



Groundwater models are only  
another type of Time Series Model

Ladies and gentleman,

I turned 65 last month and I quit working, so this is going to be my swan song, so to speak.



Here is where my working life began, on January 1 1973. Parkweg 12 in Scheveningen. The house is still there, but it once was the Geohydrological Department of the legendary

RID, the National Institute of Water Supply. The main building was across from it in the former luxury Park Hotel. But we had this charming old villa. In those days many public agencies were housed in old accommodations and I must say that I prefer them very much to the square blocks of later years, when the country became prosperous.

Here was my room. I shared it with Joep Blom who you don't know presumably, but we still have Blom's formula, although it is attributed to Ernst by the people from Wageningen. Kees van den Akker sat all the way upstairs in the attic. He shared his office with Wennemar Cramer, who was president of our Hydrological Society for many years. Wennemar had beautiful curly hair down to his shoulders. One fine day he appeared clipped short on the Institute. It turned out that his qualities were being recognized, and somebody higher up the ladder had hinted that long hair was an obstacle to his career. So you know the sole reason why I never ended up high.

This here was Bruggeman's den. Here is where he brooded on his formulae. And this is where he told me that they wanted him to write a book. That was 1973. It took him 26 years, maybe more. So Jos... (or Pieter, for that matter). I was absolutely fascinated by his work, he was my undisputed hero, but there were many other hydrologists of national fame at Parkweg 12 that I looked up against.

They had all kinds of specialisms, but they shared one thing: they had no groundwater models. Imagine! Hydrologists had been doing groundwater without computer models for more than a century. And they did it very well.

Being grown up with Modflow you might believe that the subsoil is the all important thing to start with. Well, it isn't. We may be talking groundwater, but the primary hydrologic information is not in the ground. It is in the water. It is in the observed heads. That is why time series analysis is sooo important. That is why my colleagues at the Institute always started by drawing maps of isohypses. Here is an early example.

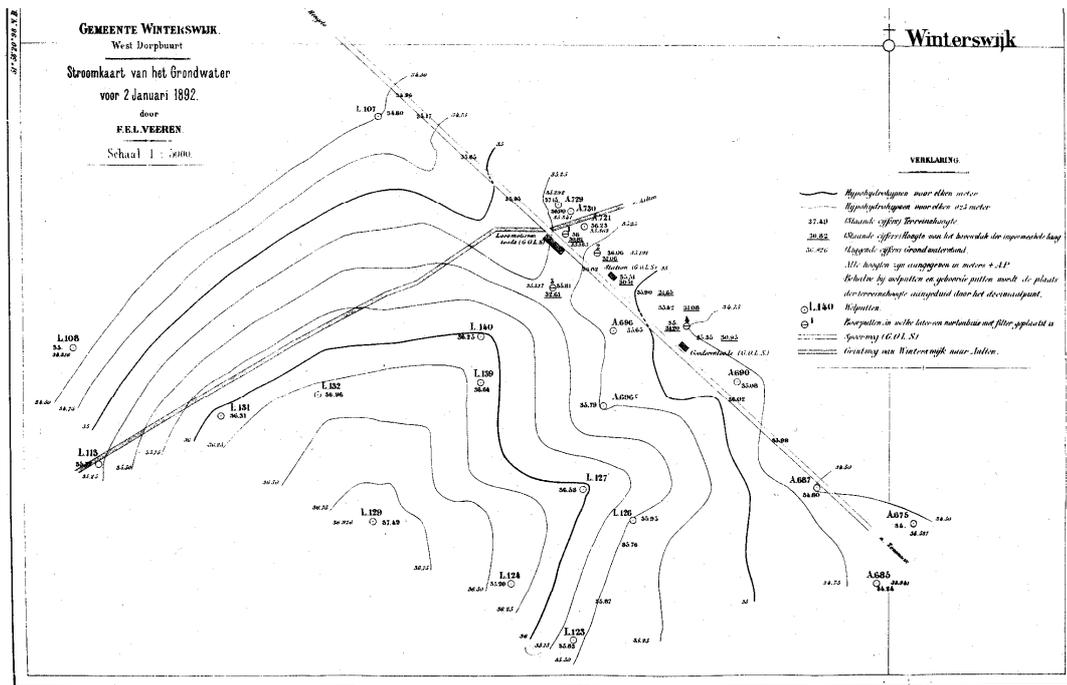
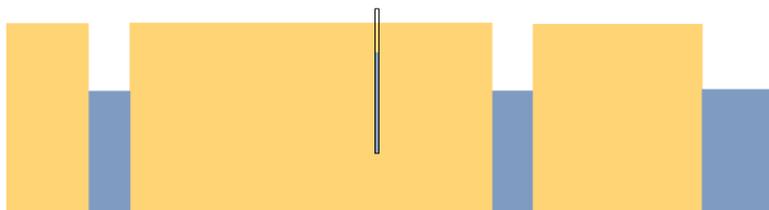


Fig. 10. Grondwaterisohypsen volgens Veeren (1893).

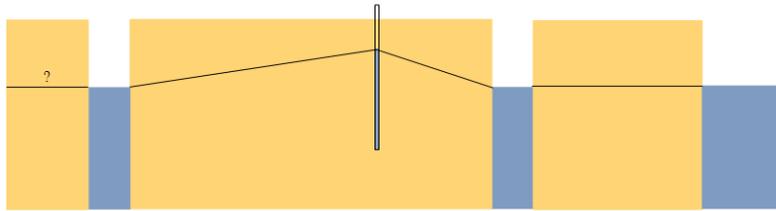
They included all kinds of information, but information on the subsoil was never an important part of it. The general perception of the subsoil, even during my study time, was rather cartoon-like. The eastern part of the country was one phreatic aquifer and the western part had what they called Het Hollandse Profiel, the Dutch Profile, which was an aquifer overlain by layers of clay and peat with ditches and canals in it, that had controlled water levels.

And that was good enough for them. Now you may pity them for being so naive and ignorant, but what I want to show you today, and leave as a message, is that they had a point. A very good point indeed.

I'll do it by first drawing a map of isohypses. Not by hand (I can hardly ask that from you) but by computer. I can keep it brief, because Mark Bakker already touched upon it.

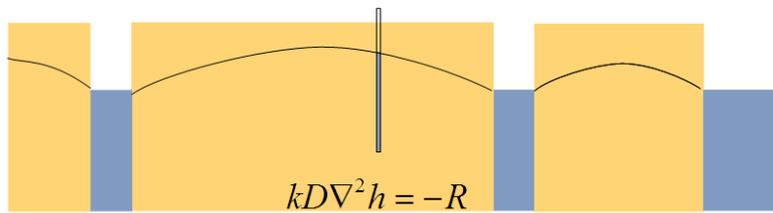
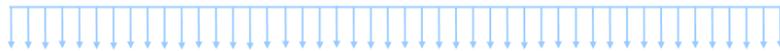


Imagine a cross section over an aquifer cut by watercourses. I said it would be cartoon like. There is one piezometer where the mean groundwater head is known and the task is to draw the phreatic table.



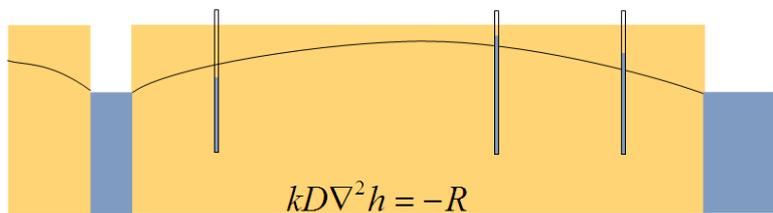
One option is to interpolate the known heads linearly, but that is not neat. It doesn't look very satisfactory.

Recharge

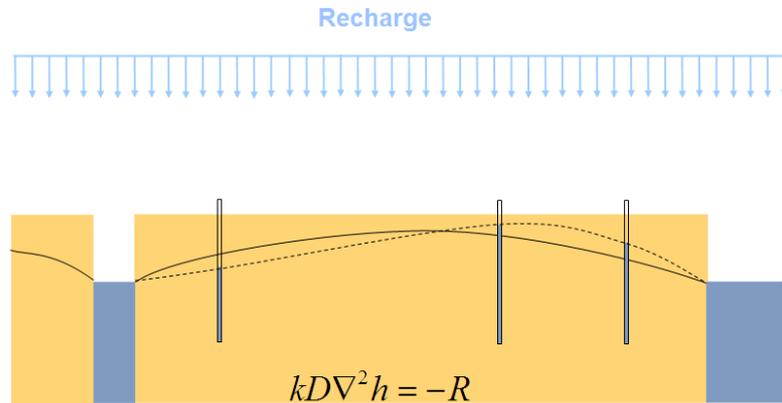


A better option is to assume a simple hydrological model and let it rain. This will cause the phreatic surface to look more realistic, and as a bonus we also get groundwater mounds in the lots that had no piezometers. The tacit assumption here is that the same simplistic differential equation applies everywhere.

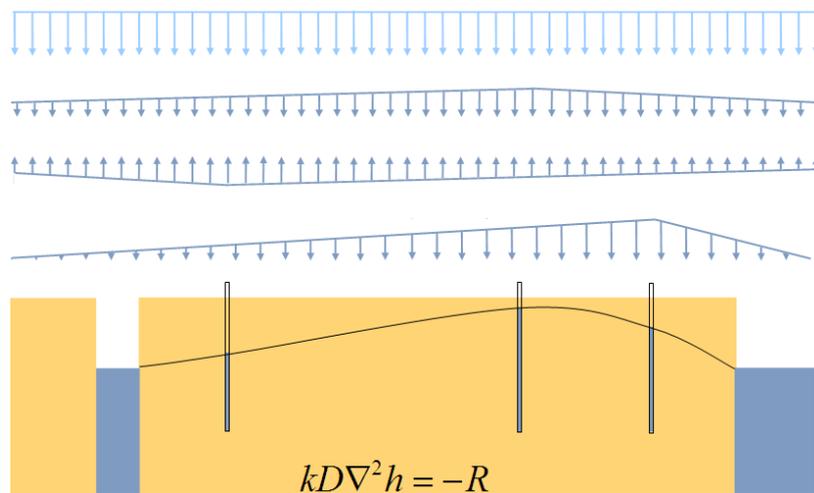
Recharge



But what if there were three piezometers? Our simple model cannot satisfy them all simultaneously.

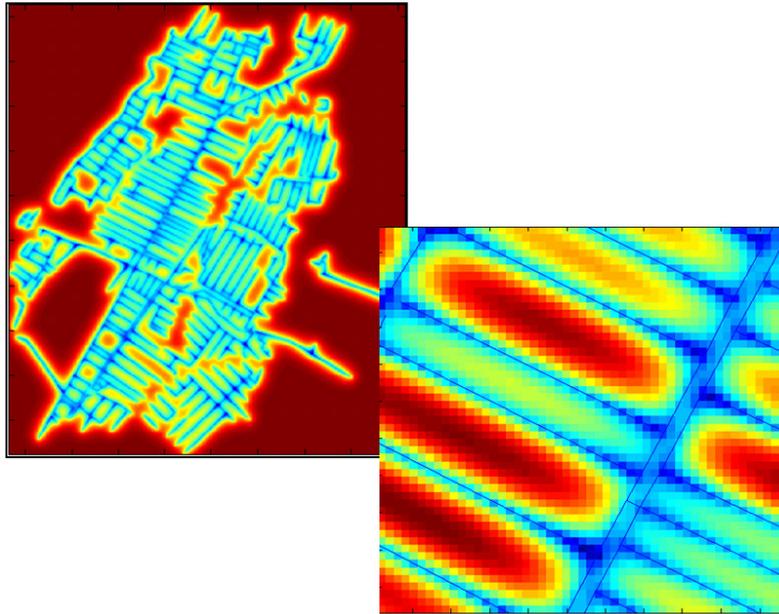


We could adjust our phreatic surface by hand, of course, and that was what my colleagues at the Insitute would do, but I promised you to do it by computer. What you could do is let it rain a little less were the head is estimated too high, and a little more were it is too low.



One way would be to assign what I would call roof-shaped precipitation patterns to each piezometer. If there is one roof function for each piezometer, then we can exactly solve for the phreatic surface. We keep the same differential equation, but we add some terms to the right hand side. Mark Bakker called them pseudo recharge. It is not real recharge, of course, because we already took the real recharge into account. It is more like time series models where you add a noise term to account for the residues, no matter what causes them.

This is the whole idea of drawing isohypses maps by computer. Here is an early result by Frans Schaars, which is also in Jos von Asmuth's thesis. It his the flower bulb region near De Zilk, close to the Amsterdam Water Supply dunes.



$$kD\nabla^2 h^* = -R - \sum P_n(x, y)$$

$$\nabla(kD\nabla h) = -R$$

$$\nabla\{kD(x, y)\nabla h\} = -R$$

$$\nabla\{\overline{kD} + dkD(x, y)\nabla h\} = -R$$

$$\overline{kD}\nabla^2 h = -R - \nabla\{dkD(x, y)\}\nabla h$$

$$\overline{kD}\nabla^2 h = -R - \nabla\{dkD(x, y)\}\nabla h$$

$$\sum P_n(x, y) \approx \nabla\{dkD(x, y)\}\nabla h^*$$

Now I can hear you thinking: Wait a minute, what nonsense it this? You can't tamper with the recharge. There is no pseudo recharge. The reason why you couldn't fit all piezometers simultaneously is that the subsoil is much more complicated. The transmissivity should have been taken heterogeneous and the differential equation should have read (2nd equation).....

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Vergelijkingen doorlopen. Concluderen dat ik  $dkD(x,y)$  kan oplossen.

(1) This is the equation I used to construct to interpolate the phreatic surface.

(2) You might say: it is not allowed tot take the transmissivity outside the gradient operator, because it depends on x and y.

(3) ...

(4) OK. Let's elaborate this correctly. I split the transmissivity in a mean value that may be taken outside the gradient operator, and a part that varies around the mean.

(5) Now I keep this part to the left and move that part over to the right hand side. Now the left hand side is identical to the left hand side of (1). Only the right hand sides differ, unless

(6) this term (blue, (6)) is the same as this one (blue, (1)).

(7) They differ unless I equate them. I know this term and that term and I can solve this equation for the unknown variations of the transmissivity. Not exactly, of course, because I don't know the real heads. I only have their interpolated values. But I can do it iteratively and it appears to converge very fast. Here is a demo.

MQdriver\_Jos  
MQdriver\_Jos2  
MQdriver\_Jos3

Aantal keren draaien en toelichten.

Demo

(1,1) Gegeneerd kD-veld

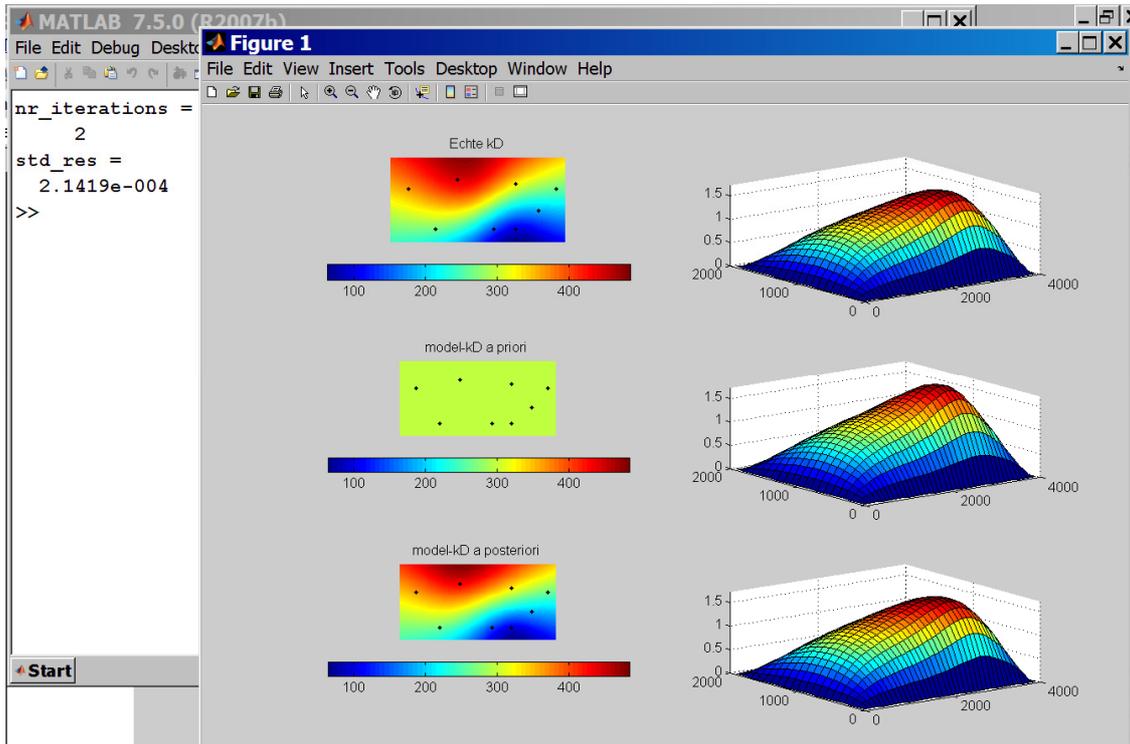
(1,2) Bijbehorend stijghoogtevlak

(2,1) Homogeen kD-veld met peilbuizen. Stijghoogten in de peilbuizen afgelezen uit  
(1,2)

(2,2) Geïnterpoleerd stijghoogtevlak volgens behandelde methode

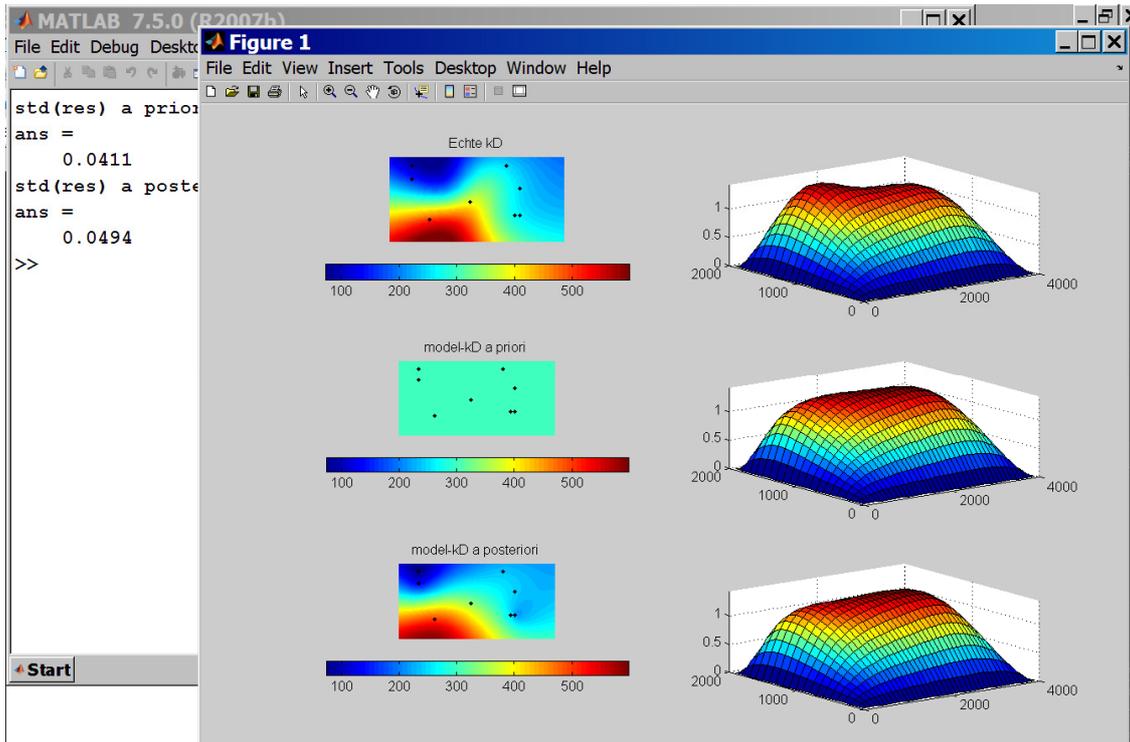
(3,1) Reconstructie van het kD-veld

(3,2) Bijbehorend stijghoogtevlak

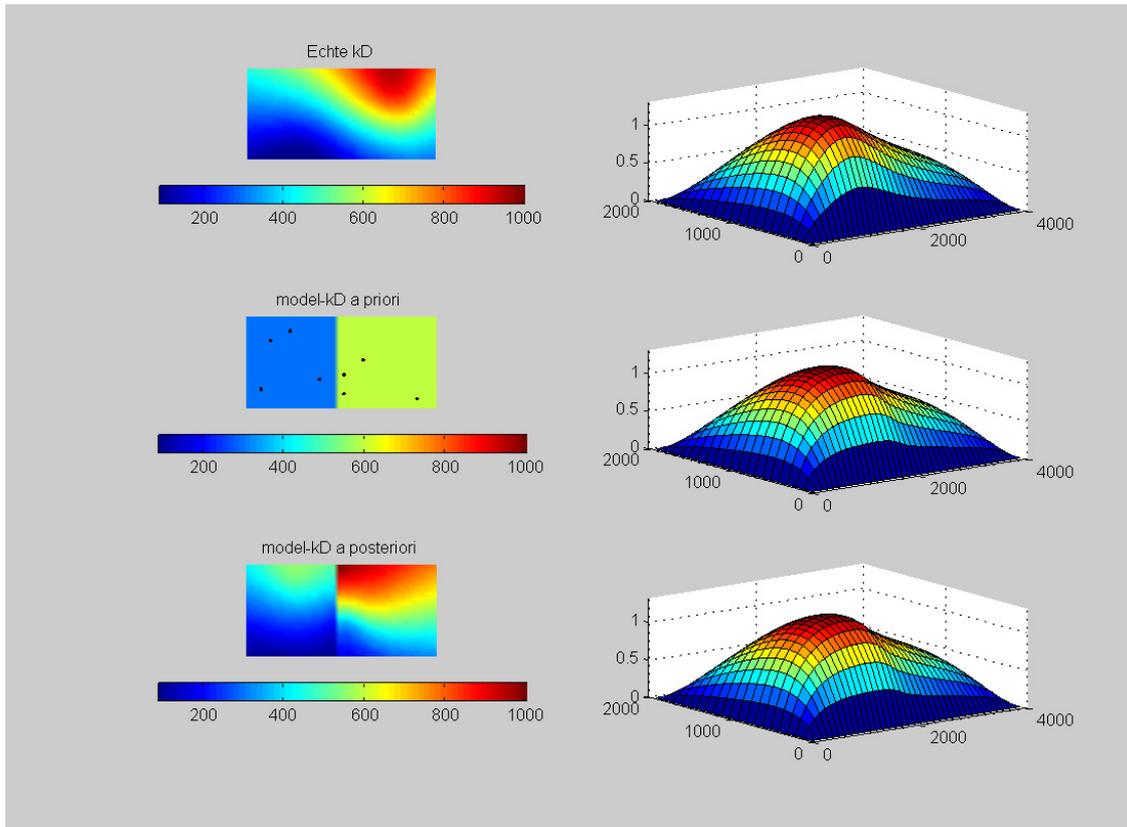


De moraal is dat het binnen een paar iteraties lukt om het oorspronkelijke kD-veld redelijk te reconstrueren, waarbij de "gemeten" stijghoogten in de peilbuizen weer kloppen. Afhankelijk van de plaatsing van de peilbuizen lukt de reconstructie beter of minder goed.

Het bijbehorende stijghoogteveld is echter slechts in iets meer dan de helft van de gevallen wat beter maar vaak dus ook slechter dan het geïnterpoleerde stijghoogtevlak. (Residuen over alle cellen genomen).



Het is mogelijk om prior-information mee te nemen. Dat zie je in het eindresultaat nog terug. In het algemeen wordt het resultaat er echter nauwelijks beter van.



## Conclusions

- Interpolation with pseudo recharge gave good results.
- Details of the subsoil did not matter much.
- It is the heads that count.
- Put your money on piezometers

In conclusion: Interpolation with pseudo recharge *and a simplistic model of the subsoil* gave good results. Details of the subsoil didn't matter much. It is the heads that count. That is where the hydrological information is. So you'd better put your money on piezometers. (And buy Menynathes...)

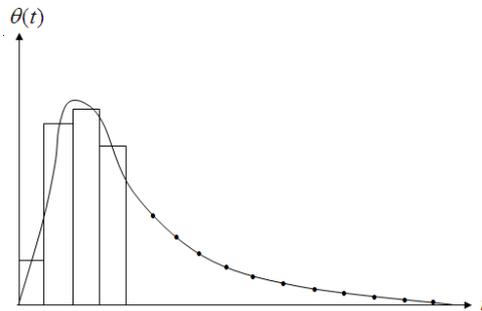
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This part of my presentation did not cover the title of my talk very well. Not at all, in fact. According to the program it ran



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This is a proposition. But don't have much time left, so I'll give an outline of the proof:



|   |   |
|---|---|
| $IR = \sum \alpha_i \exp\{\lambda_i(t)\}$   | AR-model                                    |
| $\frac{\partial}{\partial x}(kD \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(kD \frac{\partial h}{\partial y}) = S \frac{\partial h}{\partial t} - N$ | continuous groundwater model                |
| $\mathbf{A}\mathbf{h} = \mathbf{r}$   | numerical groundwater model                 |
| $\mathbf{B}\mathbf{h} = \mathbf{S} \frac{d\mathbf{h}}{dt} + \mathbf{r}$   | matrix differential equation                |
| $\mathbf{B}\mathbf{h} = \mathbf{S} \frac{d\mathbf{h}}{dt}$  | ditto, simple case                          |
| $\mathbf{h} = \exp(\mathbf{S}^{-1}\mathbf{B}t)\mathbf{h}(0)$  | matrix function (Snelle Oudjes gaan Matlab) |
| $h_i(t) = \sum \alpha_i \exp\{\lambda_i(t)\}$   | AR-model                                    |
| QED   |   |

Here is how B&J approximate a continuous IR function. The first part is discretized and the second part is approximated by a sum of decaying exponential functions. This part looks discrete, too, because the functions are evaluated at discrete times only, but they could be evaluated at any time, so they are continuous.

The first part is called MA-part, the second part is called AR-part. For instance, a ARMA(2,4) model has 4 bars and the tail consists of 2 exponential functions.

Sometimes the IR is approximated by only exponential functions. This was the case, for instance, in the program VIDENTE of Alterra. Then

$$IR = \sum \alpha_i \exp\{\lambda_i(t)\} \quad (1)$$

and t is evaluated at discrete values of time.

So this is a time series model. Now the groundwater model. This is the differential equation for a single aquifer (to keep it simple):

$$\frac{\partial}{\partial x}(kD \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(kD \frac{\partial h}{\partial y}) = S \frac{\partial h}{\partial t} - N$$

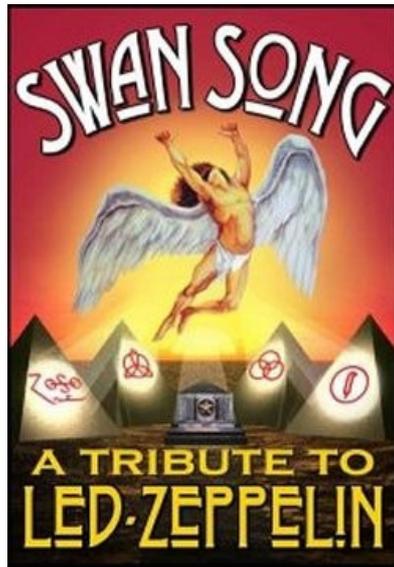
This is the equation is solved numerically by Modflow. And from this equation I derive in a number of steps eq(6), which is identical to eq(1). In theory there are as many terms as there are nodes, but in practice two or three suffice.

So yes, a groundwater model is a time series model. There is a time series model for every node, and it can be extracted from Modflow.

The equivalence of groundwater models and time series models opens beautiful vistas, but that would require another PhD student.

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This was my Swan Song...



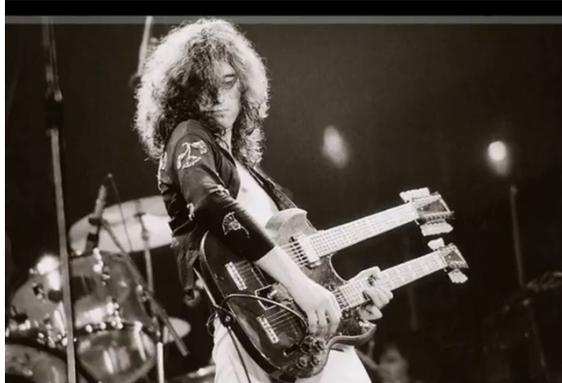
Led Zeppelin has got nothing to do with it, of course, but that was the music of my time, when I studied civil engineering in Delft. They disbanded in 1980 so I bet you never even heard of them... Oh, you still do...



This is Jimmy Page in 1957



He said he wanted to be a scientist.



I wanted to be a musician... Life can be funny, can't it?



But we grow older anyhow...

That's all. Thank you for your patience, thanks for listening and thanks for coming.

Wait a minute. Is said I quit working, but that's not quite true. I quit working for bosses. I actually registered at the Chamber of Commerce. But now the Tax Authority, in order to recognize me as a business, wants me to promote myself. That is one of their criteria. So here I am...



.Maas Geohydrologisch Advies  
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This is my business name. I skipped my initial because people are always confused whether it should be C or K. I don't have a web page yet. That might be another requirement, I don't know. For the time being you can reach me on my private email address at xs4all.

Thanks again.