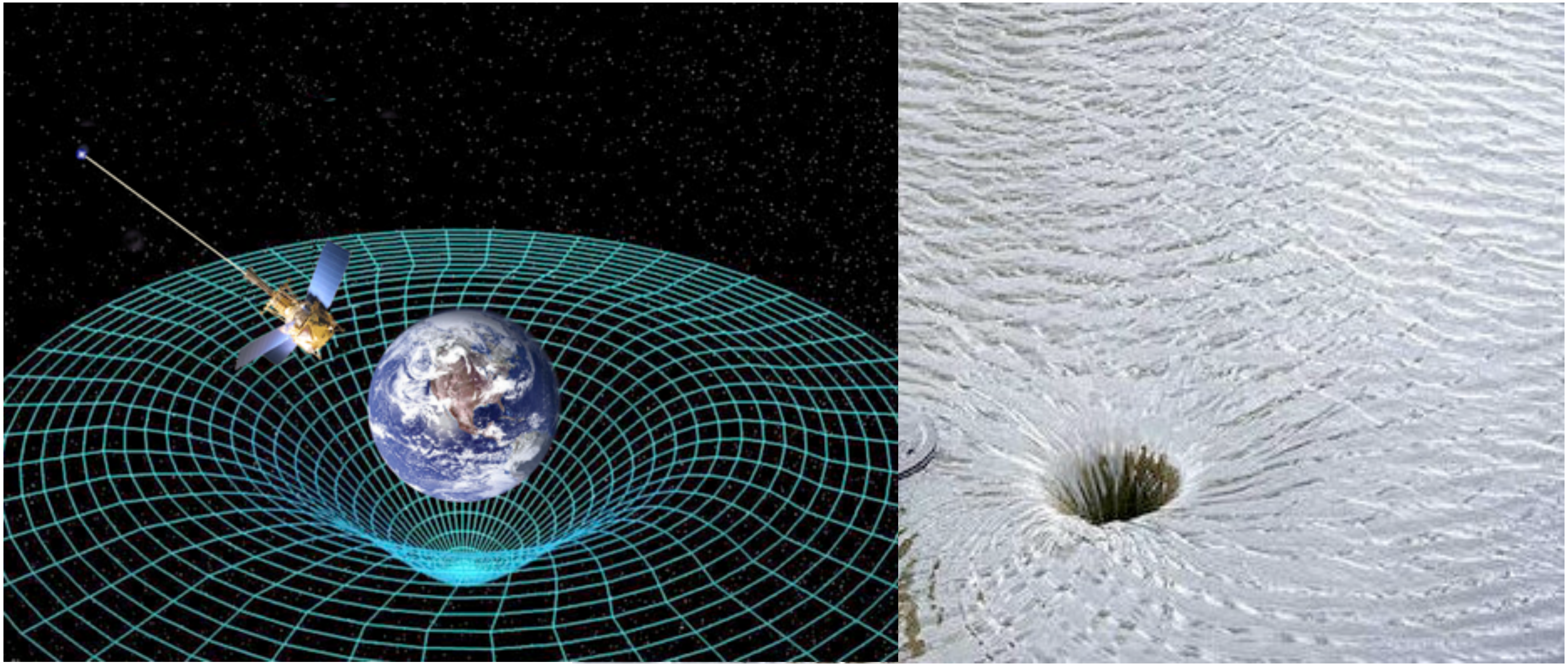
The Application of Time Series Models in Regular Groundwater Models

Extension of time series models into the space-time continuum



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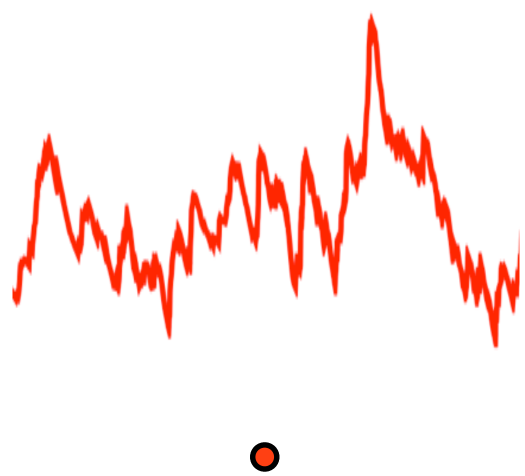
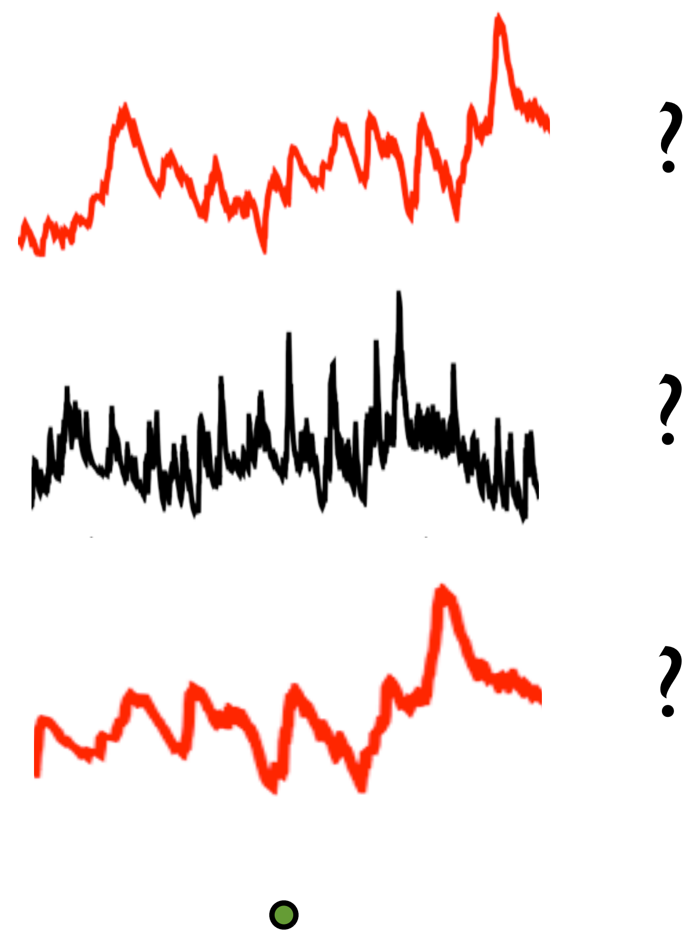
Time series analysis requires you to go to the past and then back to the future

Can we use a time series model at one observation well to simulate a time series at another observation well?



if we have a time series
of heads at this point



can we estimate a time series
of heads at this point?

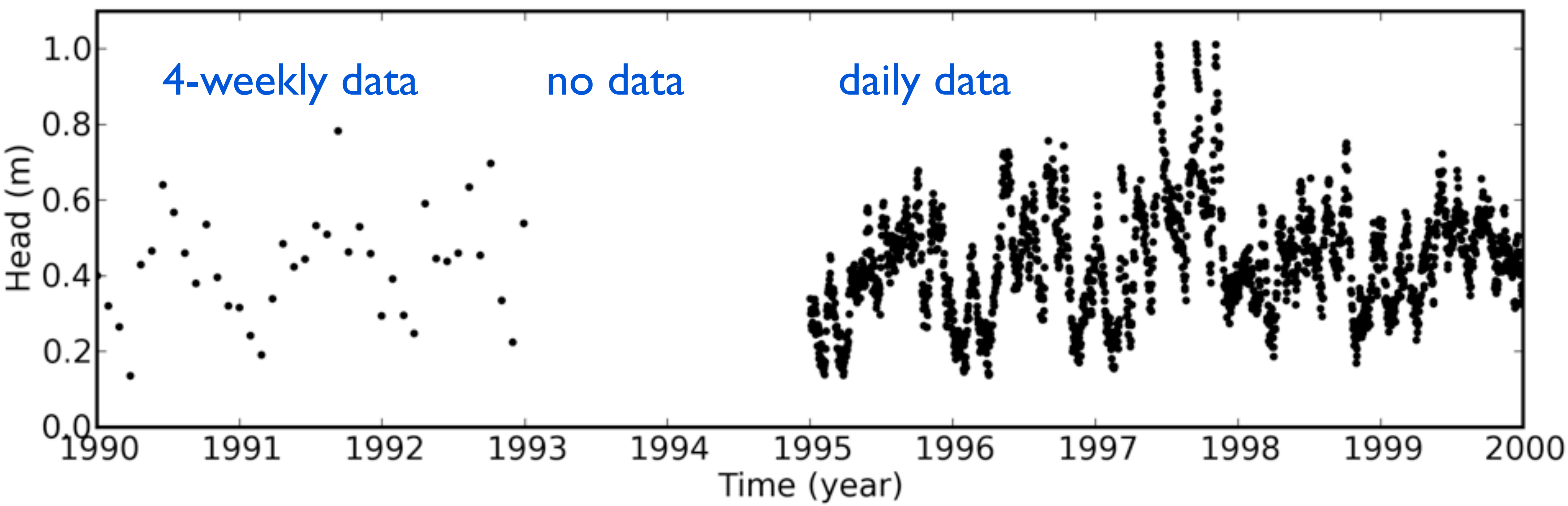
Extension of time series models into the space-time continuum

- What happens under the hood of a time series model?
- Spatial interpolation of time series models
  Generate calibration data sets for NHI
- Spatial modeling of response function characteristics
  Calibration of transient groundwater models

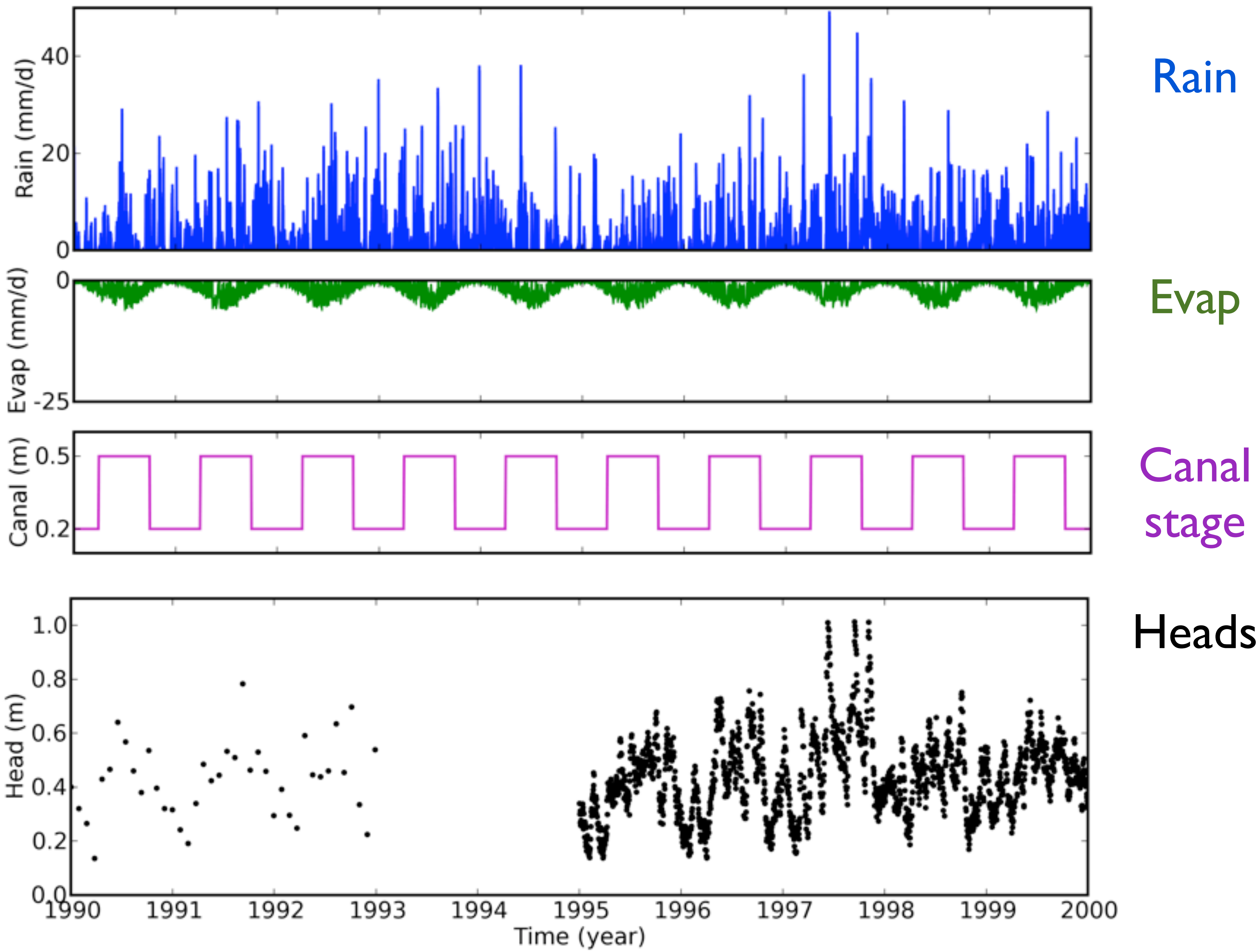


What happens under the hood of a time series model?

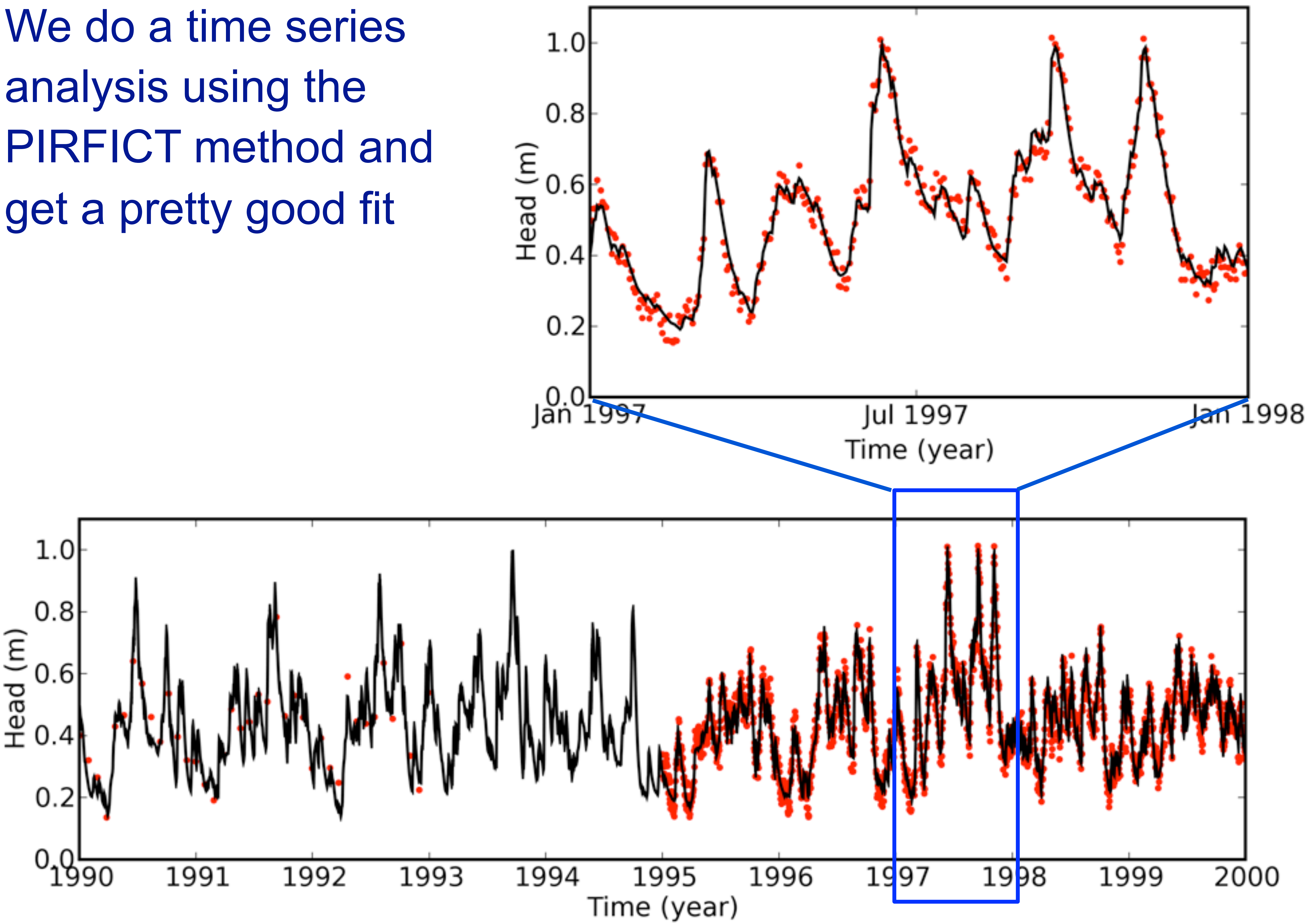
Imagine we have head observations over 10 years



And we have daily values of three measured stresses

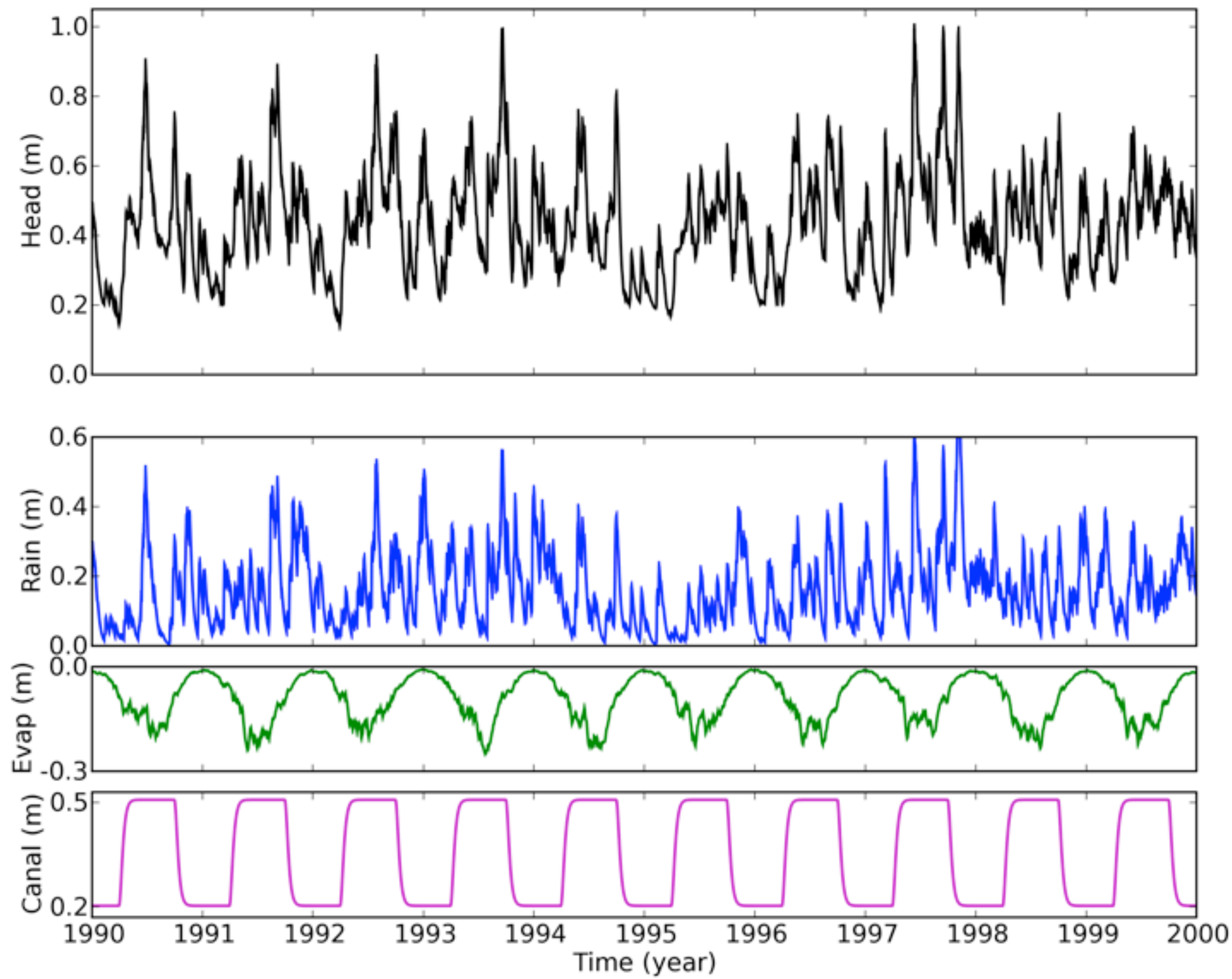


We do a time series analysis using the PIRFICT method and get a pretty good fit



Heads unravelled in contributions of all stresses

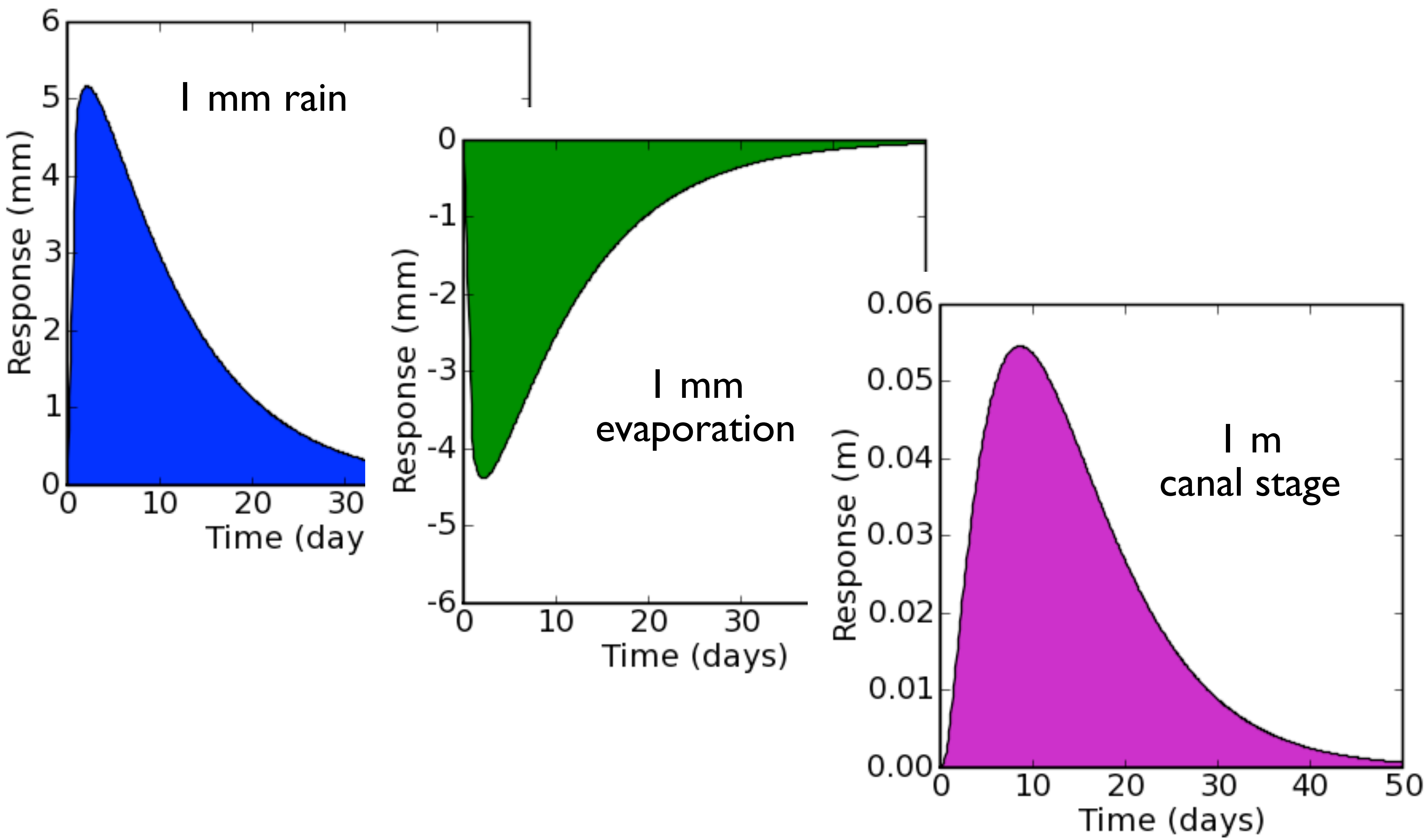
Contributions to head of three stresses



Von Asmuth et al., 2008, Ground Water

How do we do that?

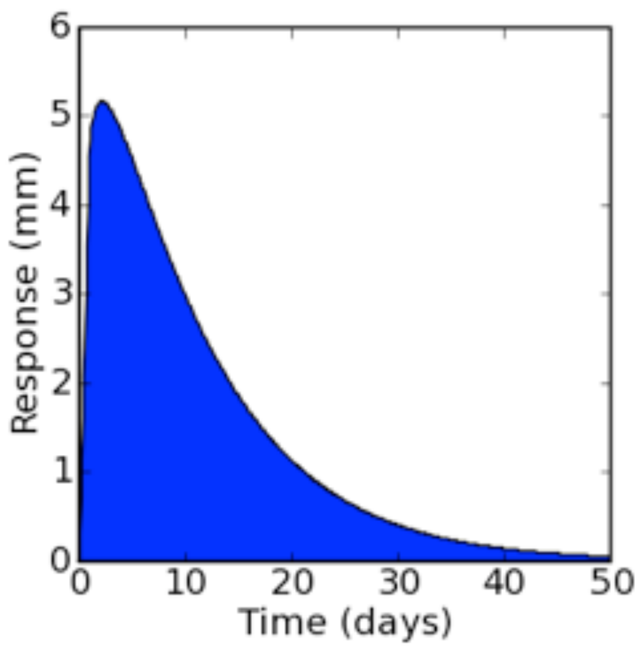
For each stress we determine an appropriate response function

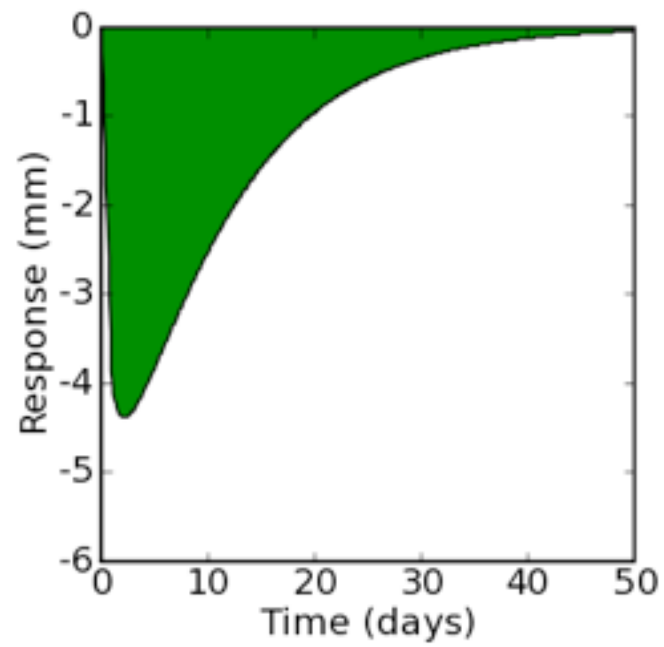


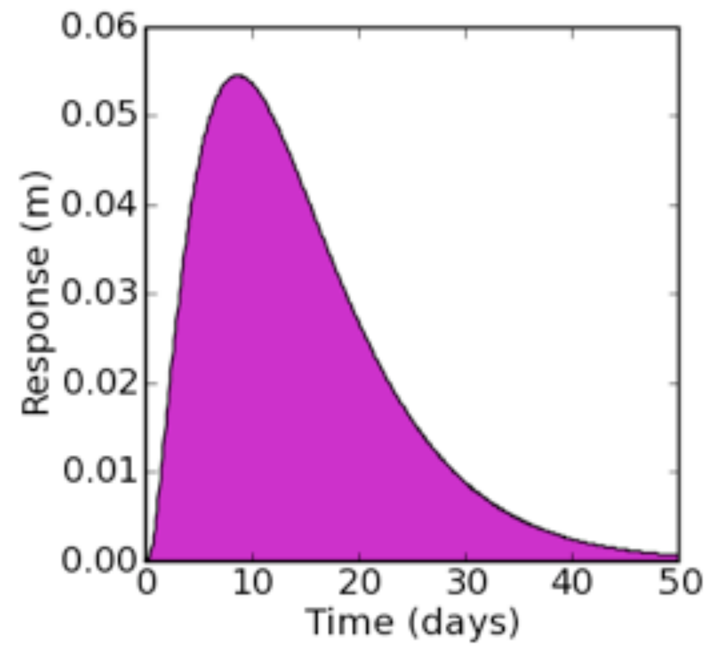
For each stress we select a function with the appropriate shape

Each response function has only a few degrees of freedom

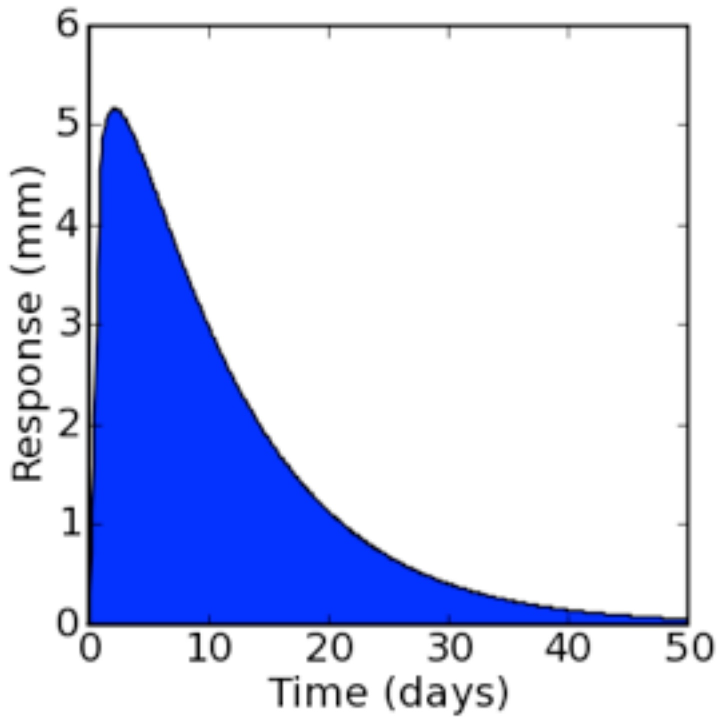
The time series model finds the parameters that give the best fit







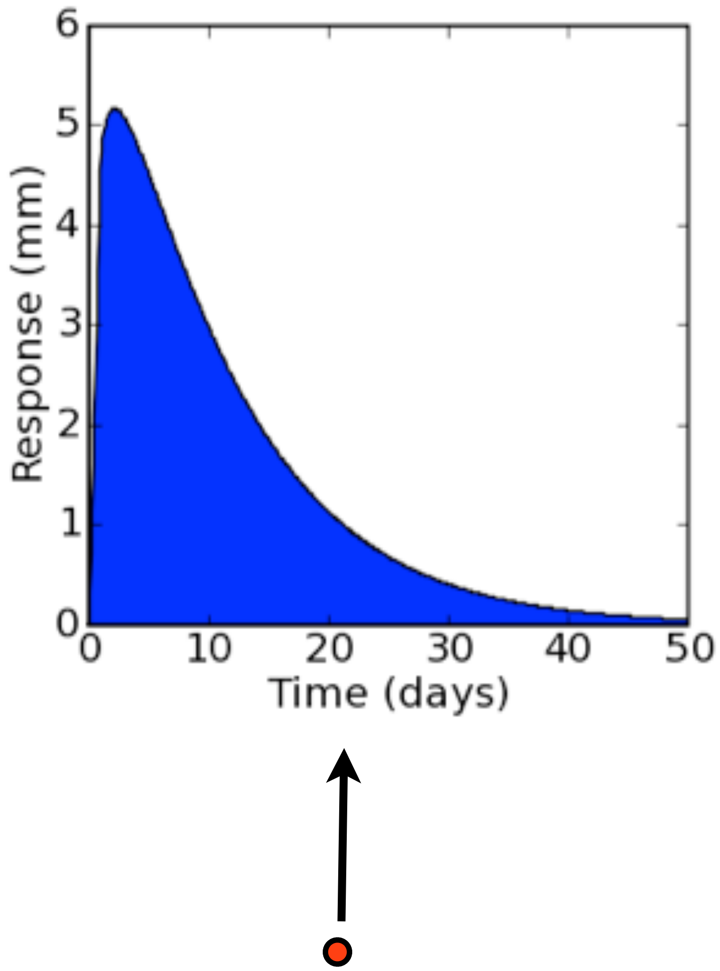
The problem has been reduced to finding the response function at any point



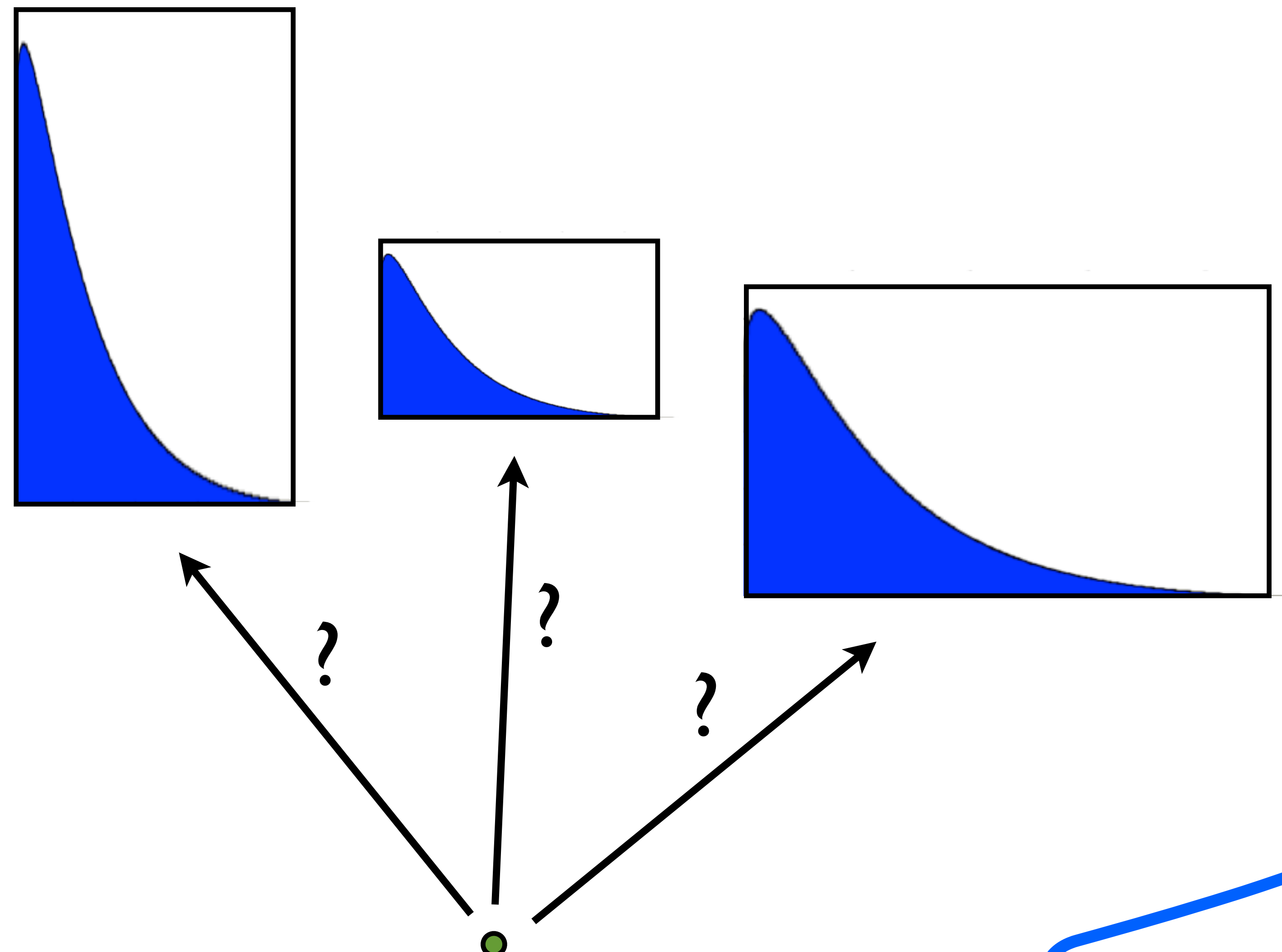
?

if we know the response function at this point

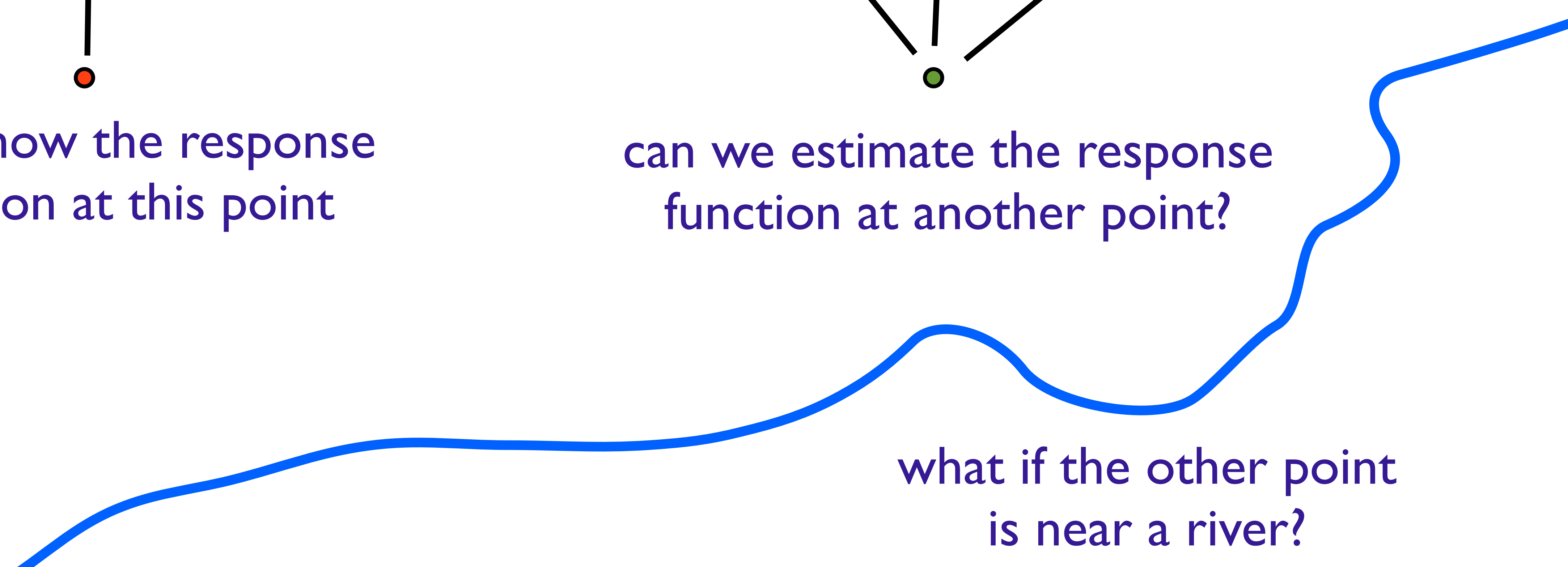
can we estimate the response function at this point?



if we know the response function at this point

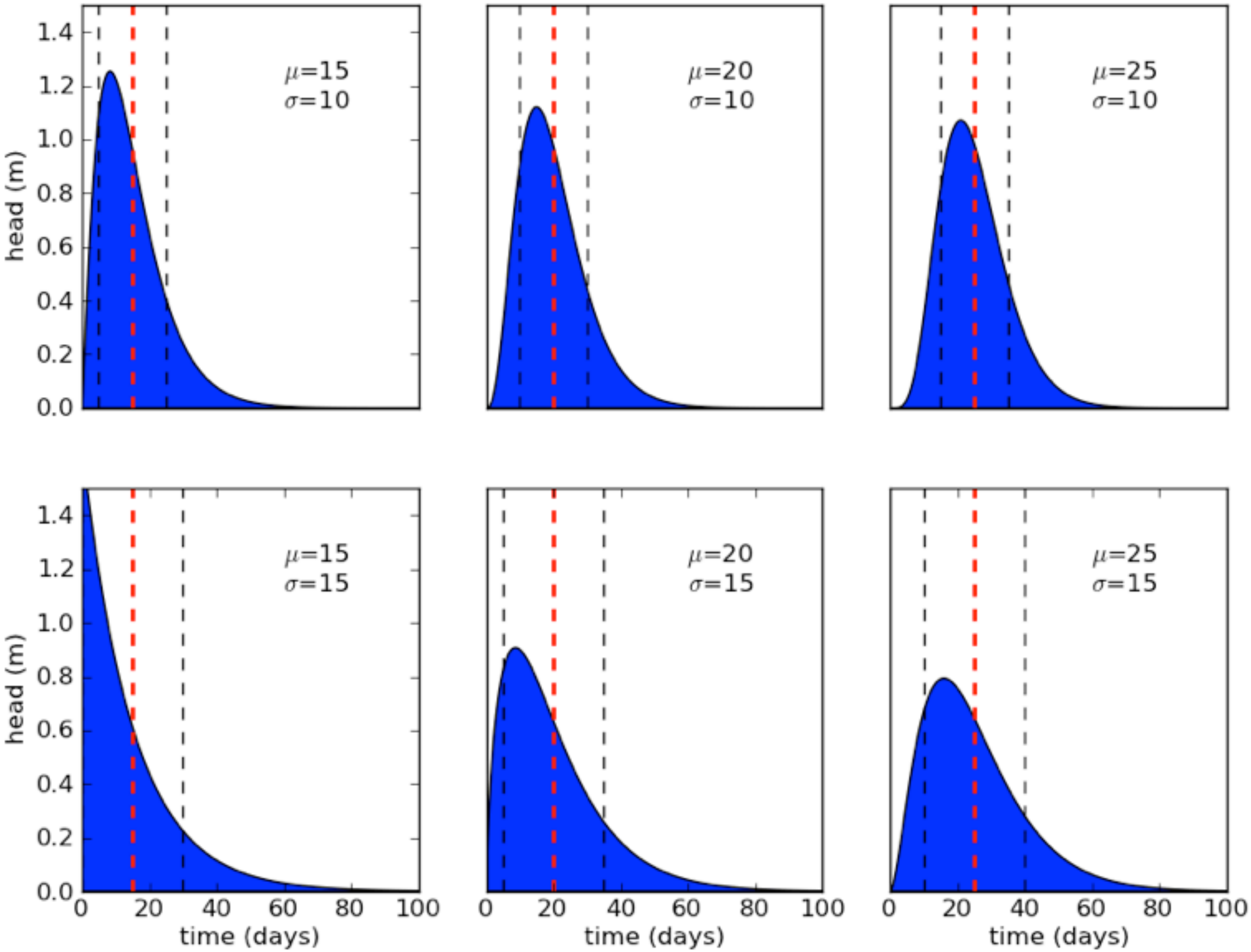


can we estimate the response function at another point?

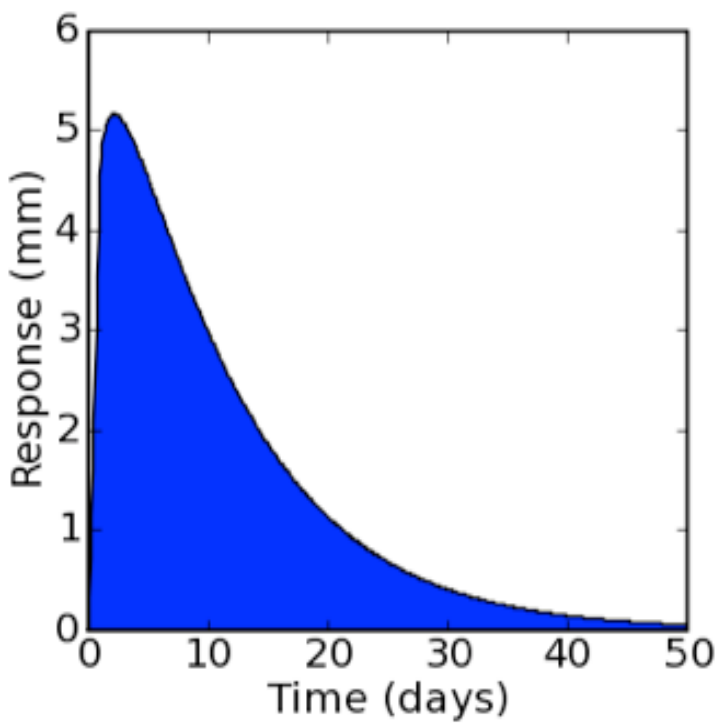


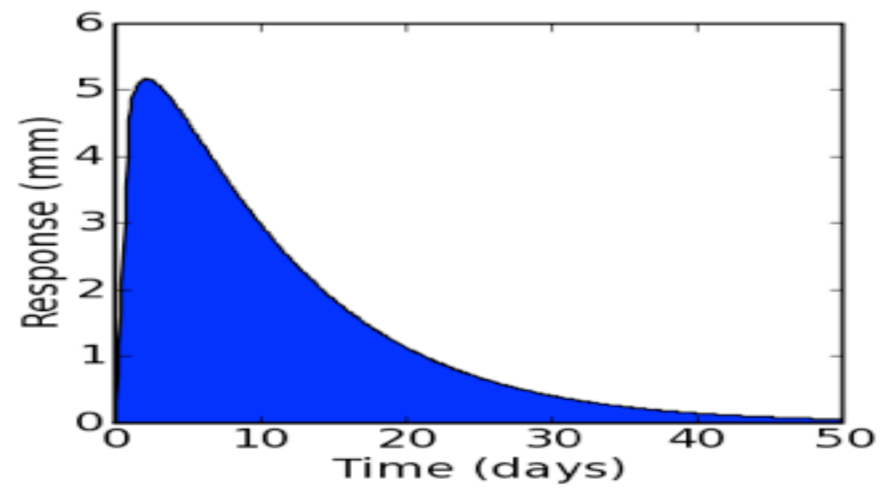
what if the other point is near a river?

A response function is a scaled probability density function and can be characterized by its area, mean and standard deviation



If you are able to estimate the area, mean and standard deviation at a point you can generate an appropriate response function





if we know the area, μ, and σ at this point

can we estimate the area, μ, and σ at this point?

Mathematical step 1: Mean and standard deviation are related to the moments of the response function $\theta(t)$

$$M_0 = \int_0^\infty \theta dt \qquad M_1 = \int_0^\infty t\theta dt \qquad M_2 = \int_0^\infty t^2\theta dt$$

area = M_0 $\mu = M_1 / M_0$ $\sigma^2 = (M_2 - \mu^2) / M_0$

So finding the moments is sufficient

Important benefit:

The moments are known along surface water features

Three easy steps to generate a head series at a new location:

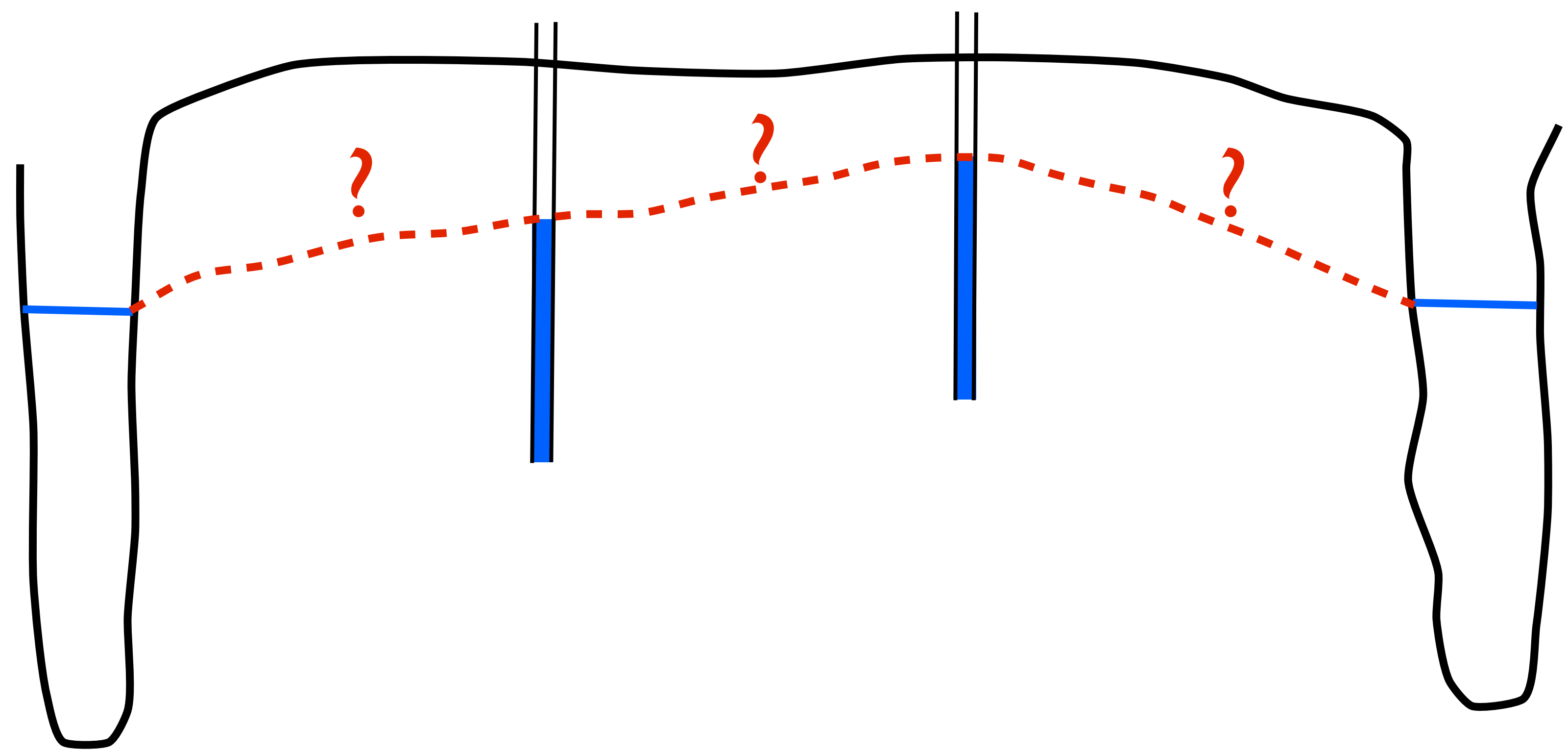
1) Estimate moments at a new location
2) Compute the response function at a new location
3) Compute the head series at a new location

Two approaches to estimate moments at a new location:

I. Spatial interpolation
II. Spatial modeling

Approach 1: Interpolation

2D interpolation that takes into account surface water features and groundwater mounding

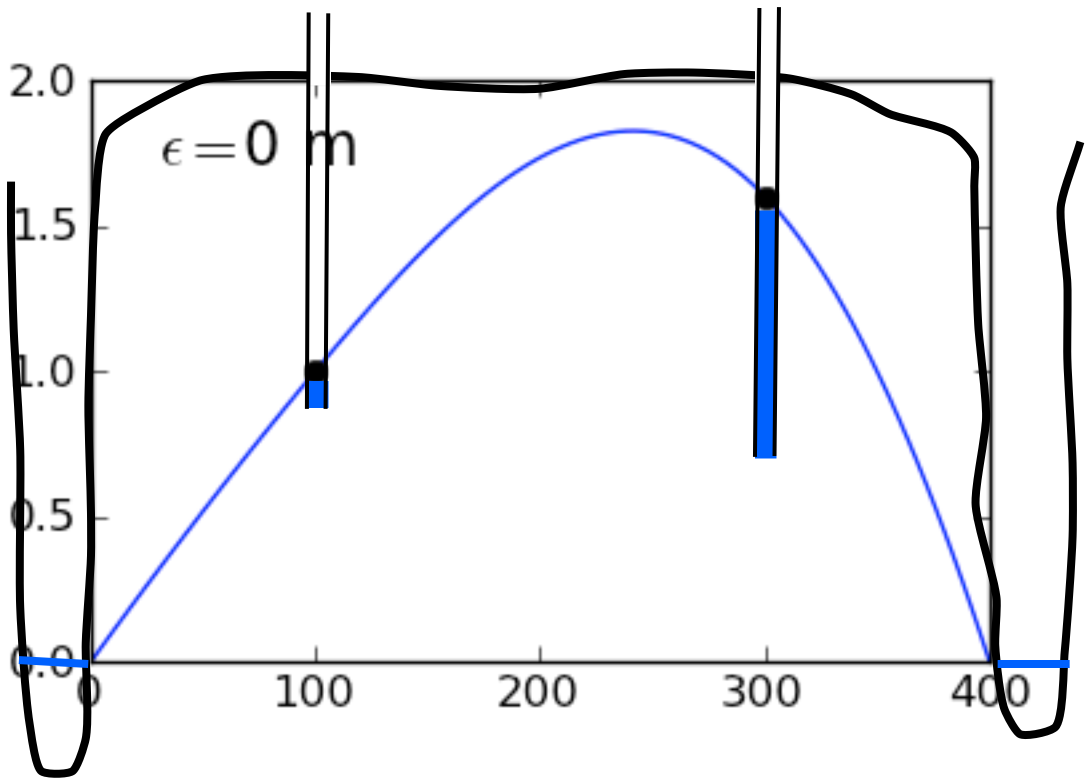


Interpolation function fulfills: $\nabla^2 M_n = -\frac{N}{T}$

Approximated with Kriging

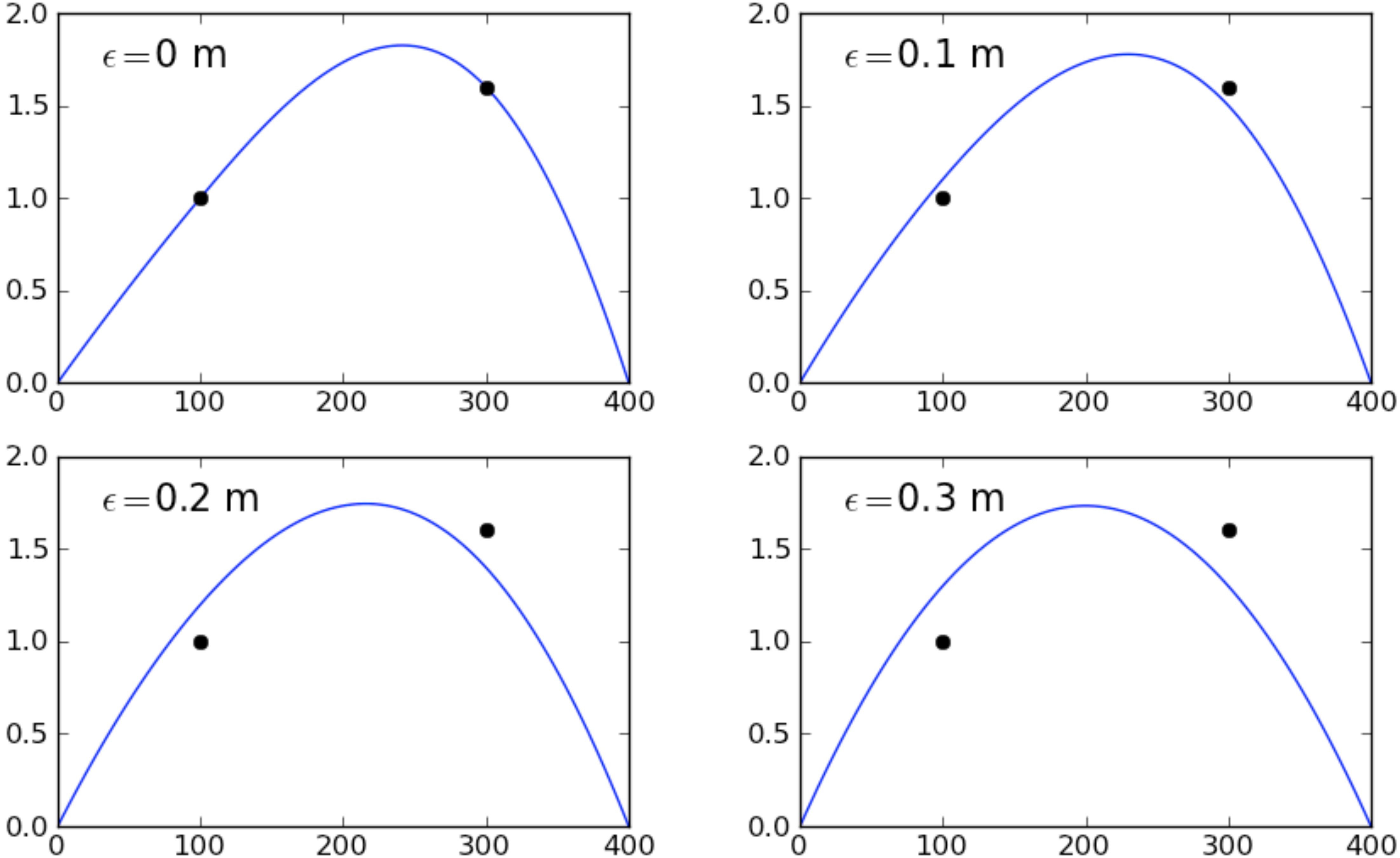
Surface water features are simulated with analytic elements

Only need to know location and water levels of surface water features, but no aquifer parameters

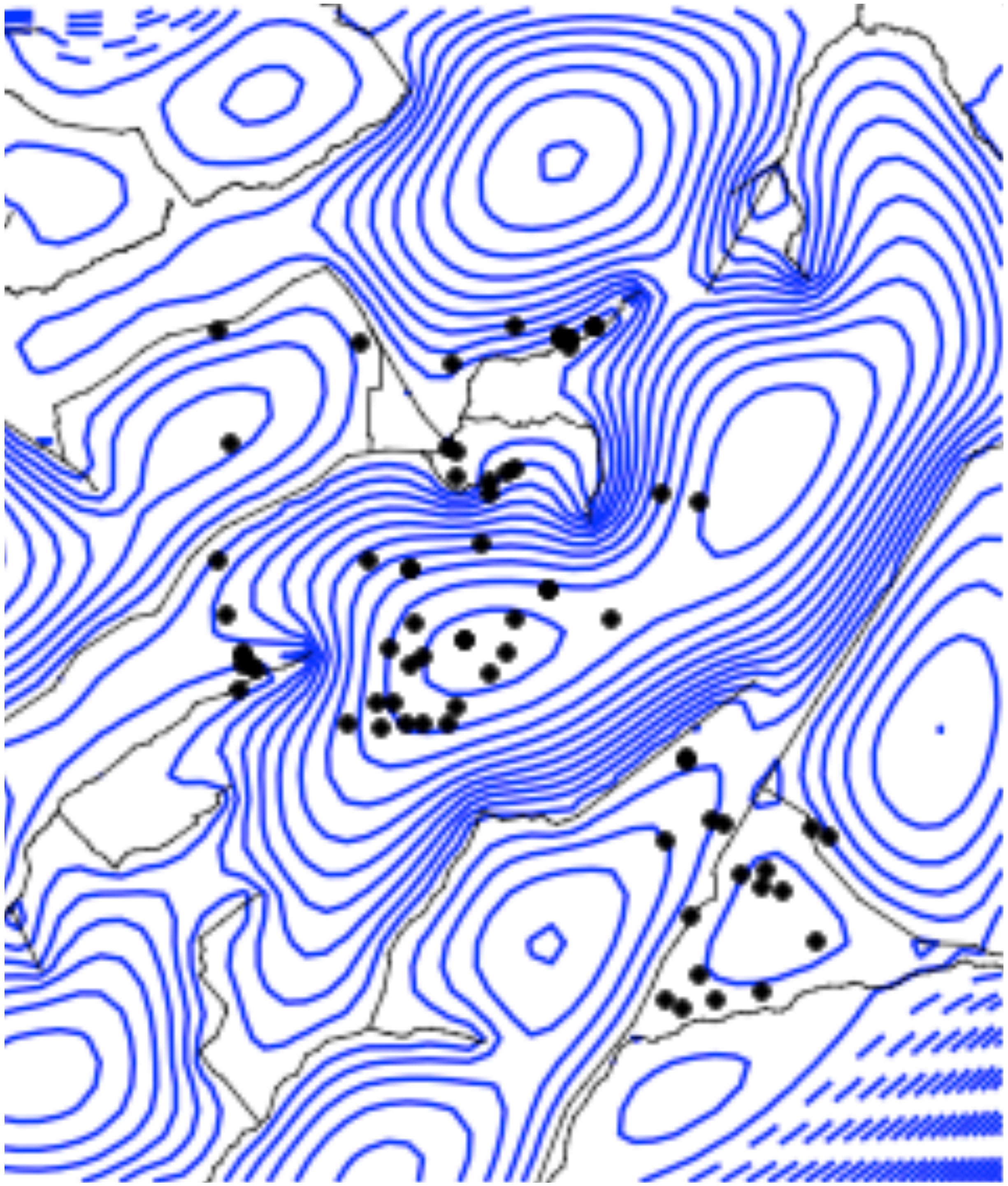


Only need to know location and water levels of surface water features, but no aquifer parameters

An average deviation may be specified for each obs. well



Interpolation goes exactly through ditches and streams and gives physically realistic groundwater mounding



Moments of response function can be computed everywhere

Time series can be generated everywhere

Example: Terwischa

Application to NHI:
Use time series analysis and interpolation to estimate cell-average behavior

- Use time-series analysis to analyze and screen head data
- Interpolate moments obtained from time series analysis (including the effect of surface water features)
- Compute average behavior for a cell

Approach II: Modeling

Solve differential equations for M_0 and M_1

Mathematical step 2

Moments fulfill (fairly simple) steady differential equations

Deq. for head

$$\nabla(T\nabla h) = S\frac{\partial h}{\partial t} - N$$

Deq. for impulse response

$$\nabla(T\nabla \theta) = S\frac{\partial \theta}{\partial t} - \delta$$

Integration with time

$$\nabla(T\nabla \int\limits_0^\infty \theta dt) = S\int_0^\infty \frac{\partial \theta}{\partial t}dt - \int\limits_0^\infty \delta dt$$

Deq. for M_0

$$\nabla(T\nabla M_0) = -1$$

Deq. for M_1

$$\nabla(T\nabla M_1) = -SM_0$$

M_0 and M_1 are known along boundaries

Approach for calibrating transient groundwater models

1. Time series analysis $\rightarrow$ M_0 and M_1 at wells
2. Construct steady models of M_0 and M_1
3. Calibrate on moments at observation wells

M_0 is groundwater model with unit infiltration
Calibrate for transmissivity T

$$\nabla(T\nabla M_0) = -1$$

M_1 is groundwater model with infiltration SM_0
Calibrate for storativity S

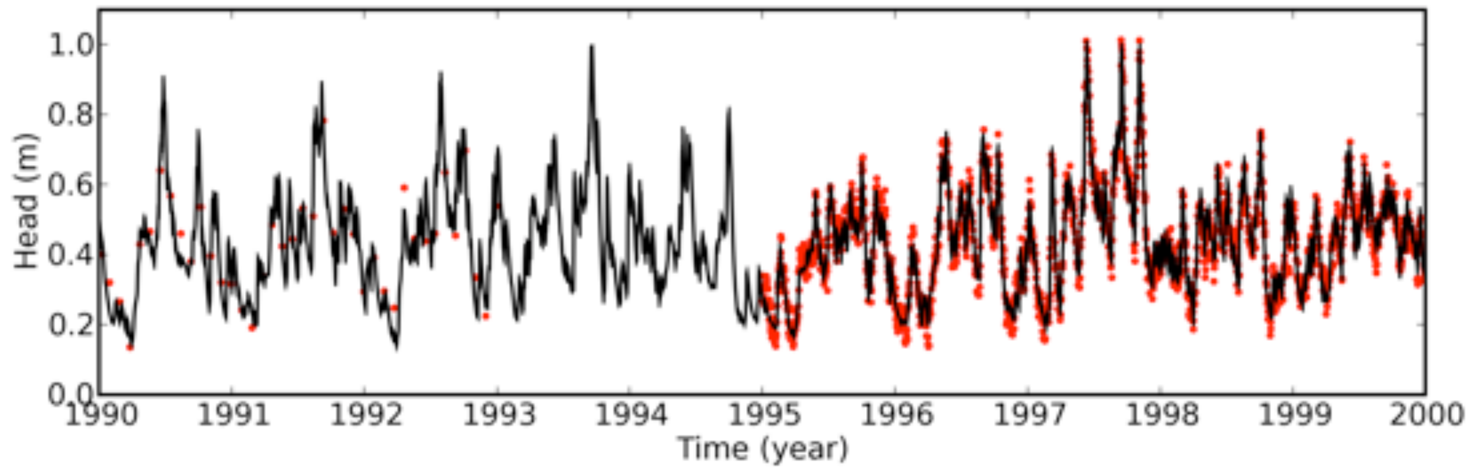
$$\nabla(T\nabla M_1) = -SM_0$$

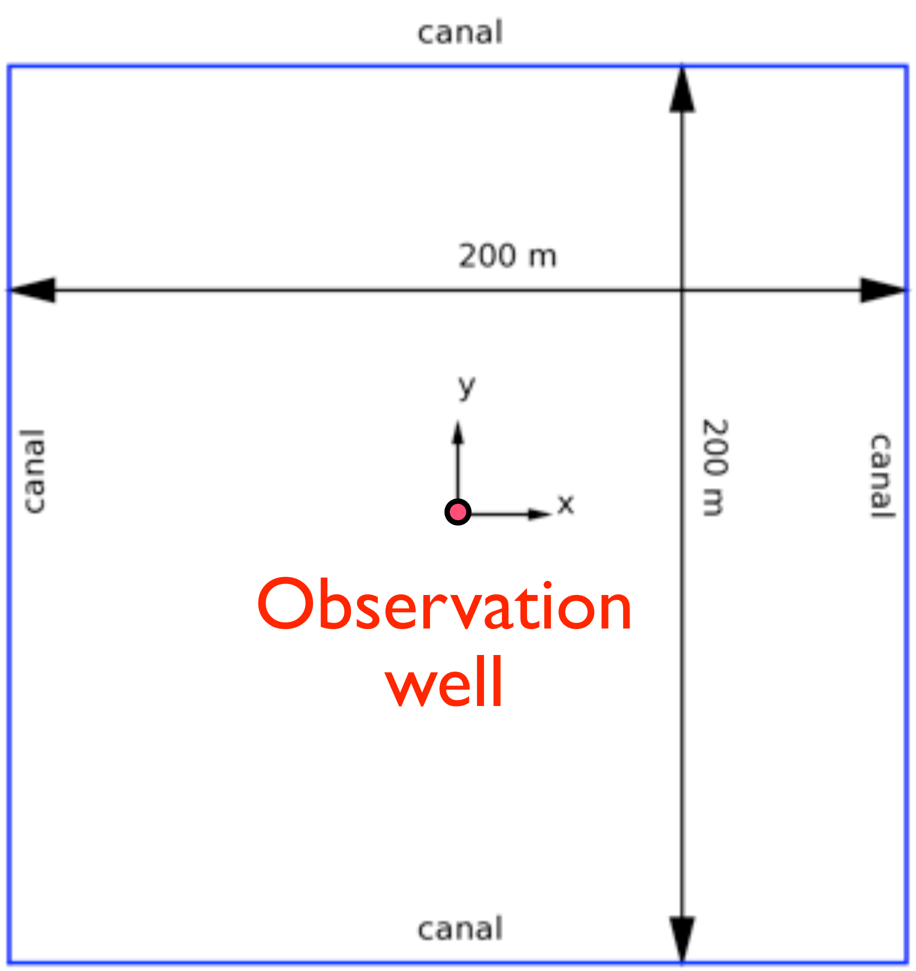
Details: Bakker et al., WRR, 2008

Series of slide 5: Rain and evaporation data were real

Heads were generated with accurate numerical model

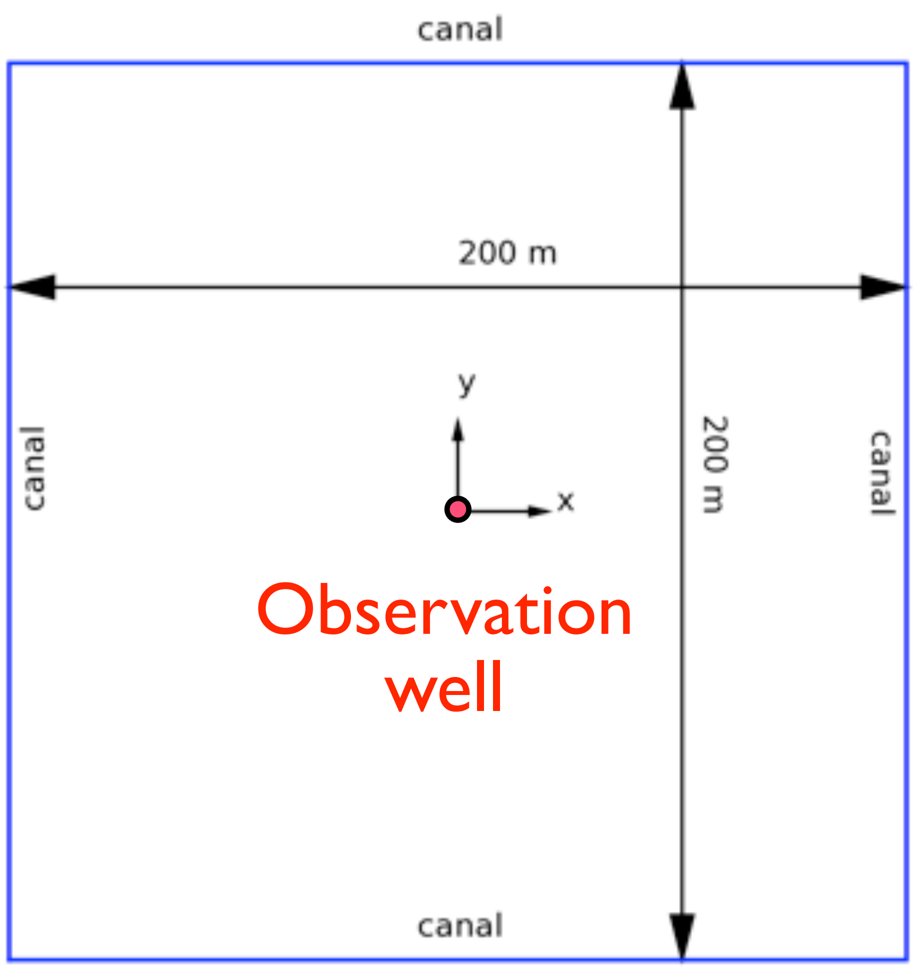
Random errors were added to recharge and heads





Head series was modeled with PIRFICT method and moments of response functions were obtained

Results of calibration of T and S using M_0 and M_1 from time series analysis as targets gives good results



Rain/Evap:
$M_0 \longrightarrow T = 41.6\ \mathrm{m^2/d}$

Truth:
$T = 40\ \mathrm{m^2/d}$

Rain/Evap:
$M_1 \longrightarrow S = 0.202$

Truth:
$S = 0.2$

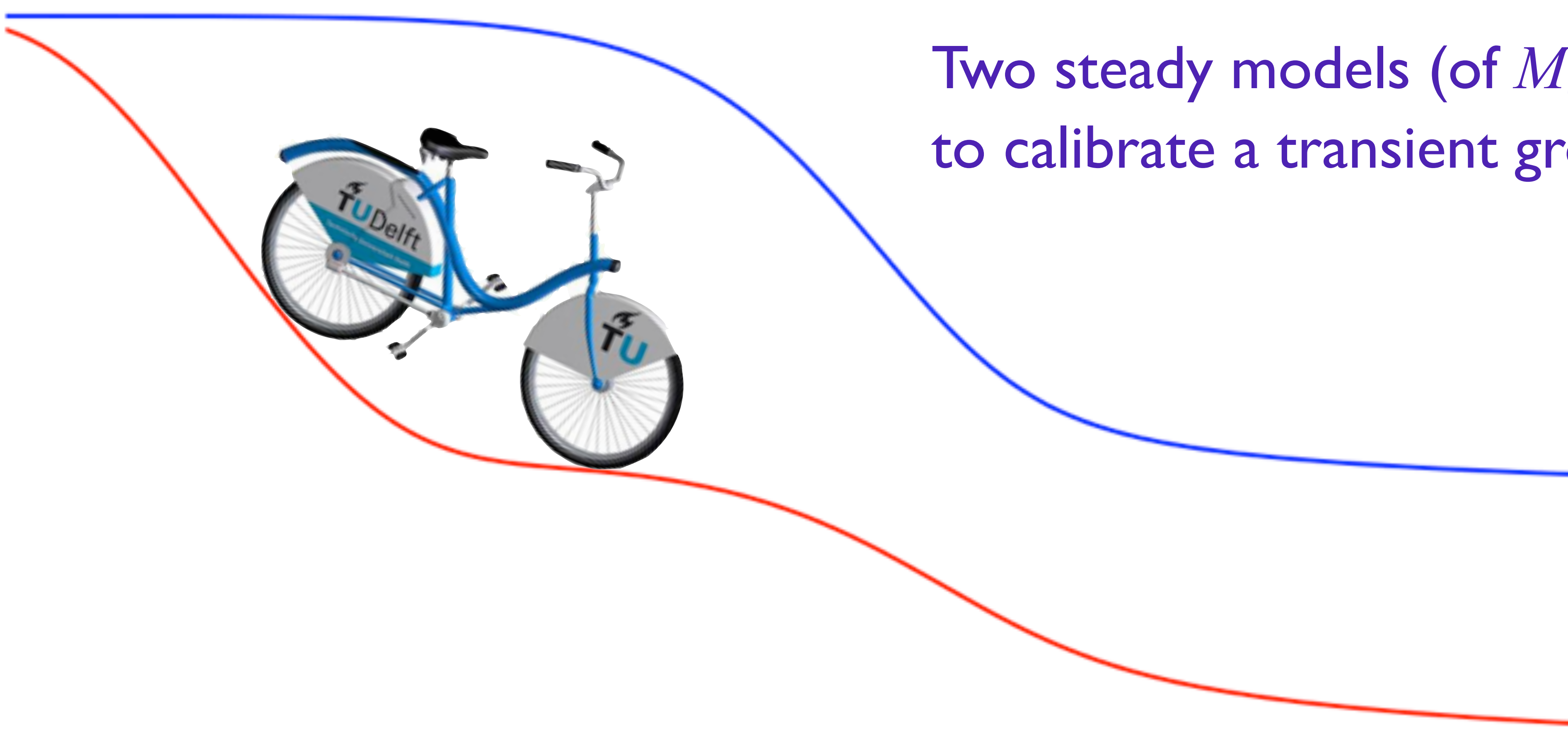
Work continued by Christophe Obergfell

Time series models may be extended to any point in the aquifer through interpolation or modeling

Selected response functions are characterized by three statistical moments (M_0, M_1, M_2)

Interpolation of moments includes mounding and surface water features; no knowledge of aquifer parameters needed

Two steady models (of M_0 and M_1) are sufficient to calibrate a transient groundwater model



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