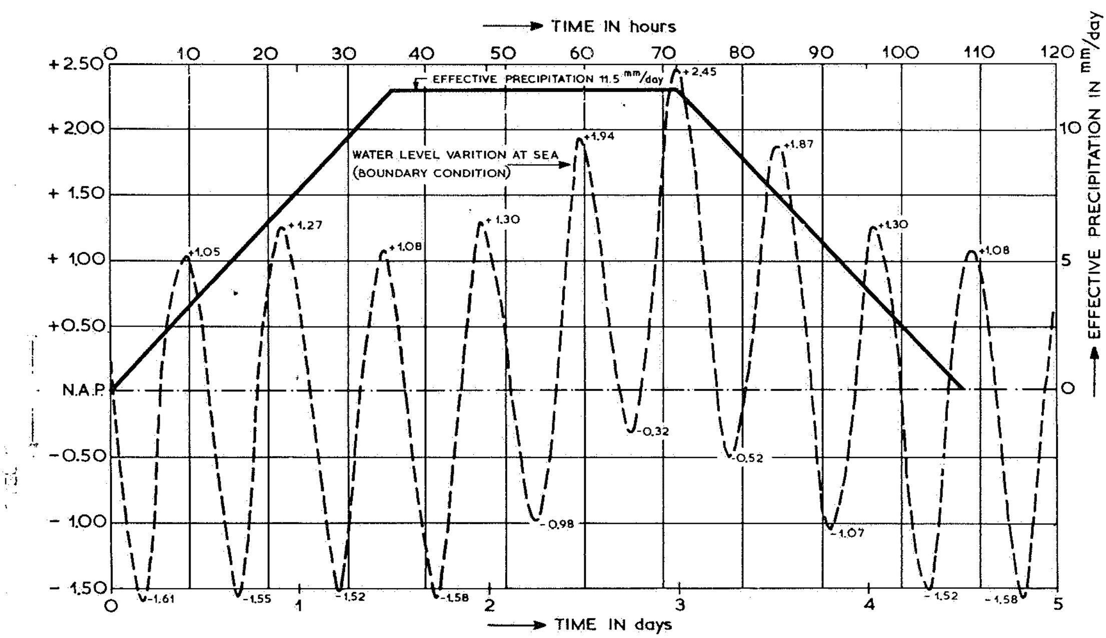


Fig. 11. Input and boundary conditions computation II.

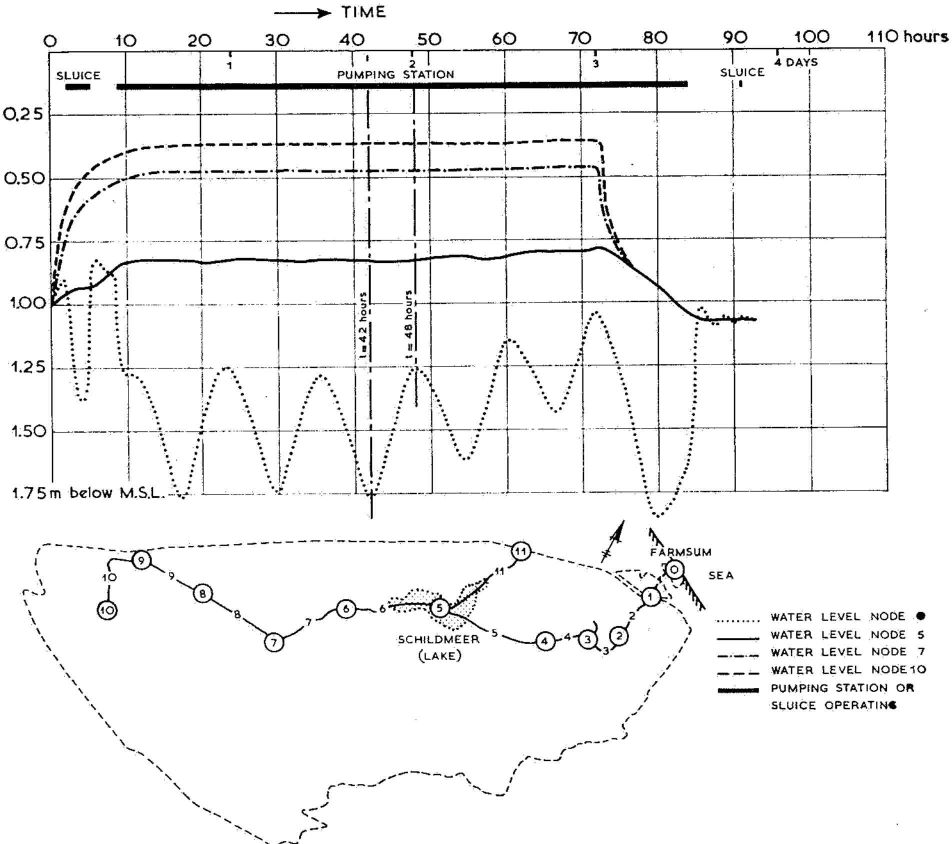


Fig. 12. Results computation I (rectangular input diagram).

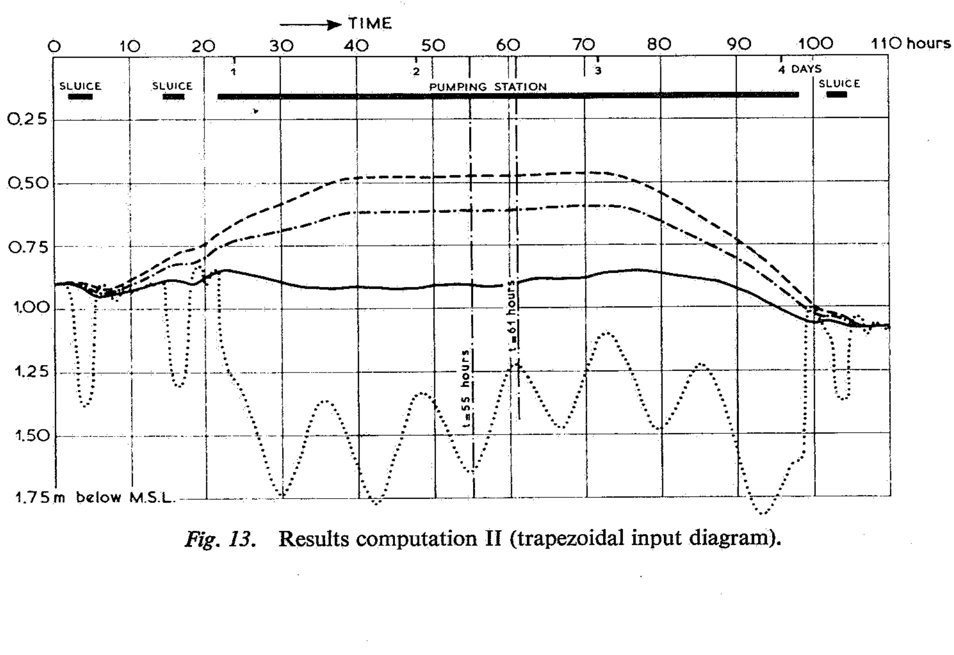


Fig. 13. Results computation II (trapezoidal input diagram).

appeared that at the end of the third day the difference in water levels between the two cases was about 0.10 m. The length of the rain period was such that during most of the time there was almost steady flow. In case of the rectangular diagram the maximum water levels occurred sooner. It should be noted that the tidal influence was flattened considerably by the presence of the lake.

If rainfall periods of short duration are considered, or if the degree of saturation of the soil is low, the retardation of the runoff might have a considerable influence on the discharge and water level patterns. For this reason the Delft Hydraulics Laboratory is developing a method for the determination of the relation between rainfall and the input into the nodes, as a part of the network computation.

Various methods can be considered:

In the first place the open-channel network can be extended by adding branches to represent the groundwater movement and surface flow. A disadvantage is that the network will quickly have a considerable magnitude, so that for the time being this method is only applicable for small areas. However, for small areas the value of the computation is restricted, because the retardation in the ground is often small, and especially because a low travel time of the high-water wave in the open water decreases the value of the open-channel computation.

A second method is the computation utilizing parameters of the area, which are determined separately. For this purpose the area under consideration is divided into sub-areas, each discharging on a node. From measurements on rainfall and runoff, and also of soil properties, relations can be established between rainfall and input in the nodes.

At present this method of computation is in an experimental stage.

3. The investigations in practice

3.1. Set-up of the mathematical model

The set-up of a mathematical model to be used for water management of a network of open-channels as described in Section 2.1 begins with the schematization of the water-courses into a system of branches and nodes.

The schematization is roughly determined by the geometry of the network in nature. Nodes are situated at those locations where water-courses come together, at locations where the cross-section of the water-course is greatly changing, and also at locations where water is supplied to or withdrawn from the system.

It should be noted that the geometry is not the only determining factor, as the nature of the problems to be investigated is also important for the design of the schematization.

If the number of branches chosen is too large, the computation will be more expensive than necessary, as both the costs of preparation (input data for all branches and nodes) and the costs of computation on the computer increase with an increasing number of branches. However, the computer time does not increase linearly if long branches are sub-divided, because in that case the iterative processes in the computation are converging more rapidly due to a decrease in the lengths of the steps from node to node.

A completely different question is whether a detailed system of side canals (smaller drainage canals and ditches, etc.), or a locally complicated system like municipal canals, should be incorporated in the model to a great extent.

A study to get an insight into the necessary degree of refinement has been made for a long main canal, with a side branch which is bifurcating into smaller branches, as is usual in polder areas. Figure 14 shows a number of schemes A...F which have a

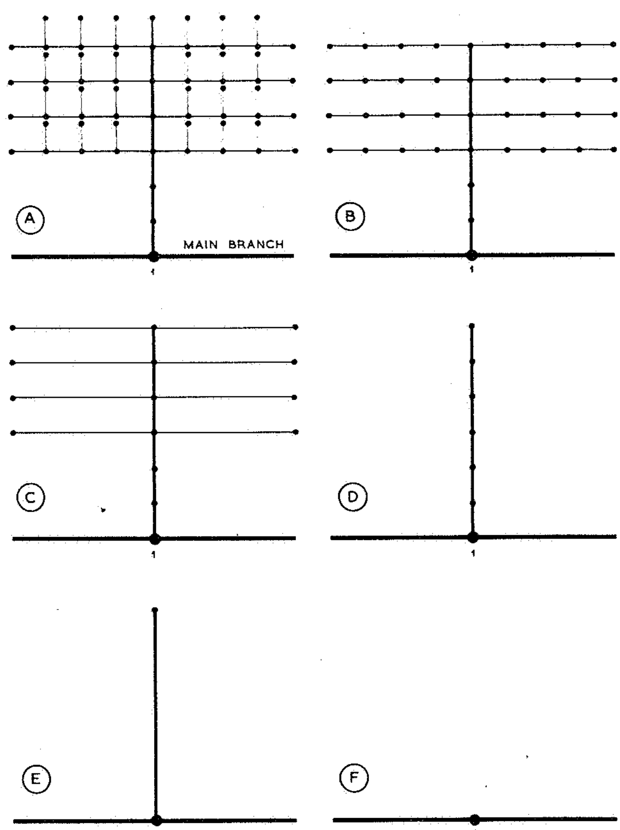


Fig. 14. Schematizations with decreasing degree of refinement.

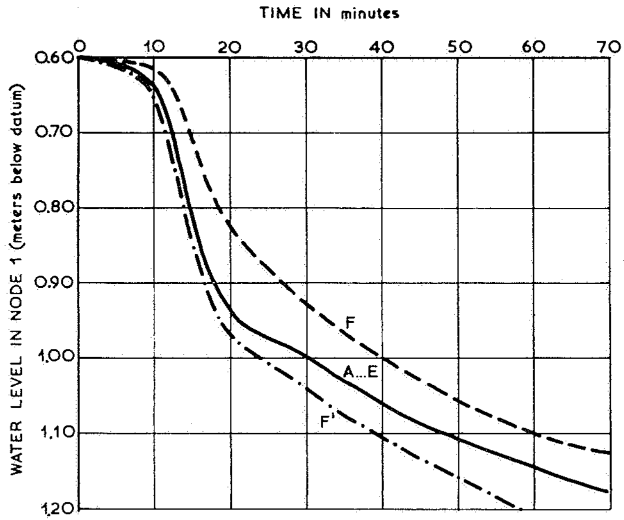


Fig. 15. Results of computation for schematizations A...F.

decreasing degree of refinement. In all cases the storage capacity of the omitted part (the so-called secondary storage) is concentrated in the nearest remaining node, which means that in case F the complete storage capacity of the side branches has been concentrated in node no. 1.

With these schemes computations were made for a negative translatory wave in the main canal. In Figure 15 the variation of the water level in node 1 is presented for various cases. It appears that the results of the computation are influenced only if all side branches are omitted. If the secondary storage capacity is concentrated completely in node 1, then the calculated water levels are too high (case F).

Omission of this storage capacity would yield water levels which are too low (case F′).

It can be concluded that for many problems schematization into a system of main branches will be sufficiently accurate if a part of the secondary storage is taken into account (solution between F and F′), or if the side branches of the first order are included in the schematization (case E).

An important factor to be taken into account during the set-up of the schematization is the influence on the accuracy of the computation of the relation between branch length L and time step Δt. In Vreugdenhil's paper (Chapter III, Par. 5) an explanation is given of the importance of a deliberate choice of L and Δt, in order to reduce the numerical damping of the waves in the computations and to avoid important errors in the wave propagation. However, these considerations do not mean that the choice of branch length and time step is completely determined by the desired accuracy. The following example of the network of Fivelingo Water Board, an artificially drained district in the north of the Netherlands, will show that there is still enough liberty to choose the branch length with a view to practical considerations.

In the original situation the main canals of Fivelingo Water Board were discharging into the sea through sluices. So the discharge was highly influenced by the tide; on an average, no discharge was possible during about half the tidal period. Therefore the basic wave-period to be considered is about 6 hours. This corresponds to a wave-length

$$l = 6 \times 3600 \times 4 \approx 8 \times 10^4 \text{ m}$$

where a wave propagation $c = \sqrt{gh} \approx 4$ m/s has been assumed, the average depth of the canals being 1.5 m.

In the main canal of the network (length about 30 km) the wave travels about 2 hours, or one-third of a wave period. During this time the wave is damped by a factor $D^{1/3}$, D being the numerical damping in a whole period. The condition that the numerical damping and the error in wave propagation will be, for example, lower than 2% yields

relative wave propagation $\quad c_r > 0.98$

damping $D^{1/3} > 0.98$, so $\quad D > 0.95$

Utilizing the graphs from Chapter III, Figure 12, various solutions can be found. The condition $D = 0.95$ is fulfilled, for example, for

μ	L/l	L	$\Delta t = \dfrac{\mu . \frac{1}{2} L}{c}$
4	0.0125	1,000 m	500 sec
2	0.025	2,000 m	500 sec
1	0.05	4,000 m	500 sec

At the same time, these combinations of μ and L/l give c_r-values which are greater than 0.98.

In Figure 16 two schematizations are given, both of which gave satisfactory results.

Although it appears that various schematizations can be used for a problem, it should be noted that for an operational model, to be used for many problems, the schematization should be considered critically for each case.

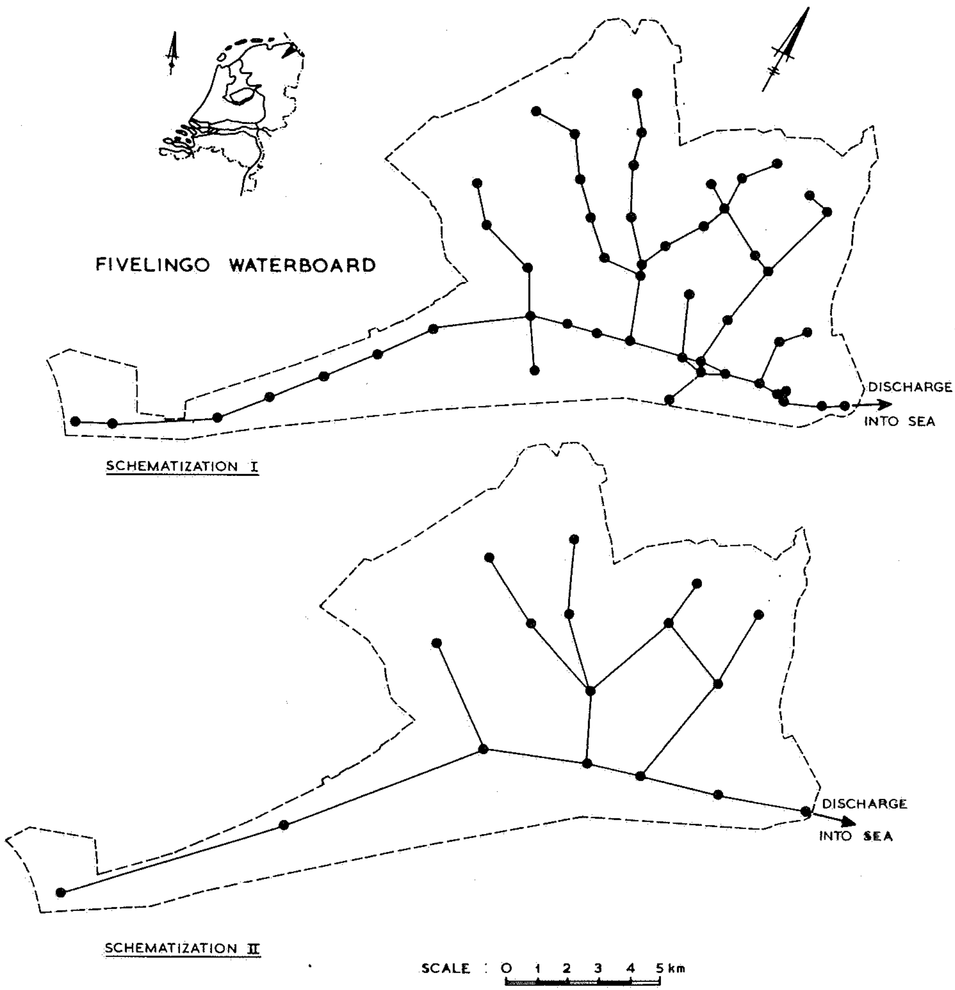


Fig. 16. Networks of Fivelingo Water Board.

The schematization being fixed, the next move is to determine all necessary input data, representing the geometry and further properties of the system.

For each branch, representing a part of a water-course, the following data should be given:

— the length L of the water-course,
— the average area A of the cross-section,
— the average width B of the cross-section, measured at the waterline, and
— the hydraulic resistance $\xi = \dfrac{L}{C^2 A^2 R}$, in which C = Chezy coefficient, to be

determined, for example, by the formula $C = 18 \log \frac{12R}{k}$ (White/Colebrook), where k = roughness coefficient, R = hydraulic radius.

In the nodes the storage of the adjacent water-courses is concentrated. Also the storage of branches which are not included in the schematization, and storage in lakes, etc., should be taken into account. (Of course, the storage is only important for non-steady flow.) Further, the level of the canal bottom at each node should be given.

For each branch or node the data can be given for various water levels above a horizontal level of reference, or above the bottom of the watercourse. This last method is often used in areas with steep slopes. Generally it is recommended to use data varying with water level, as the reduction of hydraulic resistance due to increase of water level influences the result of a computation.

It is obvious that for the set-up of a mathematical model it is necessary to deal with a large quantity of data in a very methodical way. Hence the costs of computations with mathematical models are determined to a great extent by the costs of preparation of the input, and the elaboration of the results. The results are given in the form of water levels in the nodes and discharges in the branches for each time-step. The costs of pure computation are relatively low.

3.2. *Verification of the model*

In many cases, especially for large and complicated networks, a verification of the permissibility of certain assumptions in schematizing the system is desired; in particular, if the available data are less detailed. Often the values of the hydraulic roughness have to be estimated. Further, the geometry of the system might necessitate a deviation from the theoretically desired branch lengths.

Because of these uncertainties, it might be useful to test the response of a model to a real situation of which sufficient data are known.

In the ideal case special measurements will be made to gather all necessary data, such as boundary conditions and discharges and water levels at various points of the network. Sometimes, observations from a past period can be a valuable aid.

An example of such model testing, utilizing to the best possible extent the available data from a past period, is the verification of the response of the model of the main canal system of a large area of combined polders in North Holland (Hoogheemraadschap van de Uitwaterende Sluizen in Kennemerland en West-Friesland). The schematization is presented in Figure 17. The test was made for a period of heavy rainfall in December 1960, in a period during which the water level in the system

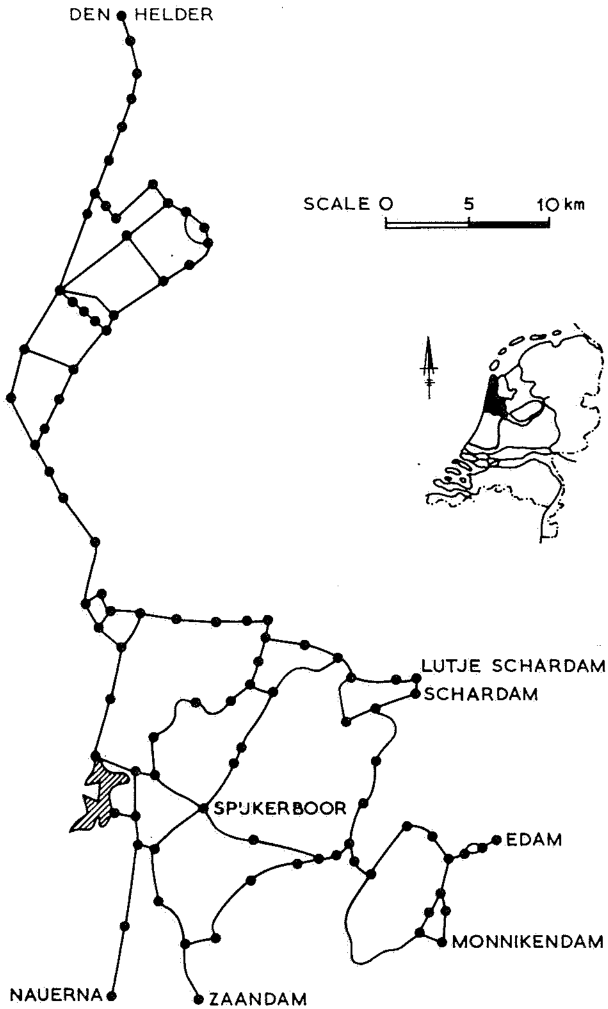


Fig. 17. Network of canals in North Holland.

several times reached the critical level at which the pumping-stations of the individual polders were no longer allowed to pump water to the main canals. The critical level was a level of NAP -0.10 m at Spijkerboor (see schematization). Discharge to the main canals was allowed again after a drop of the water level at this point below NAP -0.20 m.

Most of the area controlled by the Water Board consists of polders, with separate pumping-stations. Only a small part, e.g., the dune area, is discharging directly into the main canals. In addition, the precipitation on the open water had also to be taken into account.

In 1960 discharge of superfluous water took place via sluices at Den Helder, at the IJssel Lake, and at Nauerna and Zaandam (North Sea Canal). Boundary conditions were the water levels outside the sluices, which were partially known (Fig. 18). In Den Helder the tide was registered; at some of the other sluices the water level variations had to be determined during part of the time by interpolation or estimation. The discharge to the main canals (input in the nodes) could be determined in a fairly accurate way from precipitation figures and registered working hours of pumping-stations in the polder.

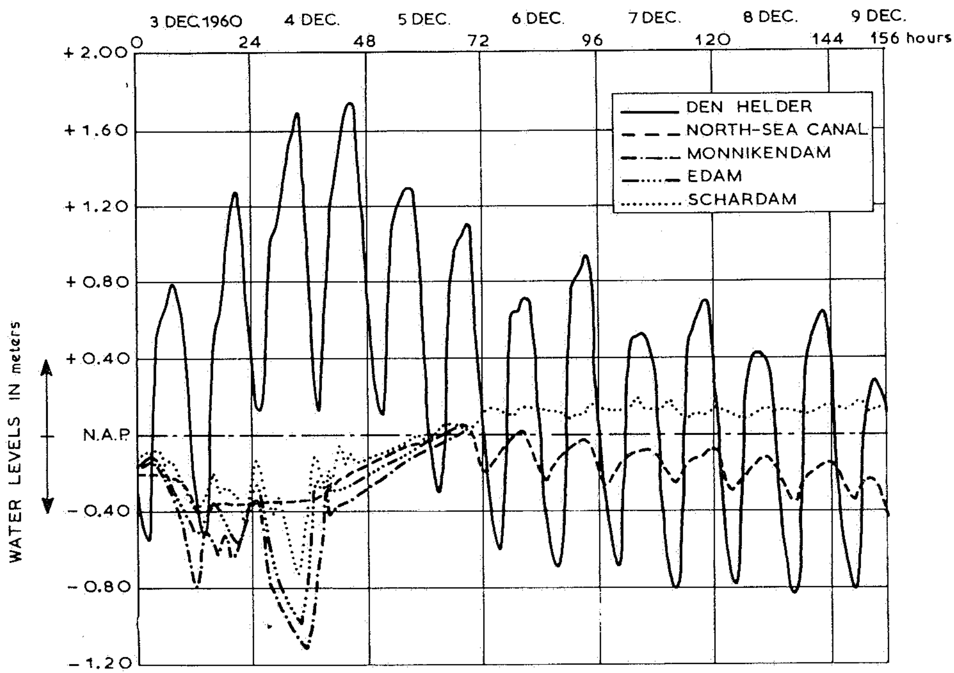


Fig. 18. Boundary conditions.

To start with, a computation was made of a 2.5-day period, applying an input in the nodes determined as accurately as possible for each time-interval of 15 minutes.

The computation gives water levels in each node and discharges in each branch as a function of time. As only in Spijkerboor water levels were measured, in Figure 19 a comparison is made between these measured points (dots in the diagram) and the calculated water level variation (dotted line). There is a good agreement, especially if the incompleteness of the boundary conditions is taken into account. The critical

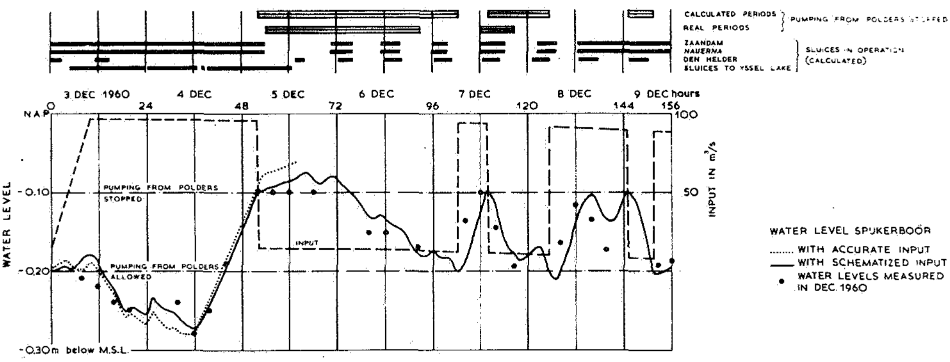


Fig. 19. Results of verification.

water level of NAP -0.10 m was reached in nature and computation almost at the same time.

In the next computation the capricious input in the nodes, being the real discharge, is replaced by a schematized trapezoid-shaped discharge, divided over the nodes according to magnitude of catchment area. Applying this input, a computation was made for a period of 6.5 days. When the water level of NAP -0.10 m was reached, the input in the nodes, as far as originating from pumping-stations, was stopped till the moment that the water level fell below NAP -0.20 m, a procedure which is also followed in nature. During this period the precipitation in the polders was stored in the polder ditches.

The result of this computation is also given in Figure 19. The first part of the line shows only a slight difference from the first computation. For the whole period the periods of no discharge last longer than in reality. However, this can be due to very small deviations in water level. If, for example, at the moment $t = 117$ hours the calculated water level would have been 0.02 m lower, the second period of no discharge would have come to a [illegible] that moment. In general, the picture of calculated water level variations is fairly similar to the observed variations, although at the end of the computational period the difference in lengths of the periods of no discharge disturbs a good comparison.

Because reasonable results were obtained for this complicated case, in spite of boundary conditions and further input data which were not quite complete, sufficient confidence was created to justify making computations for other situations. Figure 20 shows the calculated water level variations in the same period of heavy rainfall, assuming that in 1960 the present main pumping-stations in Den Helder and Zaandam had been there. Assuming some logical programme of pumping (working capacity dependent on water level and weather forecast), the lengths of the period of no discharge from the polders would have been reduced considerably.

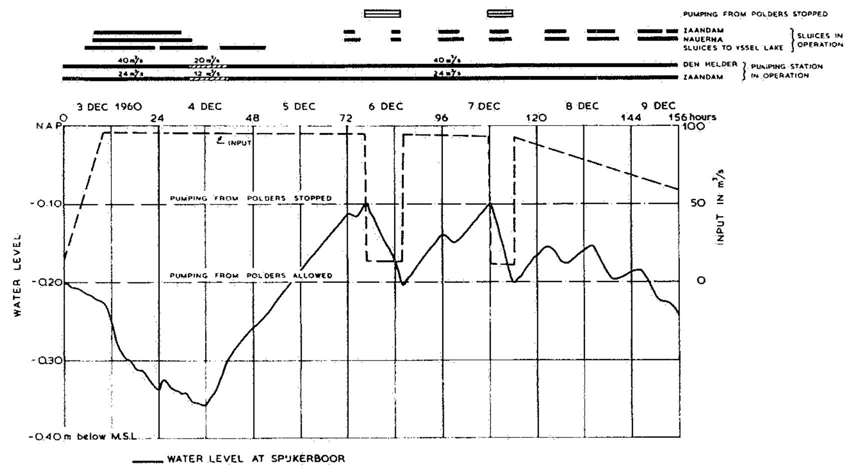


Fig. 20. Results if main pumping stations are present.

Next, various computations have been made on behalf of investigations for improvement of discharge capacity and water quality, but this extensive programme of investigations is beyond the scope of the present report.

A separate point to be taken into account when comparing computations and measurements is the wind influence on water movement in open canals. It is difficult to integrate this influence in the mathematical model, because of the great variety of winds from different directions which have influence on the various canals. Fortunately, measurements in the canal system of the Rijnland Water Board show that the tilting of the water table due to wind is almost independent of the current velocities in the canals (see Fig. 21). Hence it is possible to treat the wind influence separately.

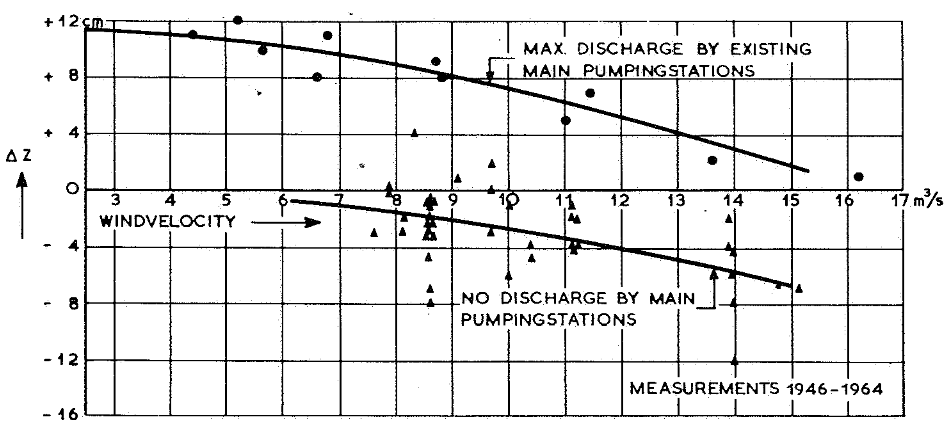


Fig. 21. Water level variation due to wind.

3.3. Steady and non-steady flow

As already stated, computations can be made for steady and non-steady flow.

In hydraulic engineering most problems are non-steady flow phenomena, although in a number of cases steady-state computations give sufficient information; hence such computations are to be preferred for reasons of simplicity.

Of course, there are problems concerning real steady flow, such as flushing of drainage canals to improve the water quality. Examples of steady-state problems are the investigations made for the canals in the cities of Haarlem and Leyden, which canals are part of the network of Rijnland. Traffic problems gave rise to a proposal to fill up several canals partly or completely, but these solutions were acceptable only if compensation could be found elsewhere in the system in such a way that the following criteria could be fulfilled for the steady-state case of flushing the system:

1. Distribution of discharges to and from the cities may not change.
2. Total difference in head at points south and north of the city may not change.

A great number of alternative solutions of such problems can be compared rapidly by steady-state computations.

The statement at the beginning of this Section that steady-state computations can sometimes give sufficient information for non-steady conditions can be illustrated by the example of Duurswold Water Board, given in Section 2.5 (Fig. 12). Although the downstream boundary condition is time-dependent, due to the tidal influence, the lake which is part of the system reduces this influence considerably. Hence, in the points of the system with the highest water levels, which are the determining points for the design of the canal cross-sections, the flow is steady during part of the time.

In such a case an investigation for an improvement of discharge capacity can be made very simply by comparing a great number of alternatives, using the results of steady-state computations, starting from a fixed water level at the downstream end (see Fig. 22).

In general, steady-state computations can often give a great deal of information concerning the effectiveness of certain solutions.

In many cases, however, the time-dependent character of a phenomenon is so important that computation of non-steady flow is essential. A clear example of a non-steady situation has been discussed already in Section 3.2, but in the following Section another specific example is given.

3.4. Special applications

3.4.1. Dike collapse in polder area

As most of the canals of a main network of a polder area are situated between dikes, the waterlevels in the canals being higher than the ground level of the polders, a special problem is the investigation of the consequences of a sudden collapse of such a dike.

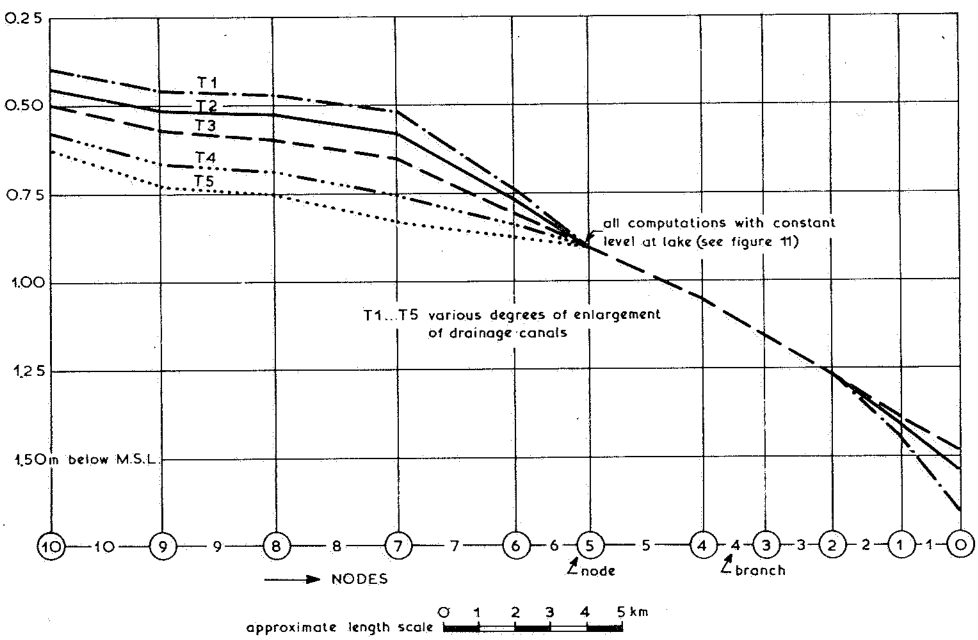


Fig. 22. Computations for steady flow.

Collapse of a dike between a canal and a low-situated large polder will cause a drop of the water levels in the whole canal system. This raises problems in the fields of hydraulics and soil mechanics. A rapid draw-down of the water levels can lead to stability problems of other dikes and thus to further dike collapses, so that it is important to determine the drop in water level as a function of time for other points of the system.

For the network of Rijnland Water Board two cases of dike collapse have been investigated:

1. Dike collapse at the end of a small dead branch (Fig. 23); and
2. dike collapse in a dike along the largest canal of the network, the Ringvaart of the Haarlemmermeerpolder (Fig. 24).

In both cases the assumption was made that the gap would be so large that the capacities of the adjoining parts of the canal would restrict the discharge through the gap. In that case the water-course itself would act as a long weir, so that a boundary condition for the computation would be formed by the assumption of critical velocity near the gap.

In the second case, of dike collapse along a water-course, two nodes were placed at the location of the gap so that the water levels and discharges at both sides of the gap were independent of each other. This approach assumed a gap which is large

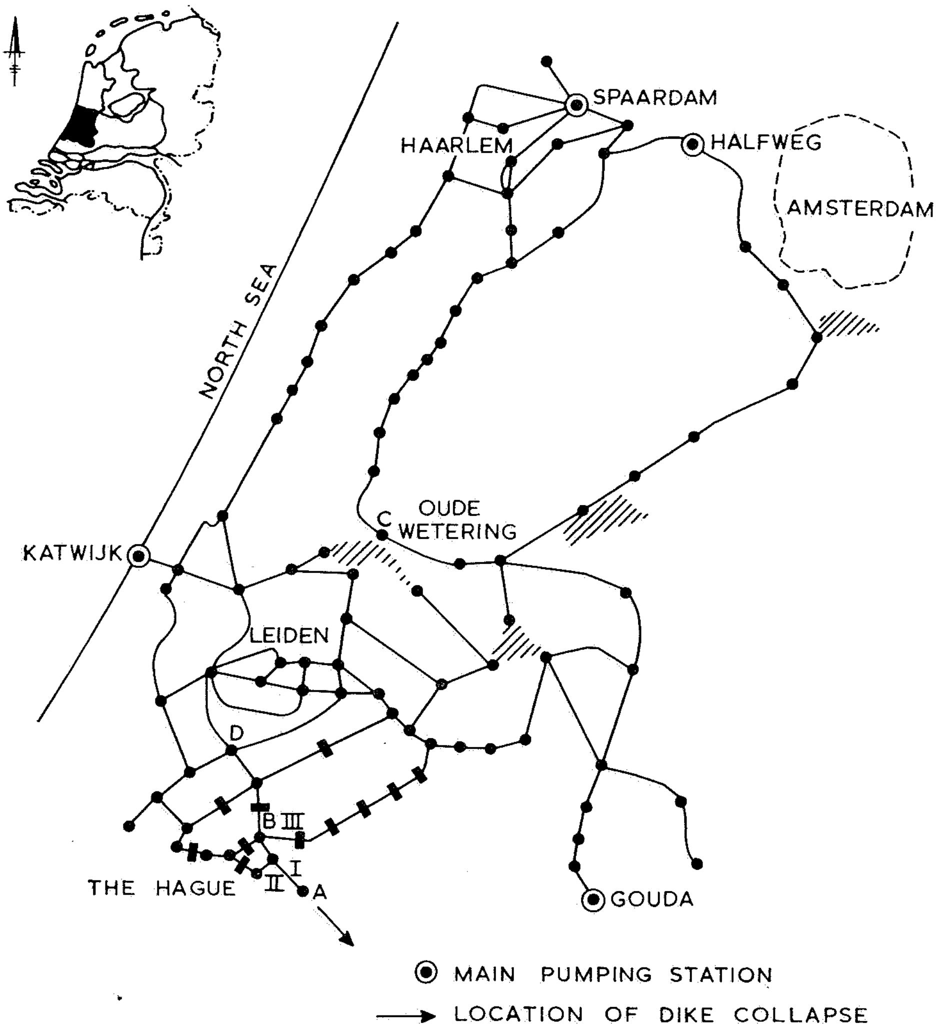


Fig. 23. Dike collapse at the end of a small branch in Rijnland network (Case I).

compared to the width of the water-course, as otherwise critical velocity could only arise in one of the two adjoining branches.

Immediately after the moment of collapse a negative wave with steep front moved through the network, and due to the greater wave depth at the front of the wave, this part of the wave had the greatest celerity, so that the steepness of the front decreased after some time. For correct reproduction of this phenomenon a dense system of nodes is necessary near the gap. Attention should be paid to the more

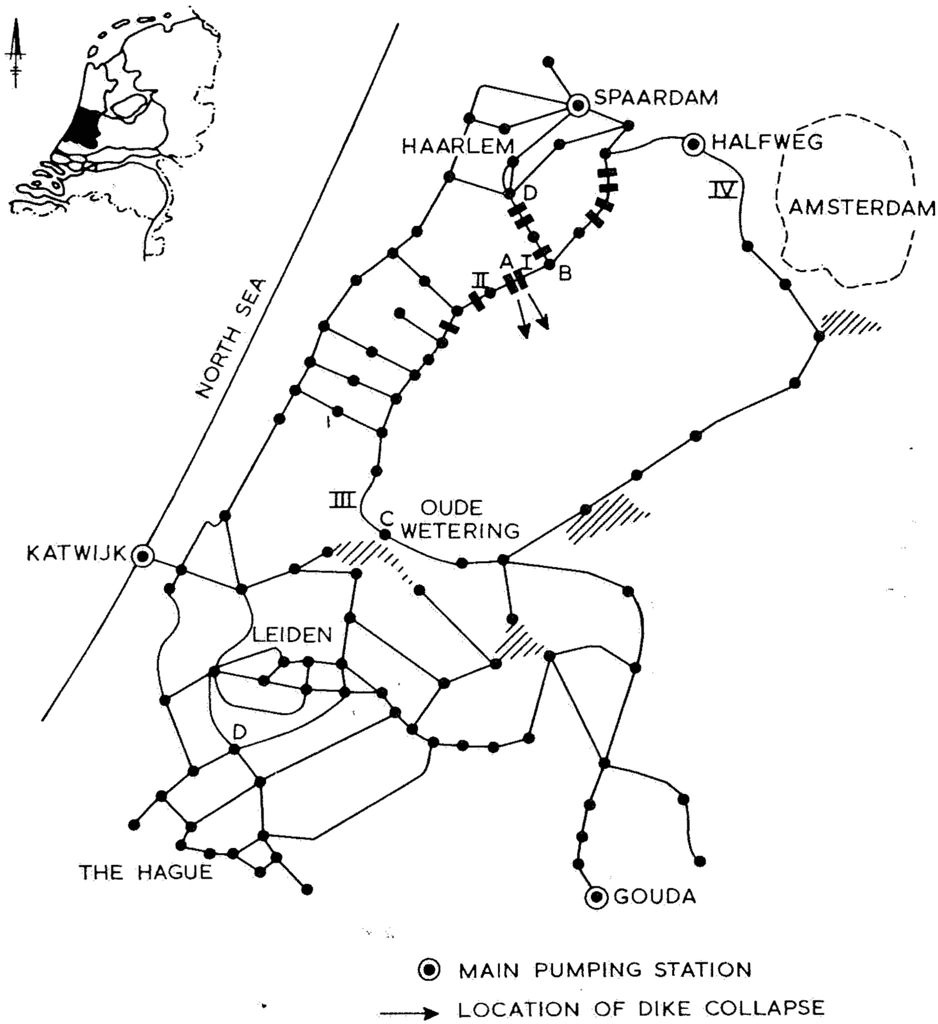


Fig. 24. Dike collapse along a large canal in Rijnland network (Case II).

refined schematization in the problem area of Figure 24, a refinement which appeared to be not necessary for previous computations for the Rijnland Water Board, like the one with the network of Figure 23.

The results of both computations are presented in Figures 25 and 26. Due to the small capacity of the adjoining water-courses in the first case, the drop in water level elsewhere in the system is in the order of several centimeters to about 10 centimeters, even after 10 hours. In the second case, the influence of the dike collapse is much greater. In both cases erosion by high stream-velocities should be expected.

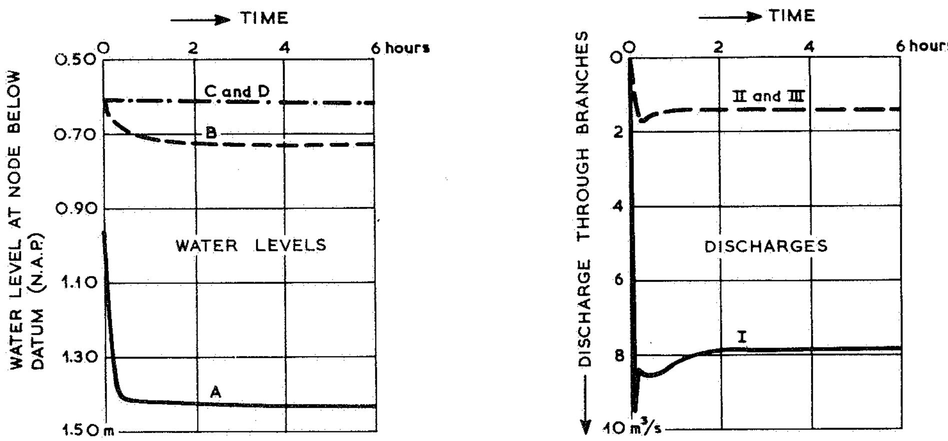


Fig. 25. Water level variations and discharges due to sudden dike collapse (Case I).

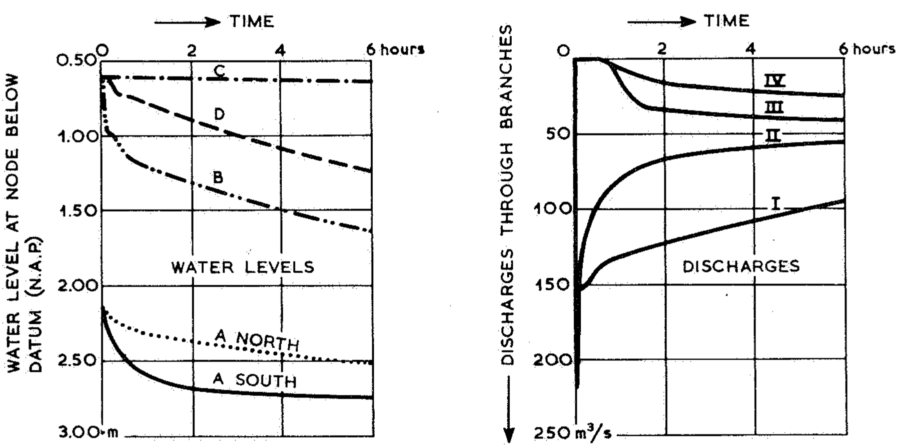


Fig. 26. Water level variations and discharges due to sudden dike collapse (Case II).

3.4.2. Dispersion of salt or pollutants

With increasing attention being paid to the control of water quality, it is important to look for methods of combined computations of water movement and dispersion of diluted matter. Whether the diluted matter consists of salt or pollutants is not really important for the method of computation. In both cases dispersion is caused by turbulent mixing, due to non-uniform flow distribution. However, the concentration gradients should be within certain limits, so that no density currents occur, because such currents are left out of consideration in the first set-up of the computations. The theoretical backgrounds of these computations are dealt with in Berkhoff's paper (Chapter IV).

In this Section some results are given of a computation of the movement of salt in the Andelse Maas. This example forms part of a current research programme being carried out by the Delft Hydraulics Laboratory, in co-operation with the Duinwaterleiding 's-Gravenhage (Municipal Water Board of The Hague).

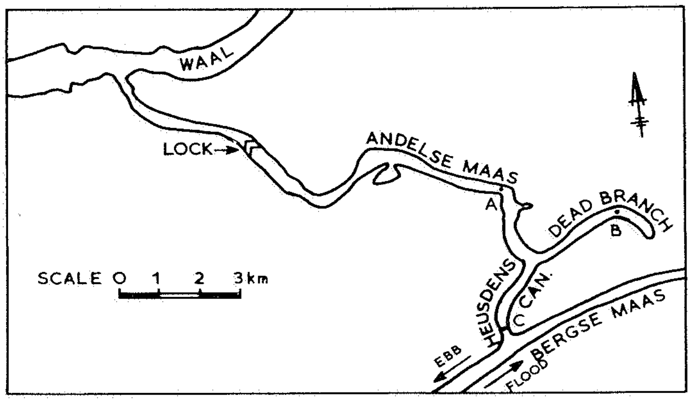


Fig. 27. Situation.

The situation is presented in Figure 27. Through a lock in the Andelse Maas relatively salt water from the River Waal, one of the branches of the River Rhine, intrudes into the Andelse Maas. This salt originates from industrial areas upstream in the River Rhine. Through the Heusdens Canal the Andelse Maas is connected with the Bergse Maas, which is a part of the River Maas (Meuse). The concentration of industrial salts in the Maas is lower than in the Waal. Further, a dead branch at the south-eastern end of the Andelse Maas plays a rôle.

The water movement in this area is influenced by the tidal movement in the Bergse Maas. Salt from the sea is not intruding into this area, however. Due to the tidal influence the salt concentrations in the Andelse Maas are varying, and so a mathematical model was made to calculate the salt movement. The calculated concentrations have been compared with measurements made in the area.

First the water movement was calculated. In this simple case this was done by means of a storage calculation. In principle, a network calculation as described in the previous Sections can also be used. However, for the computation of concentrations the schematization must be more refined than is necessary for computation of water movement.

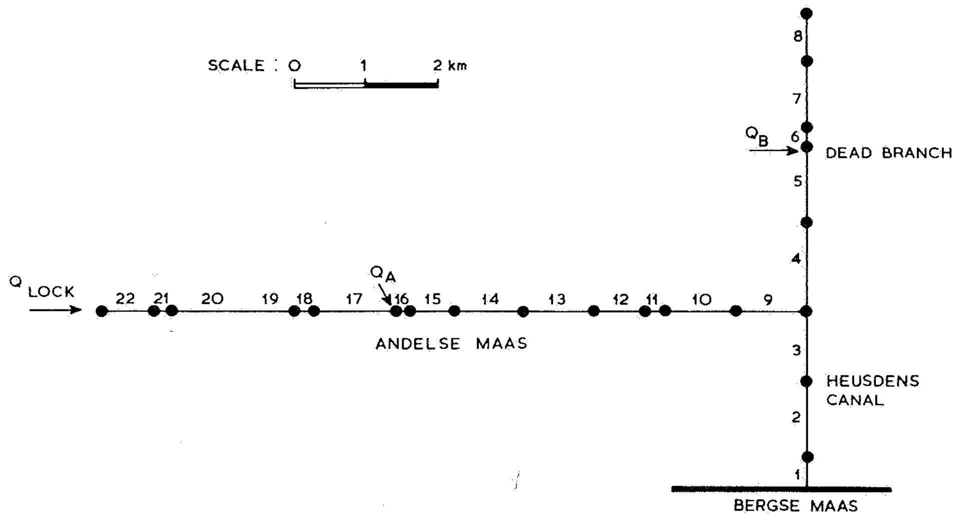


Fig. 28. Schematization.

The schematization is given in Figure 28. The location of nodes is determined to a great extent by the geometry of the water-course, i.e., by changes in area of the cross-section. Q_A and Q_B are discharges from pumping-stations.

The water movement being known (Fig. 29, upper part), the computation of concentrations can be made. Initially the distribution of concentrations in the branches must be known, e.g., from measurements or from approximations, while at the boundaries of the network, boundary conditions for the concentration should be given. In this case, the boundary conditions were chosen as follows (see Chapter IV):

At the lock: $C = C_{\text{lock}}$

End of dead branch: $\dfrac{\partial C}{\partial x} = 0$ (no mass transport)

Bifurcation Heusdens Canal-Bergse Maas:

Ebb: $\dfrac{\partial}{\partial x}\left(D\dfrac{\partial C}{\partial x}\right) = 0$ (linear variation of dispersive transport)

Flood: $C = C_{\text{Bergse Maas}}$ (complete mixing)

Further, the concentrations of the tributary inflows Q_A and Q_B should be known.

Utilizing a dispersion coefficient $D = kh|u|$, where k = constant (chosen value of $k = 20$), h = depth, and u = velocity, the variations of the concentrations with time are found as presented in Figure 29. Comparison with the measured concen-

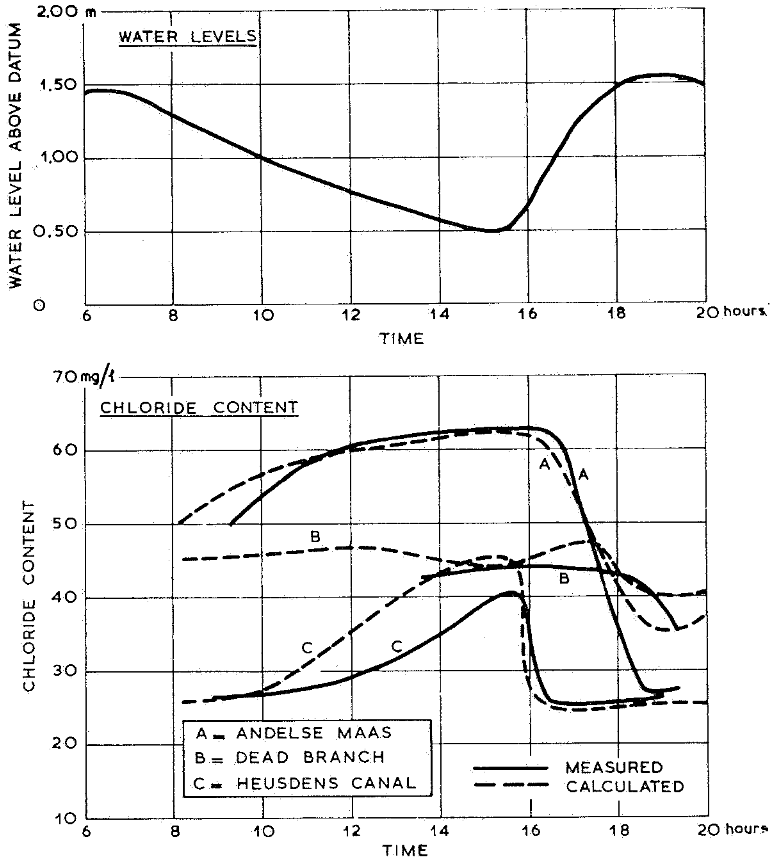


Fig. 29. Results of computations.

trations shows a reasonable agreement between computation and measurement. The phase lag could be explained by the fact that the assumption of complete mixing with the water in the Bergse Maas might be incorrect. Due to this fact, relatively salt water might enter the Heusdens Canal during the beginning of flood tide, so that the decrease of concentration in the system starts later. Also by neglecting the retardation of the water movement, as present in a storage calculation, the results might have been influenced. However, the results have been sufficiently encouraging to warrant the continuation of the research programme.

4. Conclusion

The previous paragraphs give an idea of the possibilities of network computations.

Further developments should be found in the field of optimization of networks, so that the computations can be applied more directly for design purposes.

Water management in polder areas to-day is a matter of taking decisions based on experience and sound judgment. However, these decisions are also based on data and expectations, so that it should be possible to "translate" the procedure of taking decisions mathematically. Thus a model of a channel network can be utilized as an operational model, producing quick solutions to problems arising in all kind of circumstances, so that a sound base for optimal water management will be obtained.

LITERATURE

M. DE VRIES — Het rekentuig als hulpmiddel bij rioleringsberekeningen, Delft Hydraulics Laboratory, Publ. no. 75, December 1969.

SAMENVATTING

PRAKTISCHE TOEPASSINGEN VAN NETWERKBEREKENINGEN

In dit hoofdstuk wordt het toepassingsgebied van netwerkberekeningen, met betrekking tot problemen van waterbeheersing in de meest ruime zin, geïllustreerd aan de hand van een aantal voorbeelden. Tevens wordt een inzicht gegeven in de opzet van een wiskundig model, de benodigde gegevens, en de mogelijke resultaten.

Een overzicht van de verschillende soorten problemen waarvoor de rekenmethode reeds werd toegepast, is gegeven in paragraaf 2:

Par. 2.1. Netwerken van open leidingen.

Een belangrijk toepassingsgebied hierbij is de waterbeheersing in stelsels van polder- en boezemwateren. Enkele problemen die zich voor berekening lenen zijn de volgende:

- optimale keuze van plaats en capaciteit van gemalen en sluizen;
- berekening van de benodigde capaciteit van boezemwateren, alsmede berekening van de invloed van uit te voeren werken in deze wateren en de wijze waarop compensatie elders in de boezem kan worden verkregen;
- onderzoek naar de waterloopkundige gevolgen van kadebreuk;
- onderzoek naar de verspreiding van zout of afvalstoffen, bijvoorbeeld als gevolg van industriële lozingen;
- onderzoek naar maatregelen die moeten worden genomen in geval van een calamiteit, zoals de lozing van gifstoffen.

Eveneens kunnen wiskundige modellen worden gemaakt van koelwater-circuits, waarbij zowel de waterbeweging als het temperatuurverloop worden berekend.

Par. 2.2. Hoogwatergolven in beken en rivieren.

De voortplanting van de golf kan worden berekend, evenals een eventuele inundatie als gevolg van de hoogwatergolf.

Par. 2.3. Getijberekening in estuaries en rivieren.

Als voorbeeld worden enkele problemen behandeld die zijn opgelost met behulp van het wiskundig model van de lagune bij Abidjan, Ivoorkust.

Par. 2.4. Rioleringsberekeningen.

Hiervoor is de rekenmethode aangepast ten behoeve van de berekening van gesloten leidingen en de verdere karakteristieke eigenschappen van rioleringsstelsels.

Par. 2.5. Koppeling van stroming in open water en grondwaterstroming.

Voor Nederlandse omstandigheden kunnen voor de berekening van waterbezwaar in poldergebieden redelijke aannamen worden gedaan voor de vertraging tussen regenval en belasting van het open water, daar in de maatgevende omstandigheden de grond veelal met water is verzadigd. Voor korte neerslagperioden of onverzadigde grond worden rekenmethoden ontwikkeld om de vervorming en vertraging door afvoer over en in de grond vast te stellen.

In paragraaf 3 wordt de opzet van het wiskundige model behandeld. Achtereenvolgens wordt ingegaan op de benodigde gegevens voor schematisering van de waterlopen en de invloed van de schematisering op de nauwkeurigheid van de resultaten (par. 3.1.), de ijking van het model aan de hand van metingen (par. 3.2.) en de mogelijkheid om in bepaalde gevallen niet-permanente verschijnselen vereenvoudigd weer te geven door middel van permanentie-berekeningen (par. 3.3.). Hierbij wordt een aantal voorbeelden gegeven van berekeningen voor de boezems van hoogheemraadschappen en waterschappen.

Paragraaf 3.4. geeft tenslotte voorbeelden van enkele speciale toepassingen, respektievelijk een onderzoek naar de gevolgen van kadebreuk in de boezem van het Hoogheemraadschap van Rijnland, en de berekening van zoutdoordringing in de Andelse Maas, tengevolge van de aanvoer van relatief zout water door de sluis die de Andelse Maas verbindt met de Waal.

Als conclusie wordt gesteld dat het wiskundige model als operationeel model ten dienste kan staan van de waterbeheerder, om in voorkomende situaties op adequate wijze een optimaal beleid te kunnen voeren.

COMMITTEE FOR HYDROLOGICAL RESEARCH T.N.O.

Proceedings and Informations

No. 1. Proceedings of Technical Meetings 1–6 (with summaries in English), 1952.
1. Investigations into the water balance of the Rottegatspolder
2. The water supply for crops I
3. Observations of groundwater levels
4. Investigations by drain gauges in the Netherlands
5. The water supply for crops II
6. The problem of the increasing salinity of ground and surface water in the Netherlands

No. 2. Proceedings of Technical Meetings 7–10 and Report on the evaporation research in the Rottegatspolder 1947–1952 (with summaries in English), 1955.
7. The study of precipitation data
8. Model research on groundwater flows
9. Measurements and improvement works in basin of brooks
10. Geo-electrical research

No. 3. Proceedings of Technical Meetings 11–12 (with summaries in English) and Report on the lysimeters in the Netherlands (in English), 1958.
11. The water supply of sandy soils
12. Quality requirements for surface waters

No. 4. Evaporation Symposium and Report on the Lysimeters in the Netherlands II (with summaries in English), 1959.

No. 5. Proceedings of Technical Meetings 13–14 (with summaries in English), 1960.
13. Groundwater levels and groundwater movement in the sandy areas of the Netherlands
14. Water in unsaturated soil

No. 6. Proceedings of Technical Meeting 15 (with summaries in English), 1961.
15. The regimen of the Rhine, the Ysselmeer and Zealand lake

No. 7. Proceedings of Technical Meeting 16 (with summaries in English), 1962.
16. The dry year 1959

No. 8. Proceedings of Technical Meeting 17 (with summaries in English). 1963.
17. The laws of groundwater flow and their application in practice

No. 9. Proceedings of Technical Meeting 18 (with summaries in English), 1963.
18. Water nuisance

No. 10. Steady flow of ground water towards wells, compiled by the Hydrologisch Colloquium (in English), 1964.

No. 11. Proceedings of Technical Meeting 19 (with summaries in French and German), 1964.
19. Geo-hydrological cartography

No. 12. Water balance studies, Proceedings of Technical Meeting 20 (in English), 1966.

No. 13. Recent trends in hydrograph synthesis, Proceedings of Technical Meeting 21 (in English), 1966.

No. 14. Proceedings of Technical Meeting 22 (with summaries in English), 1968.
22. Precipitation Data (II)
Report on the Lysimeters in the Netherlands (III).

No. 15. Soil — Water — Plant, 1969 (in English).

No. 16. Not yet published.

No. 17. Proceedings of Technical Meeting 25, (in English), 1973.
25. Automatic Processing of Hydrological Data.

COMMISSIE VOOR HYDROLOGISCH ONDERZOEK T.N.O.

Verslagen en Mededelingen

No. 1. Verslagen van de Technische bijeenkomsten 1–6 (with summaries in English, avec des résumés en français), 1952.
1. Het waterhuishoudkundig onderzoek in de Rottegatspolder
2. De watervoorziening der gewassen I
3. Waarneming van grondwaterstanden
4. Lysimeteronderzoek in Nederland
5. De watervoorziening der gewassen II
6. Het vraagstuk van de verzouting van grond- en oppervlaktewater in Nederland

No. 2. Verslagen van de Technische Bijeenkomsten 7–10 en Verslag inzake het verdampingsonderzoek in de Rottegatspolder over de jaren 1947–1952 (with summaries in English, avec des résumés en français), 1955.
7. Bewerking van regenwaarnemingscijfers
8. Modelonderzoek van grondwaterstromingen
9. Metingen en verbeteringen in het stroomgenied van beken
10. Geo-electrisch bodemonderzoek

No. 3. Verslagen van de Technische Bijeenkomsten 11–12 (with summaries in English, avec des résumés en français) en Rapport inzake de lysimeters in Nederland (in de Engelse taal), 1958.
11. Watervoorziening van zandgronden
12. Kwaliteitseisen van oppervlaktewater

No. 4. Verdampingssymposium Agrohydrologisch Colloquium C.O.L.N. en Rapport inzake de lysimeters in Nederland II (with summaries in English), 1959.

No. 5. Verslagen van de Technische Bijeenkomsten 13–14 (with summaries in English), 1960.
13. Grondwaterstanden en grondwaterbeweging in de Nederlandse zandgronden
14. Het water in onverzadigde grond

No. 6. Verslag van de Technische Bijeenkomst 15 (with summaries in English, avec des résumés en français), 1961.

15. Enige onderzoekingen ten behoeve van de grondslagen van het beheer van de Rijn, het IJsselmeer en het Zeeuwse Meer

No. 7. Verslag van de Technische Bijeenkomst 16 (with summaries in English), 1962.
16. Het droge jaar 1959

No. 8. Verslag van de Technische Bijeenkomst 17 (with summaries in English), 1963.
17. De stromingswetten van het grondwater en de toepassing daarvan in de praktijk

No. 9. Verslag van de Technische Bijeenkomst 18 (with summaries in English), 1963.
18. Wateroverlast

No. 10. Steady flow of ground water towards wells, compiled by the Hydrologisch Colloquium, 1964.

No. 11. Verslag van de Technische Bijeenkomst 19 (avec des résumés en français, mit Zusammenfassungen in Deutsch), 1964.
19. Geo-hydrologische karteringen

No. 12. Water balance studies, Proceedings of Technical Meeting 20, 1966.

No. 13. Recent trends in hydrograph synthesis, Proceedings of Technical Meeting 21, 1966.

No. 14. Verslag van de Technische Bijeenkomst 22 (with summaries in English), 1968.
22. Regenwaarnemingscijfers (II), Rapport inzake Lysimeters in Nederland (III).

No. 15. Soil — Water — Plant, 1969

No. 16. Nog niet verschenen.

No. 17. Verslag van de Technische Bijeenkomst 25 (in Engelse taal), 1973.
25. Automatic Processing of Hydrological Data.